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Effect of random telegraph noise on entanglement and nonlocality of a qubit-qutrit system

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Abstract. We study the evolution of entanglement and nonlocality of a non-interacting qubit-qutrit system under the effect of random telegraph noise (RTN) in independent and common environments in Markovian and non-Markovian regimes. We investigate the dynamics of qubit-qutrit system for different initial states. These systems could be existed in far astronomical objects. A monotone decay of the nonlocality (entanglement) is found in the Markov regime, while for non-Markovian noise, phenomena of sudden change (death) and revival occurs. We find that the preserving nonlocality (entanglement) depends on initial state of the system in common and independent environments; so, we can not strictly conclude that independent or common environments are more robust against the noise.

1 Introduction

A distinctive property of the quantum world is nonlocality which differs it from the classical world. Nonlocality could be used in quantum information processing tasks to improve the efficiency. It prepares a new classification for bipartite states, which might be useful in quantum state steering [1]. In quantum mechanics the nonlocality shows itself in two different scenarios, the Aharonov-Bohm effect and the entanglement (the Einstein-Podolsky-Rosen correlations). Nonlocality in quantum usually refers to correlations which local hidden variable theory can not describe them while nonlocality has been shown by Bells inequalities widely [2]. Different measures have been proposed for detecting the nonclassical correlations which can not be found by entanglement measures [3, 4, 5, 6]. On the other hand, there are unavoidable interactions between system and its surround environment which induces decoherence. Decoherence is the main factor which destroys quantum properties as correlations, so investigation of evolution of quantum correlations on these systems sounds interesting for making quantum device with high performance. The combination of general relativity and quantum field theory predicts that black hole emits Hawking radiation [7, 8]. In some numerical simulations, spontaneous Hawking radiation emanates from an analogue black hole in an atomic Bose-Einstein condensate which is stimulated by quantum vacuum fluctuations[7, 8]. They observe some correlations and entanglement between the Hawking particles outside the black hole and the partner particles inside. They find that the pairs with high energy are entangled, while the pairs with low energy are not entangled so in this paper we consider both states (entangled and separable states) [8]. The study of the thermodynamics of interaction of black holes with entangled particles is very interesting [8, 9, 10, 11, 12, 13].

2 The model

Particles with spin-$\frac{1}{2}$ (two-level systems) and spin-1 (three-level systems) are called qubit and qutrit respectively. Karpat and Gedik have been studied the evolution of quantum correlations and entanglement for a qubit-qutrit system in common and independent environments under the effect of classical dephasing noise in the Markovian regime [14]. So, it seems interesting to study the dynamics of nonlocality and the entanglement of a qubit-qutrit system under other classical noise sources as RTN. We consider a model of a qubit-qutrit system which has been prepared in different initial states as entangled (ρ_E), separated (ρ_S) and Bell-like (ρ_B) states. The system is under a random telegraph noise (RTN) of common and independent environments in Markovian and non-Markovian regimes.
The Hamiltonian of the dynamics of the qubit-qutrit system under RTN is defined as [15]

$$H\left(t\right) = H^{A}\left(t\right)\otimes\mathbb{I}^{B} + \mathbb{I}^{A}\otimes H^{B}\left(t\right), \tag{1}$$

where $\mathbb{I}^{A}$ ($\mathbb{I}^{B}$) is the identity operator in the subspace of qutbit (qutrit) and H^{A} (H^{B}) is the single qubit (qutrit) Hamiltonian which is defined as

$$\begin{aligned} H^{A}\left(t\right) &= \omega_0^{A}\mathbb{I}^{A} + \nu\,\zeta^{A}\left(t\right)s_z{}^{A} \\ H^{B}\left(t\right) &= \omega_0^{B}\mathbb{I}^{B} + \nu\,\zeta^{B}\left(t\right)S_z{}^{B}. \end{aligned} \tag{2}$$

