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Research Paper

Investigation of the Effect of Density Gradients in Fuel Pellets on Filamentation Electromagnetic Instability during Beam-Plasma Interactions in Inertial Confinement Fusion

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Abstract. The transfer of a beam of high-energy particles, such as electrons, to the center of a fuel pellet is a significant aspect of plasma fusion processes. When the beam is directed toward the center of the fuel pellet, a counter-current is generated by the plasma electrons surrounding the pellet. This current leads to the production of an electromagnetic field. The growth of this electromagnetic field results in the appearance of instabilities, including filamentation instability within the plasma medium. Furthermore, the gradual growth of these instabilities disrupts energy transfer to the fuel pellet, hindering the achievement of ideal ignition conditions. The present study investigated the effects of parameters such as the density gradient of the fuel pellet, thermal anisotropy and the relativistic mass factor on filamentation instability in a beam-plasma system that includes non-relativistic background electrons and a relativistic mono-energetic electron beam. By linearizing Maxwell-Vlasov equations, the dispersion relation for filamentation instability was derived. While solving the dispersion equation and calculating the instability growth rate, it was observed that with increasing the scale length of the density gradient, due to the higher collision rate and the increase in energy transfer to the plasma particles, the growth rate of filamentation instability decreased. Additionally, it was found that increasing the relativistic mass factor and thermal anisotropy fraction leads to an increase in the instability growth rate due to increased internal energy dissipation.

Keywords: Electromagnetic, Instability, Dispersion relation, Density gradient, Filamentation instability, Thermal anisotropy

1 Introduction

In the process of plasma fusion, the transfer of a beam of high-energy particles to the center of a fuel pellet through focusing a short, high-energy laser pulse is of particular importance

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[1].The energy deposition of electron beams near the center of the target pellet is associated with the appearance of various instabilities, leading to significant challenges. In laser-plasma interactions, the generated high-energy particle beam disrupts the quasi-neutrality of the plasma. Therefore, the target plasma, while generating a counter-current against the electron beam, attempts to maintain the neutral state of its medium and minimize the beam currents, ultimately leading to the generation of electromagnetic fields [2]. Due to the production of strong magnetic fields, electromagnetic instabilities will arise, playing a crucial role in stopping electron beams, increasing energy losses, and preventing energy deposition from the beam. Plasma filamentation due to thermal anisotropy was first introduced by Eric Weibel in 1995. Over recent decades, numerous research studies have focused on transverse instabilities as well as the effects of plasma resistivity and thermal anisotropy on filamentation instability [3,4]. When a high-intensity laser pulse is directed at a fuel pellet, a beam of relativistic electrons is formed at critical density levels. Small-scale density fluctuations generate a net current, during which perturbations and disturbances perpendicular to the beam direction lead to the appearance of electromagnetic filamentation instability. This instability arises from the repulsion between two opposing currents that tend to amplify initial transverse disturbances [5,6].

When a high-power electromagnetic wave enters a plasma, instabilities appear within the plasma [7,8]. The filamentation instability mode occurs when a low-density electron beam interacts with a high-density ionized plasma [9]. This instability is classified among electromagnetic instabilities and transverse modes, arising from vertical perturbations affecting the beam [10]. The processes of filamentation and filamentation instability have been studied in laboratory settings through simulations. This instability has been observed in large-scale cosmic plasmas, solar corona, and also in laboratory plasmas such as focus and magnetohydrodynamic generators. Furthermore, these types of instabilities play a significant role in X-ray lasers, plasma lasers, particle accelerators, and laser-induced fission [**?**]. Understanding the instabilities present in the beam-plasma system and their consequences leads to improved simulations and designs for ignition and combustion of fuel pellets. Therefore, in this study, based on the kinetic description of plasma, we investigate electromagnetic filamentation instability in the presence of density gradients and the effects of thermal anisotropy in plasma. Then, we analytically evaluate the impact of parameters influencing the growth rate of filamentation instability.