Where $\hbar = 1$, so ω_0^{A} (ω_0^{B}) is the energy of system A (B) in absence of noise, s_z^{A} (S_z^{B}) is spin-$\frac{1}{2}$ (spin-1) operator in z direction and the system-environment coupling constant is ν. The Hamiltonian given in Eq. (1) is stochastic because of randomness of the parameter $\zeta^{A(B)}(t)$. This parameter describes a fluctuator which flips between the values ±1 randomly with rate γ so the quantum states evolves stochastically.
Random phase factor is $\varphi^{A(B)}(t) = -\nu\int_0^t ds\,\zeta^{A(B)}(s)$. One can get the qubit-qutrit density matrix by the average over the random phase factor as $\rho(t) = \left\langle\left\langle\rho(\varphi^{A}(t),\varphi^{B}(t))\right\rangle_{\varphi^{A}}\right\rangle_{\varphi^{B}}$ [15]. The autocorrelation function is $\left\langle\delta\zeta^{A(B)}(t)\delta\zeta^{A(B)}(0)\right\rangle = e^{-2\gamma t}$. The ratio of the system-environment coupling (ν) and the switching rate (γ) shows the non-Markovian and Markovian behaviors of system. For $\nu \gg \gamma$ (or $\alpha = \frac{\nu}{\gamma} \gg 1$), the system is in non-Markovian regime, while for $\nu \ll \gamma$ (or $\alpha = \frac{\nu}{\gamma} \ll 1$), it is in Markovian regime. The noise parameters of the qubit and qutrit ($\zeta^{A}(t)$ and $\zeta^{B}(t)$) are different for independent environments ($\zeta^{A}(t) \neq \zeta^{B}(t)$) while they are same for common environment ($\zeta^{A}(t) = \zeta^{B}(t) = \zeta(t)$). The average phase factor expresses as

$$\begin{aligned} \Delta_{n\nu}(t) = \quad & e^{-\gamma t}\quad\left[cosh(\delta_{n\nu}t) + \frac{\gamma}{\delta_{n\nu}}sinh(\delta_{n\nu}t)\right]\qquad \gamma > n\nu \\ & e^{-\gamma t}\quad\left[cos(\delta_{n\nu}t) + \frac{\gamma}{\delta_{n\nu}}sin(\delta_{n\nu}t)\right]\qquad \gamma < n\nu, \end{aligned} \tag{3}$$

where $\delta_{n\nu} = \sqrt{|(n\nu)^2 - \gamma^2|}$ and $n \in \mathbb{N}$.

The initial state of qubit-qutrit system could be defined as Bell-like state (ρ_B), entangled state (ρ_E) and separable state (ρ_S).

$$\rho_B(x) = \left|\psi\right\rangle\left\langle\psi\right|, \qquad \left|\psi\right\rangle = \sqrt{x}\left|11\right\rangle + \sqrt{1-x}\left|33\right\rangle, \tag{4}$$

$$\begin{aligned} \rho_E(0) \quad = \quad & \frac{p}{2}(\left|11\right\rangle\left\langle 11\right| + \left|12\right\rangle\left\langle 12\right| + \left|11\right\rangle\left\langle 23\right| + \left|22\right\rangle\left\langle 22\right| + \left|23\right\rangle\left\langle 23\right| + \left|23\right\rangle\left\langle 11\right|) \\ + \quad & \frac{1-2p}{2}(\left|13\right\rangle\left\langle 13\right| + \left|13\right\rangle\left\langle 21\right| + \left|21\right\rangle\left\langle 13\right| + \left|21\right\rangle\left\langle 21\right|) \end{aligned} \tag{5}$$

and

$$\begin{aligned}\rho_S(0) &= \frac{r}{2}(|11\rangle\langle 11| + |12\rangle\langle 12| + |11\rangle\langle 23| + |22\rangle\langle 22| + |23\rangle\langle 23| + |23\rangle\langle 11| \\ &+ |13\rangle\langle 21| + |21\rangle\langle 13|) + \frac{1-2r}{2}(|13\rangle\langle 13| + |21\rangle\langle 21|)\end{aligned} \tag{6}$$

where $0 \leq x \leq 1$, $0 \leq p \leq \frac{1}{2}$ and $0 \leq r \leq \frac{1}{3}$. For $p = \frac{1}{3}$, the state ρ_E is separable.
The qubit-qutrit system is under RTN, its density matrices evolution for Bell-like initial state and common and independent environments are

$$\rho_B^{ie(ce)}(t) = \begin{bmatrix} x & 0 & 0 & 0 & 0 & \sqrt{x(1-x)}F^{ie(ce)} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \sqrt{x(1-x)}F^{ie(ce)} & 0 & 0 & 0 & 0 & 1-x \end{bmatrix}. \tag{7}$$

Qubit-qutrit system evolution for entangled initial state in independent and common environments are

$$\begin{aligned}\rho_E^{ie}(t) &= \frac{1}{2}\begin{bmatrix} p & 0 & 0 & 0 & 0 & pF^{ie} \\ 0 & p & 0 & 0 & 0 & 0 \\ 0 & 0 & 1-2p & (1-2p)F^{ie} & 0 & 0 \\ 0 & 0 & (1-2p)F^{ie} & 1-2p & 0 & 0 \\ 0 & 0 & 0 & 0 & p & 0 \\ pF^{ie} & 0 & 0 & 0 & 0 & p \end{bmatrix}, \\ \rho_E^{ce}(t) &= \frac{1}{2}\begin{bmatrix} p & 0 & 0 & 0 & 0 & pF^{ce} \\ 0 & p & 0 & 0 & 0 & 0 \\ 0 & 0 & 1-2p & 1-2p & 0 & 0 \\ 0 & 0 & 1-2p & 1-2p & 0 & 0 \\ 0 & 0 & 0 & 0 & p & 0 \\ pF^{ce} & 0 & 0 & 0 & 0 & p \end{bmatrix}.\end{aligned} \tag{8}$$

Evolution of the system for an initial separable state will be

$$\rho_S^{ie}(t) = \frac{1}{2}\begin{bmatrix} r & 0 & 0 & 0 & 0 & rF^{ie} \\ 0 & r & 0 & 0 & 0 & 0 \\ 0 & 0 & 1-2r & rF^{ie} & 0 & 0 \\ 0 & 0 & rF^{ie} & 1-2r & 0 & 0 \\ 0 & 0 & 0 & 0 & r & 0 \\ rF^{ie} & 0 & 0 & 0 & 0 & r \end{bmatrix}, \tag{9}$$

$$\rho_S^{ce}(t) = \frac{1}{2}\begin{bmatrix} r & 0 & 0 & 0 & 0 & rF^{ce} \\ 0 & r & 0 & 0 & 0 & 0 \\ 0 & 0 & 1-2r & r & 0 & 0 \\ 0 & 0 & r & 1-2r & 0 & 0 \\ 0 & 0 & 0 & 0 & r & 0 \\ rF^{ce} & 0 & 0 & 0 & 0 & r \end{bmatrix}$$

with $F^{ie} = \Delta_{2\nu}^2(t)$, $F^{ce} = \Delta_{4\nu}(t)$ and $n = 2$ ($n = 4$) for independent (common) environments.

Evolution of entanglement

At first, we apply negativity as an estimator to quantify entanglement of the system which is defined as

$$E(\rho^{AB}) = \frac{\sum_i(|\eta_i| - \eta_i)}{2}, \tag{10}$$

where η_is are the eigenvalues of the partial transpose of ρ^{AB} with respect to the subsystem "A" [16].