2 Calculation of the Theoretical Model

In the fast ignition scheme, it is assumed that a mono-energetic beam with a number density n_b and mean relativistic speed v_b is propagating in a plasma with stationary background ions, and a return current with a number density $n_p$and speed v_p is generated in the opposite direction of the beam while maintaining charge neutrality and electric current balance, leading to the relation $n_b v_b = n_p v_p$ A relativistic electron beam propagates from a plasma halo with a density of $n_e \sim 10^{21} cm^{-3}$ toward the center of a deuterium-tritium fuel pellet with a density of $n_e \sim 10^{25} cm^{-3}$ Since the center of the fuel pellet is approximately four times denser than the halo at the edge of the pellet, the electron beam encounters a density gradient and is subjected to an unstable system [11,12]. Consequently, the energy transfer of the electron beams in the plasma will depend on the density gradient.

The plasma electron density n_e at the critical surface is equal to n_0, which is the lowest plasma density: $n_e\,(z=0) = n_0 = 10^{21} cm^{-3}$ As one moves away from the critical surface towards the center of the fuel disk, n_e increases beyond its critical value, reaching its maximum at the center of the disk, denoted as $n_e\,(z=c) = n_c = 10^{25} cm^{-3}$ The characteristic length scale of density gradient (λ) can be expressed as; $\lambda = \frac{n_e}{\frac{dn_e}{dz}} = \frac{z_e}{\ln \frac{n_e}{n_0}}$ and the electron density, as a function of position, is given by; $n_e\left(z\right) = n_0 e^{\frac{z}{\lambda}}$ Since changes in electron density lead to changes in medium temperature; $T_e\left(z\right) = \frac{n_0^2(z)T_{e0}}{n_e^2}$ [13,14], the plasma thermal anisotropy, which is due to the density gradient, can be investigated in the filamentary instability. In order to study the factors affecting the growth rate of the filamentation electromagnetic instability, it is assumed that the electron number density of the medium changes along the z axis, and the incoming electron beam moves along the density gradient from the outer region of the fuel pellet towards the center of the Deuterium-Tritium (DT) fuel pellet. Based on the kinetic model describing the plasma medium and combining the Maxwell-Wallace equations, the dispersion relation can be extracted. Then, by solving the dispersion relation, the growth rate of the filamentation electromagnetic instability can be calculated. The beam particle distribution function in the presence of the gradient density scale parameter $\eta = \frac{n_0^2}{n_p^2(z)}$ has been chosen as a Maxwell-Delta distribution to express the non-relativistic nature of the return current electrons and also the relativistic nature of the high-energy electrons in the beam [15]

$$f_0^b = \left(\frac{m_0\gamma_b\eta}{2\pi T_z^b}\right)^{\frac{1}{2}} \exp\left(\frac{-m_0\gamma_b v_x^2}{2T_\perp^b}\right)\delta\left(p_y\right)\exp\left[\frac{-m_0\gamma_b\left(v_z - v_b\right)^2\eta}{2T_z^b}\right]. \tag{1}$$

In the Maxwellian distribution, η and T represent the density and temperature of the plasma environment, respectively, m and v represent the particle mass and velocity, respectively.v_z and v_b represent the velocities of the background particles and beam particles, respectively. The distribution function governing the plasma particles will be expressed as follows:

$$f_0^p = \left(\frac{m_0}{2\pi}\right)^{\frac{3}{2}} \frac{1}{T_\perp^p}\left(\frac{\eta}{T_z^p}\right)^{\frac{1}{2}} \exp\left[\frac{-m_0\left(v_x^2 + v_y^2\right)}{2T_\perp^p} - \frac{m_0\left(v_z + v_p\right)^2\eta}{2T_z^p}\right]. \tag{2}$$

The thermal spread is in the perpendicular and parallel directions and is defined as:

$$T_z^{b(p)} = \int m_0\gamma_{b(p)} v_z^2 f_0^{b(p)} d^3v, \tag{3}$$

and

$$T_\perp^{b(p)} = \frac{1}{2}\int m_0\gamma_{b(p)} v_\perp^2 f_0^{b(p)} d^3v. \tag{4}$$