The negativity has been obtained for different initial conditions of the system in independent (common) environments. For initial Bell-like state, the negativity has been obtained as

$$E_B^{ce(ie)} = 2\,|F^{ce(ie)}|\sqrt{-x^2 + x} \tag{11}$$

and for initial entangled state in independent (common) environments, the negativity is

$$\begin{aligned} E_E^{ce} &= \frac{1}{2}(p(F^{ce} - 1) + |3p - 1| + |2p + pF^{ce} - 1|), \qquad (12)\\ E_E^{ie} &= \frac{1}{2}(p((F^{ce})^2 - 1) + |2p - 1 + p(F^{ce})^2| + |p - (F^{ce})^2 + 2p(F^{ce})^2| \\ &+ |2p(F^{ce})^2 - p - (F^{ce})^2|). \end{aligned}$$

For an initial Bell-like state ($x = \frac{1}{2}$), Fig. 1.a shows negativity for independent and common environments for Markovian regime. It demonstrates when environments are independent and the entanglement of the system is more robust. One can say when the qubit-qutrit is coupled with a common environment, there is a kind of interaction mediator between qubit subsystem and qutrit subsystem. For non-Markovian regime, phenomena of sudden death and revival are demonstrated in Fig1.b. One can see that in the common environment, entanglement is more robust in comparison with case of the independent environments.
As Figs. 1.c and 1.d show for the case $p = \frac{1}{4}$, entanglement in common environment is constant while in independent environments for Markovian regime has sudden death and for non-Markovian regime has sudden death and revival entanglement. This behavior of system seems a remarkable result of our work, because in before studies have been emphasized that robustness of entanglement happens for independent environments in Markovian regime and it also happens for common environment in non-Markovian regime. We investigate entanglement of the system, for initial entangled state with $p = 0.45$. The evolution of entanglement has been shown in Fig.1.c and 1.d for Markovian and non-Markovian regime respectively. One can see that these figures verify previous studies.