Considering the Maxwell equations governing the electromagnetic fields of the medium,

$$\overrightarrow{\nabla} \times \overrightarrow{E} = -\frac{1}{c}\left(\frac{\partial \overrightarrow{B}}{\partial t}\right), \tag{5}$$

$$\overrightarrow{\nabla} \times \overrightarrow{B} = -\frac{4\pi}{c}\overrightarrow{J} + \frac{1}{c}\left(\frac{\partial \overrightarrow{E}}{\partial t}\right). \tag{6}$$

The Vlasov equation governing plasma particles is given by:

$$\frac{\partial f}{\partial t} + \overrightarrow{v}.\frac{\partial f}{\partial \overrightarrow{r}} + \frac{q}{m_0\gamma}\left(\overrightarrow{E} + \frac{\overrightarrow{v}}{c} \times \overrightarrow{B}\right).\frac{\partial f}{\partial \overrightarrow{v}} = 0. \tag{7}$$

In equation (7), the parameter f represents the distribution function governing the plasma particles, and is defined as a combination of the equilibrium distribution$f_0\left(\overrightarrow{v}\right)$and the perturbation $f_1\left(\overrightarrow{r}, t\right)$; $f_1\left(\overrightarrow{v}, \overrightarrow{r}, t\right) = f_0\left(\overrightarrow{v}\right) + f_1\left(\overrightarrow{r}, t\right)$ m_0 denotes the rest mass of the electron; γ is the relativistic mass factor; $\overrightarrow{E}$ and $\overrightarrow{B}$ represent the electromagnetic fields in the non-magnetized plasma medium and are defined as:$\overrightarrow{E} = \overrightarrow{E}_0 + \delta\overrightarrow{E}$and $\overrightarrow{B} = \overrightarrow{B}_0 + \delta\overrightarrow{B}$where $\delta\overrightarrow{E}$ and $\delta\overrightarrow{B}$ are the perturbative electric and magnetic fields, respectively, and are denoted as $\overrightarrow{E}_1$ and $\overrightarrow{B}_1$Considering the propagation direction of the beam in the plasma along the $\widehat{x} - axis$, $\left(\overrightarrow{k} = k\widehat{x}\right)$ and the electric field direction along the $\widehat{z} - axis$, $\left(\overrightarrow{E} = E_1\widehat{e_z}\right)$ we use Maxwell's equations to express the magnetic field B as: $\overrightarrow{B_1} = -\frac{ck}{w}E_1\widehat{e_y}$. Taking into account the perturbative fields and the distribution function, f is expressed as:$f_1\left(\overrightarrow{r}, t\right) \propto e^{i\left(\overrightarrow{k}.\overrightarrow{r} - wt\right)}$ where k indicates the wave propagation direction and w represents the oscillation frequency. The current density is given by; $\overrightarrow{J} = -n_e e \int v_z f_1 d^3v\widehat{e_z}$ The current density is perpendicular to the beam direction, therefore it only has a z-component. The general dispersion relation governing the plasma-beam interaction can be derived by combining Maxwell's equations with the Vlasov equation, leading to a comprehensive understanding of wave dynamics in this context;

$$\begin{aligned} k^2c^2 =& w^2 + w_p^2 \int \frac{kv_z^2}{(w - kv_x)} \left(\frac{\partial f_{0p}}{\partial v_x}\right) d^3v + w_p^2 \int v_z \left(\frac{\partial f_{0p}}{\partial v_z}\right) d^3v \\ &+ w_b^2 \int \frac{kv_z^2}{(w - kv_x)} \left(\frac{\partial f_{0p}}{\partial v_x}\right) d^3v + w_b^2 \int v_z \left(\frac{\partial f_{0b}}{\partial v_z}\right) d^3v. \end{aligned} \tag{8}$$

In equation (8), f_0p and f_0b represent the equilibrium distribution functions governing the plasma and beam, respectively. Since the electron number density of the beam is much smaller than that of the return current, the dynamics governing the plasma return current and the beam flow are considered in the non-relativistic regime. By substituting the distribution functions of the beam and plasma into the dispersion relation (equation (8)) and calculating the integrals involved, the dispersion relation can be rewritten as follows:

$$\begin{aligned} -w_p^2 + k^2c^2 =& w^2 + w_p^2 \left(\frac{T_\parallel^p}{\eta T_\perp^p}\right) \xi z(\xi) + v_p^2 m_0 w_p^2(\xi) z(\xi) + w_p^2 \left(\frac{T_\parallel^p}{\eta T_\perp^p}\right) \\ &+ \frac{v_p^2 m_0 w_p^2}{T_\perp^p} w_b^2(\xi') \left(\frac{T_\parallel^b}{\eta T_\perp^b}\right) z(\xi') + \frac{v_b^2 m_0 \gamma_b}{T_\perp^b} w_b^2 \xi' z(\xi') + w_b^2 \frac{T_\parallel^b}{\eta T_\perp^b} \\ &+ \frac{v_b^2 m_0 \gamma_b}{T_\perp^b} w_b^2 - w_b^2. \end{aligned} \tag{9}$$

In this relation, T represents the thermal temperature of the electrons, while $z(\xi)$ and $z(\xi')$ denote the dispersion functions for the plasma and the beam, respectively, and are defined as follows:

$$z(\xi) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} \frac{e^{\left(-x^2\right)}}{(x - \xi)} dx, \tag{10}$$

where $\xi' = \frac{w\sqrt{m_0\gamma}}{k\sqrt{2T_\perp^b}}$ and $\xi = \frac{w\sqrt{m_0}}{k\sqrt{2T_\perp^p}}$.

To solve the dispersion relation, it is necessary to consider appropriate boundary conditions for the dispersion variables of the plasma and the beam. Therefore, we will have the boundary condition $|\xi| \ll 1$ for the short wavelength limit, and $|\xi| \gg 1$ for the long one. For the plasma, we have the following expression:

$$\begin{aligned} &|\xi| \gg 1 \to z(\xi) \approx -\tfrac{1}{\xi} - \tfrac{1}{2\xi^3} + \cdots \\ &|\xi| \ll 1 \to z(\xi) \approx -2\xi + \cdots i\sqrt{\pi}\exp\left(-\xi^2\right), \end{aligned} \tag{11}$$

and for the beam:

$$\begin{aligned} |\xi'| \gg 1 &\to z(\xi') \approx -\tfrac{1}{\xi'} - \tfrac{1}{2\xi'^3} + \cdots \\ |\xi'| \ll 1 &\to z(\xi') \approx -2\xi' + \cdots i\sqrt{\pi}\exp\left(-\xi'^2\right). \end{aligned} \tag{12}$$

Considering the characteristics of the unstable modes in the short wavelength limits $|\xi| \ll 1$ and $|\xi'| \ll 1$ terms with second-order and higher powers of ξ can be neglected. Thus, the final form of the dispersion relation for short wavelengths is obtained as follows:

$$\begin{aligned} k^2c^2 =& w_p^2 i\sqrt{\pi}\frac{T_\parallel^p}{\eta T_\perp^p}\sqrt{\frac{m_0}{2T_\perp^p}}\left(\frac{w}{k}\right) + w_p^2 i\frac{\sqrt{\pi}m_0 v_p^2}{T_\perp^p}\sqrt{\frac{m_0}{2T_\perp^p}}\left(\frac{w}{k}\right) + w_p^2\frac{T_\parallel^p}{\eta T_\perp^p} \\ &+ \frac{m_0 v_p^2 w_p^2}{T_\perp^p} - w_p^2 + w_b^2 i\sqrt{\pi}\frac{T_\parallel^b}{\eta T_\perp^b}\sqrt{\frac{m_0\gamma_b}{2T_\perp^b}}\left(\frac{w}{k}\right) + \frac{i\sqrt{\pi}w_b^2 m_0 v_b^2\gamma_b}{T_\perp^b}\sqrt{\frac{m_0\gamma_b}{2T_\perp^b}}\left(\frac{w}{k}\right) \\ &+ w_b^2\left(\frac{T_\parallel^b}{\eta T_\perp^b}\right) + w_b^2\left(\frac{m_0\gamma_b v_b^2}{T_\perp^b}\right) - w_b^2. \end{aligned} \tag{13}$$