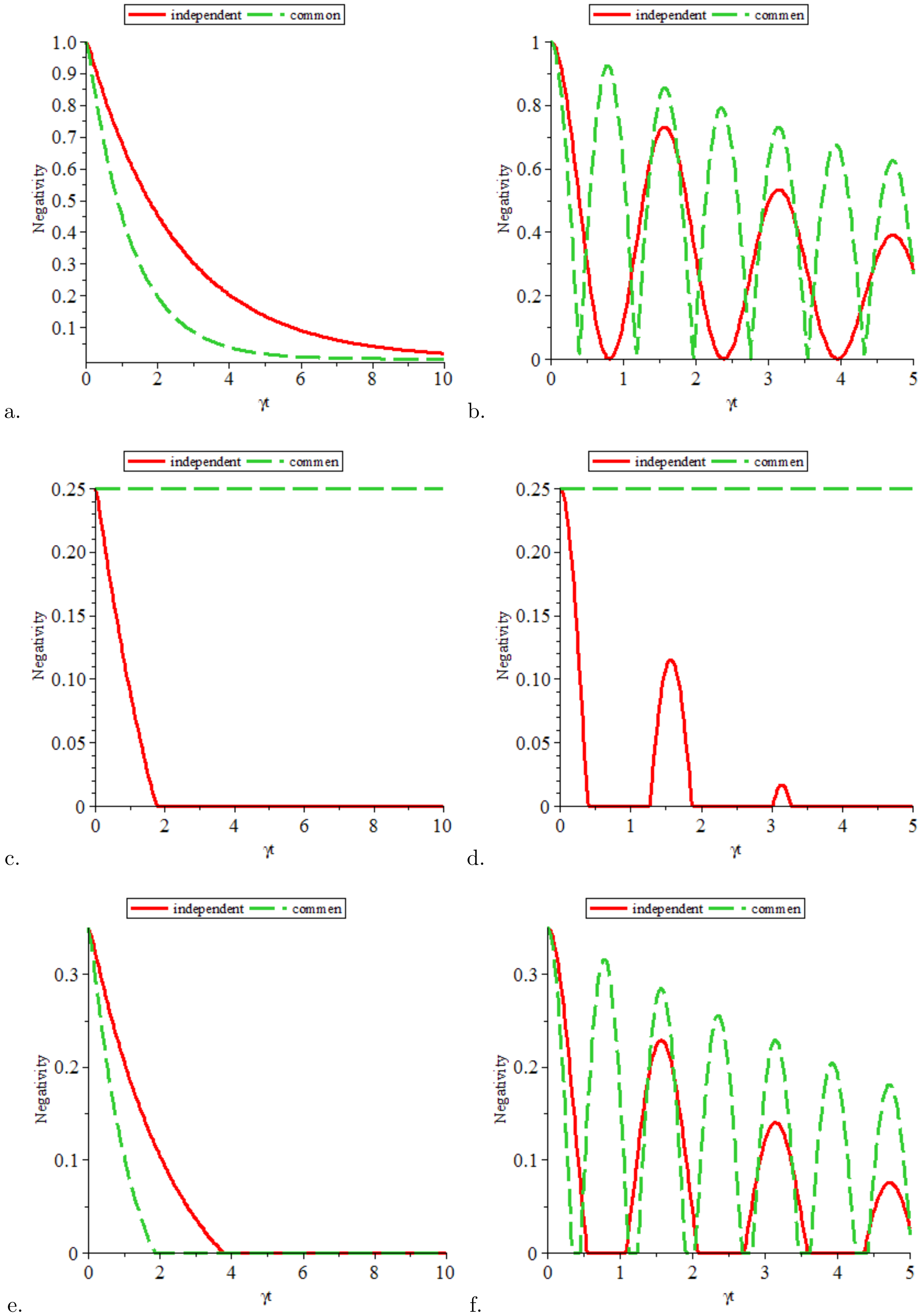

Figure 1: Comparision of negativity for common and independent environments. a,b: $x = \frac{1}{2}$, c,d:$p = \frac{1}{4}$ and e,f:$p = 0.45$. a,c,e $\alpha = 0.1$ and b,d,f $\alpha = 10$.

Evolution of nonlocality

The nonlocality of a physical system shows quantumness which could be measured by measurement induced nonlocality (MIN). This measure is defined as

$$N(\rho^{AB}) = \max_{\Pi^A} ||\rho^{AB} - \Pi^A(\rho^{AB})||^2, \tag{13}$$

the $\{\Pi^A\}$ is projective measurements set which has been made of eigenstates of ρ^A. For any operator, Hilbert-Schmidt norm is defined as $||K|| = \sqrt{tr(K^\dagger K)}$. The spaces that have all linear operators on H^A and H^B respectively are $L(H^A)$ and $L(H^B)$. The $L(H^A)$ and $L(H^B)$ have orthonormal bases as $X_i(i = 1, 2, ..., m^2)$ and $Y_j(j = 1, 2, ..., n^2)$ respectively. A bipartite state in $H^A \otimes H^B$ can be written as

$$\rho = \frac{1}{mn}\mathbb{I}^A \otimes \mathbb{I}^B + \sum_{i=2}^{m^2} x_i X_i \otimes \frac{\mathbb{I}^B}{\sqrt{n}} + \sum_{j=2}^{n^2} \frac{\mathbb{I}^A}{\sqrt{m}} \otimes y_j Y_j + \sum_{i=2}^{m^2}\sum_{j=2}^{n^2} t_{ij} X_i \otimes Y_j. \tag{14}$$

Where $X_1 = \frac{\mathbb{I}}{\sqrt{m}}$, $Y_1 = \frac{\mathbb{I}}{\sqrt{n}}$, $\{X_i, i = 2, ..., m^2\}$ and $\{Y_j, j = 2, ..., n^2\}$ are defined as generators of $SU(m)$ and $SU(n)$, respectively. Coefficients in Eq. (14) are

$$x_i = \frac{m}{2}Tr(\rho X_i \otimes \mathbb{I}_n), y_j = \frac{n}{2}Tr(\rho \mathbb{I}_m \otimes X_j) \text{ and } t_{ij} = \frac{mn}{4}Tr(\rho X_i \otimes Y_j) \tag{15}$$