In this expression, the frequency of the beam electrons, w_b , is given by: $w_b = \sqrt{\frac{n_b e^2}{\varepsilon_0 m_0 \gamma}}$ For the non-relativistic case, we have: $w'_b = w_b.\gamma^{\frac{1}{2}} = \sqrt{\frac{n_b e^2}{\varepsilon_0 m_0}}$ To calculate the growth rate, it is necessary to consider the frequency as complex; that is; $w = w_r + iw_m$ where the imaginary part of the frequency represents the growth rate of the electromagnetic instability of the filamentation. By substituting the complex frequency into the dispersion relation and separating the imaginary part of the frequency, we obtain:

$$w_m = k\frac{v_{th.\perp}^p}{c}\sqrt{\frac{2}{\pi}}\left\{\frac{-\frac{k^2c^2}{w_b^2}\left[\frac{\beta'}{\eta} + \frac{m_0\gamma_b v_b^2}{T_\perp^b} - 1\right] + \frac{w_p^2}{w_b^2}\left[\frac{\beta}{\eta} + \frac{m_0 v_p^2}{T_\perp^p} - 1\right]}{\frac{v_{th.\perp}^p c}{v_{th.\perp}^b c}\left[\frac{\beta'}{\eta} + \frac{m_0\gamma_b v_b^2}{T_\perp^b}\right] + \frac{w_p^2}{w_b^2}\left[\frac{\beta}{\eta} + \frac{m_0 v_p^2}{T_\perp^p}\right]}\right\}. \tag{14}$$

Here, $\beta = \left(\frac{T_\parallel^p}{T_\perp^p}\right)$ represents the thermal anisotropy ratio for the plasma, while $\beta' = \left(\frac{T_\parallel^b}{T_\perp^b}\right)$ denotes the thermal anisotropy ratio for the beam. Additionally, $v_{th.\perp}^p = \left(\frac{T_\perp^p}{m_0}\right)^{\frac{1}{2}}$ represents the thermal velocity.

3 Analysis of Results

In this section, the effects of the parameters influencing the growth rate of electromagnetic filamentation instability will be examined. Based on the growth rate relation calculated in equation (14), the variations of the normalized growth rate of filamentation $\frac{w}{w_b}$ as a function of the normalized light frequency $\frac{kc}{w_b}$ are presented for different density gradient values, while keeping the anisotropy ratio and relativistic mass factor constant, as shown in Figure (1-a). It was observed that with increasing the density gradient, the growth rate of the filamentation instability decreased due to the increase in the collision rate of the beam with the fuel pellet plasma particles and the transfer of energy from the beam particles to the plasma medium. The spatial variation in plasma particle density enhances collisions between electron beam particles and plasma particles. As the collision rate increases, a greater portion of the beam particles' kinetic energy is transferred to the plasma, resulting in significant energy loss of the electron beam. This energy transfer from beam to plasma acts as an effective damping mechanism that suppresses instability-driving oscillations, thereby reducing the instability growth rate. The decreasing effect of the plasma density gradient on the maximum growth rate of instability is clearly evident in Figure (1-b).