$\vec{x} = [x_i]$, $\vec{y} = [y_j]$ and $T = [t_{ij}]$. For a $2 \otimes n$ dimensional system from Eq. (14) MIN, has following form

$$N(\rho^{AB}) = Tr(TT^t) - \lambda_{min} \tag{16}$$

Where TT^t is a 3×3 dimensional matrix, and λ_{min} is its minimum eigenvalue.
For initial Bell-like state in common and independent environments, we calculate nonlocality as

$$\begin{aligned} N_B^{ce(ie)} &= 8\,x(F^{ce(ie)})^2 - 8\,x^2(F^{ce(ie)})^2 \\ &+ \frac{4}{3}(x^2 - x + 1 - \min\left(3\,x(F^{ce(ie)})^2 - 3\,x^2(F^{ce(ie)})^2, x^2 - x + 1\right)). \end{aligned} \tag{17}$$

The evolution of MIN for an initial entangled state in common and independent environments is

$$\begin{aligned} N_E^{ce} &= 2\,p^2(F^{ce})^2 + 3 - 14\,p + 17\,p^2 - \min(9\,p^2 - 6\,p + 1, p^2(F^{ce})^2 - 2\,pF^{ce} + 4, \\ &\quad p^2F^{ce} + 1 - 4\,p + 4\,p^2, p^2(F^{ce})^2 + 2\,pF^{ce} - 4\,p^2F^{ce} + 1 - 4\,p + 4\,p^2) \\ N_E^{ie} &= 10\,p^2(F^{ie})^2 - 8\,(F^{ie})^2p + 2\,(F^{ie})^2 + 9\,p^2 - 6\,p + 1 \\ &- \min\left(9\,p^2 - 6\,p + 1, p^2(F^{ie})^2 - 2\,(F^{ie})^2p + (F^{ie})^2, 9\,p^2(F^{ie})^2 - 6\,(F^{ie})^2p + (F^{ie})^2\right). \end{aligned} \tag{18}$$

For an initial state with zero entanglement (separable) in common and independent environments, MIN will be

$$\begin{aligned} N_S^{ce} &= 2\,r^2(F^{ce})^2 + 11\,r^2 - 6\,r + 1 \\ &- \min\left[9\,r^2 - 6\,r + 1, r^2(F^{ce})^2 - 2\,r^2F^{ce} + r^2, r^2(F^{ce})^2 + 2\,r^2F^{ce} + r^2\right] \\ N_S^{ie} &= 4\,r^2(F^{ie})^2 + 9\,r^2 - 6\,r + 1 - \min\left[0, 4\,r^2(F^{ie})^2, 9\,r^2 - 6\,r + 1\right]. \end{aligned} \tag{19}$$

One can find from Fig 2.a, evolution of nonlocality for initial Bell-like state with $x = \frac{1}{2}$ in Markovian regime which shows that nonlocality in independent environments are more robust than common type. With the passage of time nonlocality goes to a fixed value in both types of environments. For non-Markovian case, Fig. 2.b shows that in common and independent environments, MIN has sudden change and revival. In the case of common environment, MIN is more robust than independent environments against the noise. These behaviors of MIN are the same with entanglement behaviors which have been studied in previous section.

We demonstrate in Fig 2.c that nonlocality decreases monotonically and at the final stage, it goes to a fixed value in Markovian regime. In non-Markovian regime, MIN has a sudden change and revival behavior. In the common environment, nonlocality of the system is more robust than independent environments in two regimes (Markovian and non-Markovian). We remind that for a separable initial state, entanglement is zero while there is nonlocality.

Result

We have shown the evolution of entanglement and nonlocality for different initial states and different conditions. Our finding shows that for a separable state, there is nonlocality but entanglement is zero. Robustness of entanglement and nonlocality, in common or independent environment, depend on initial state for Markovian regime. This result is different from results of previous studies. While for non-Markovian case in common environment, entanglement and MIN are more robust than independent environments which are interesting results of our work.