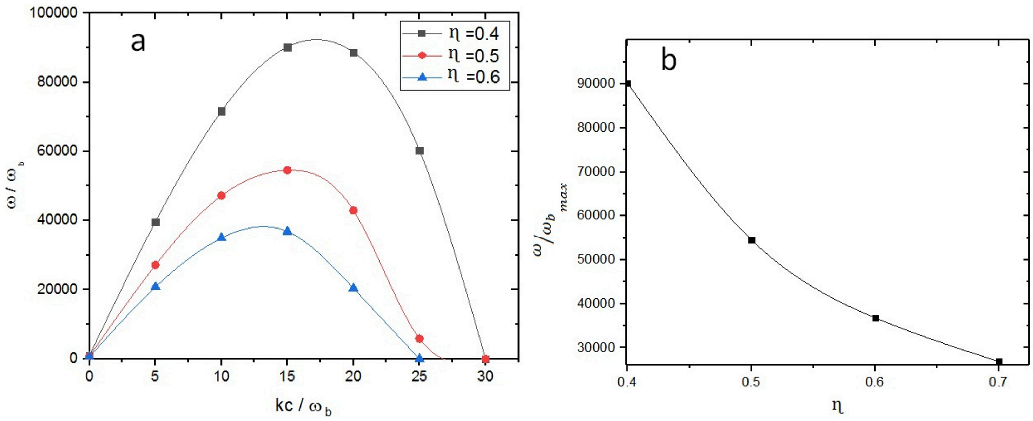

Figure 1: (a)The variations of the normalized growth rate of filamentation instability $\left(\frac{w}{w_b}\right)$ as a function of the normalized light frequency $\left(\frac{kc}{w_b}\right)$ for different values of the density gradient, with a thermal anisotropy ratio of $\beta = 2$ and a relativistic mass factor of $\gamma = 3$. (b) The variations of the normalized maximum growth rate of instability as a function of the plasma density gradient.

To investigate the impact of the relativistic mass factor, Figure (2-a) presents the variations of the normalized growth rate of filamentation instability as a function of the normalized light frequency for different relativistic mass factors while maintaining constant values for the density gradient and anisotropy ratio. Results show that with increasing the relativistic mass factor, the growth rate also shows an increasing trend as a result of the increase in internal energy. As the internal energy of particles increases, oscillations grow with greater speed and intensity, leading to an increase in the instability growth rate. This trend of increasing maximum growth rate with changes in relativistic mass factor is clearly visible in Figure (2-b).

Similarly, to study the effect of the thermal anisotropy fraction on the growth rate of filamentation instability, the changes in the normalized filamentation instability growth rate in terms of the normalized light frequency for different anisotropy fractions and constant values of the density gradient and relativistic mass factor have been presented in Figure (3-a). It can be seen that an increase in the thermal anisotropy ratio leads to an increase in the growth rate of instability due to the thermal dispersion of the plasma medium and the increase in internal energy of the plasma particles. This trend is also evident for the maximum growth rate, as shown in Figure (3-b).

4 Conclusions

In the process of transporting a beam of high-energy particles such as electrons to the center of the fuel pellet, we encountered instabilities including filamentation instability. The growth of these instabilities not only prevents the transfer of energy to the center of the fuel pellet but also complicates the ignition process, leading to an increase in the amount of compensatory energy required. Utilizing the Vlasov equations and dispersion relations, the growth