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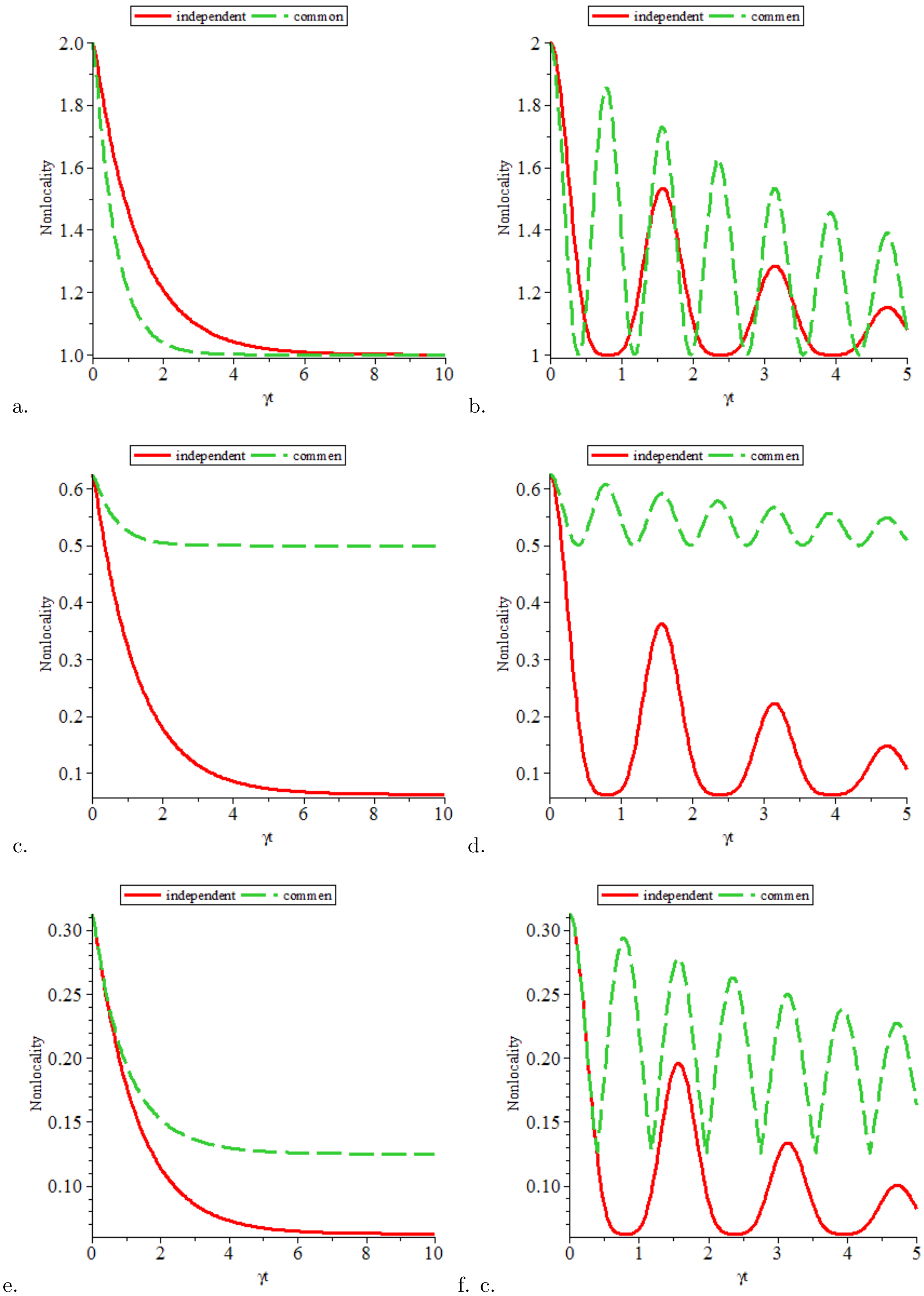

Figure 2: Comparision of nonlocality for common and independent environments. a,b: $x = \frac{1}{2}$, c,d:$r = \frac{1}{4}$ and e,f:$s = \frac{1}{4}$. a,c,e: $\alpha = 0.1$ and b,d,f: $\alpha = 10$.

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