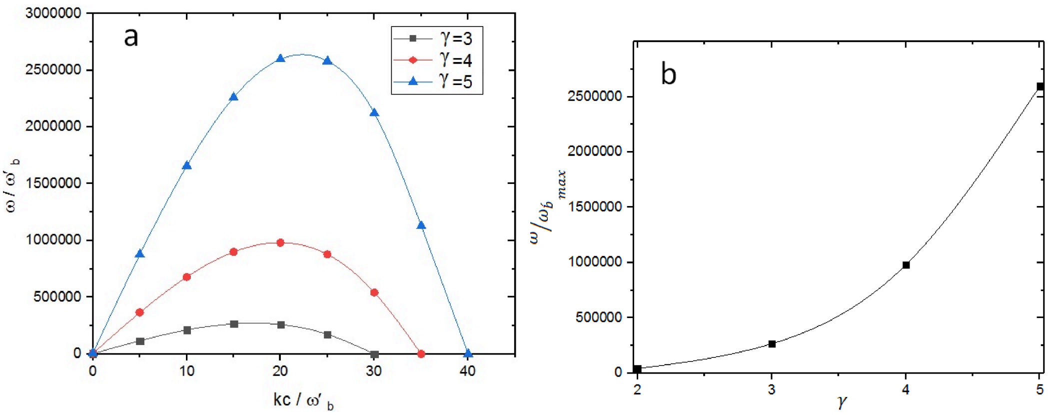

Figure 2: (a) The variations of the normalized growth rate of filamentation instability $\left(\frac{w}{w_b}\right)$ as a function of the normalized light frequency $\left(\frac{kc}{w_b}\right)$ for different values of the relativistic mass factor, with a density gradient of $\eta = 0.4$ and a thermal anisotropy ratio of $\beta = 2$ (b) The variations of the normalized maximum growth rate of instability as a function of the relativistic mass factor.

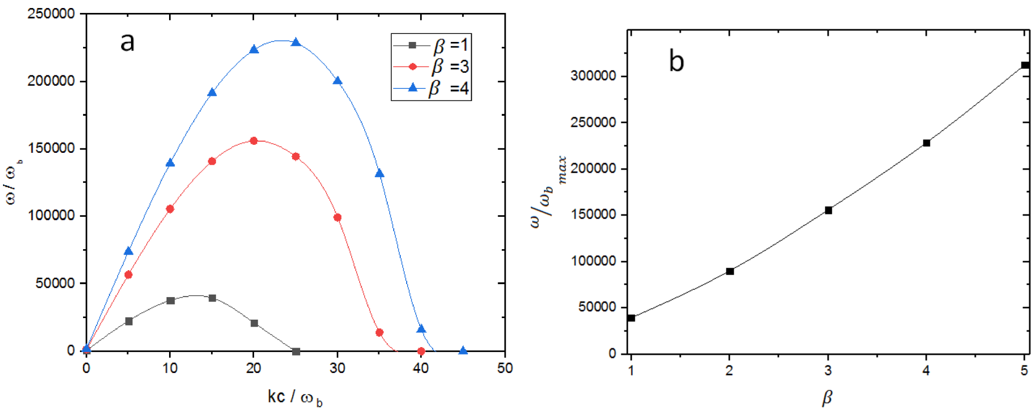

Figure 3: (a) The variations of the normalized growth rate of filamentation instability $\left(\frac{w}{w_b}\right)$ as a function of the normalized light frequency $\left(\frac{kc}{w_b}\right)$ for different values of thermal anisotropy ratios, with a density gradient of $\eta = 0.4$ and a relativistic mass factor of $\gamma = 3$. (b) The variations of the normalized maximum growth rate of instability as a function of the thermal anisotropy ratios.

rate of filamentation instability was calculated. Figs. (1-3) illustrate the dependence of the normalized growth rate on the normalized light frequency with respect to various parameters affecting this instability, such as thermal anisotropy, density gradient, and relativistic mass factor. The results indicate that the maximum growth rate of instability increases with a decrease in the density gradient. Near the center of the fuel pellet, instabilities are maintained over large regions of wave numbers within the system. In fact, as the density gradient scale parameter increases, the growth rate of filamentation instability decreases due to enhanced collision rates and energy transfer to plasma particles. Furthermore, with an increase in both components, the relativistic mass factor and thermal anisotropy, there will be a corresponding increase in the growth rate due to increased internal energy losses.

Authors' Contributions

All authors have the same contribution.

Data Availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare no potential conflicts of interest.

Ethical Considerations

The authors have diligently addressed ethical concerns, such as informed consent, plagiarism, data fabrication, misconduct, falsification, double publication, redundancy, submission, and other related matters.

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