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Research Paper

Observational Status of Zero-Point Length Cosmic Inflation

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Abstract. We examine observational status of cosmological inflation with a zero-point length. By considering the zero-point length correction to the geometric part of the field equations, specially the Hubble parameter, we derive modified background equations, slow-roll parameters and inflation observables including scalar spectral index and tensor-to-scalar ratio in this setup. We conduct numerical analysis on a power-law inflation as a toy model and also some other inflation potentials to assess the impact of a minimum length on the inflationary cosmology. In this regard, we compare our results with recent data from Planck 2018 TT, TE, EE +lowE +lensing, Planck 2018 TT, TE, EE +lowE +lensing+BK15, and Planck 2018 TT, TE, EE +lowE +lensing+ BK15+ BAO at the %68 and %95 levels of confidence. We find that the impact of the zero-point length varies across different potentials and its characteristic value is of different orders of magnitude (in units of the Planck length), determined based on the various types of the potentials. We show that, while some inflation potentials fall outside the mentioned datasets' confidence levels in the absence of the zero-point length, they are in good agreement with the same datasets in the presence of the zero-point length. In this comparison, we obtain a range of consistency for the zero-point length with the mentioned observational data.

Keywords: Cosmological inflation, Quantum gravity phenomenology, Zero-Point length, Observational data

1 Introduction

Cosmic inflation is a fundamental theory for understanding the universe's origins in its very early stages. According to its definition, it is a period of quick expansion that happened in the initial stages of the universe at very high energies. It provides solutions for some problems of the standard hot big bang model such as the flatness, the horizon, the monopole, and also the origin of the cosmic structures [1–3]. Inflation pairs with quantum mechanics and offers

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the fundamental seeds for the density fluctuations in the Universe and the causal process for creating the Large-scale structure of the Universe [4–6]. According to the data collected by the Planck satellite and its consistency with multiple essential and broad forecasts of this theory, it is widely accepted that inflation provides a precise representation of the physical environment for the early time era of the universe [7].

The zero-point length refers to a theoretical concept that arises from attempts to unify quantum mechanics and general relativity [8–10]. It could provide insights into the early universe and the Big Bang singularity, where classical descriptions of gravity and spacetime become inadequate. Some theories suggest that spacetime and its properties may emerge from more fundamental "pre-geometric" entities or structures, rather than being a fundamental feature of reality [11]. The zero-point length could be a signature of this underlying pre-geometric structure.

To explore the effect of zero-point length on cosmic inflation, we first briefly overview the relationship between cosmological inflation, thermodynamics, gravity, quantum mechanics, and the existence of a minimal length (zero-point length). So, we begin with Einstein's gravitational field equations. After presenting the gravitational field equations by Einstein, Carl Schwarzschild obtained the first exact solution of these equations for a symmetric spherical mass in a vacuum. This solution shows a direct relationship between Einstein's field equations and the structure of space-time in the presence of a concentrated mass and led to obtaining the Schwarzschild metric and as a result, obtaining the Schwarzschild radius with its unique properties. This result led to many achievements in the field of physics in connection with horizons and thermodynamics. Linking thermodynamic properties to black hole horizons requires the integration of quantum mechanics, general relativity, and thermodynamics. This integration has the potential to offer valuable insights into the underlying structure of quantum spacetime geometry [12]. A key achievement in this field is the Bekenstein-Hawking entropy-area relation in the 1970s, which incorporates quantum mechanics to state that "Hawking radiation emanates from black holes and behaves like hot bodies, with a temperature related to the black hole's surface gravity. Additionally, black holes have an entropy proportional to the area of their horizons area" [13–16],

$$\begin{aligned} S_{BH} &= \frac{k_B A}{4L_p^2}, \\ T_H &= \frac{\hbar\kappa}{2\pi G k_B c}, \\ \kappa &= \frac{c^4}{4GM}, \end{aligned} \tag{1}$$

where A is the surface area, L_p is planck length and κ is the surface gravity of the event horizon. These equations demonstrate that S and T as thermodynamic properties, are directly related to the surface area of the event horizon A and the surface gravity κ, which are geometric properties. After that, Jacobson obtained the relationship between thermodynamics and gravity for the first time and indicated the primary equations of gravity describing the geometry of spacetime, are similar to the first law of thermodynamics. He was able to derive Einstein field equations by invoking the Clausius relation [17],

$$\delta Q = TdS. \tag{2}$$

Furthermore, many physicists argue that gravity and the structure of space-time are emergent phenomena and gravity arises from the information content associated with substance and its position, measured by entropy. It is argued that changes in this entropy, as matter moves, lead to an entropic force that manifests as gravity. "An entropic force is a macroscopic force that emerges in a macroscopic system with several degrees of freedom due to the statistical drive to augment its entropy. The force equation is explained based on entropy alteration and is unrelated to the specifics of the microscopic dynamics" [18]. The central assumption is that information about a portion of space obeys the holographic principle, which states that degrees of freedom can be encoded on a boundary of the region [19]. This theory is supported by the AdS/CFT correspondence and black hole physics [20,21]. Incorporating these ideas, Eric Verlinde proposed a holographic framework to explain the origin of space and discussed the fundamental connections between gravity and inertia, linked by the equivalence principle. He has demonstrated the feasibility of deriving Newton's laws by treating entropy, temperature, and energy as spatially independent consequences through the application of first principles. He viewed gravity as an entropic force that emerges from altering the amount of information in the substance's location,

$$F\Delta X = T\Delta S. \tag{3}$$

It is an entropic force created when a particle has an entropic reason to be on one side of a membrane and the membrane experiences a temperature change. In this framework with

$$S = \frac{A}{4}, \tag{4}$$

and

$$A = 4\pi r^2, \tag{5}$$

it is easy to derive

$$F = G\frac{Mm}{R^2}. \tag{6}$$

To study more details see Ref. [18].

In general relativity, there are anomalies like a gravitational singularity at the core of a black hole, thought to have infinite density and curvature, and also, the Big Bang singularity in the early universe, making them beyond our current understanding of physics. To better grasp or eliminate these singularities, approaches to quantum gravity, such as string theories, loop quantum gravity, and non-commutative geometry suggest a minimum measurable length scale (zero-point length) for spacetime, establishing a lower limit on measuring spacetime intervals [22]. This concept deviates from the standard uncertainty principle and is governed by the Generalized Uncertainty Principle (GUP), a proposed modification to Heisenberg's Uncertainty Principle in quantum mechanics. Consequently, the Hawking equation's standard relationship between entropy and black hole area changes according to the GUP. Particles emitted from the black hole adhere to this principle [23–30]. The basis of cosmic inflation begins with the Friedman equation derived from general relativity. According to Jacobson's framework, Friedman's equations can be considered as thermodynamic relations. Any change in thermodynamics due to the generalized uncertainty principle (GUP) will consequently cause changes in the Friedman equations in Jacobson's approach [17]. In String T-duality and considering the zero-point length correction in the gravitational potential, it is feasible to derive modified Friedmann equations by utilizing the first law of thermodynamics [24,31,32].

In this paper, we intend to investigate cosmological inflation within the context of zero-point length by considering the zero-point length correction in the gravitational potential. Our main purpose is to see the observational status of cosmic inflation with a zero-point length. In this regard, the work will be outlined as follows. In Section 2, we briefly introduce the context of cosmic inflation in order to specify our notations. Then, in Section 3, we review the calculations of the corrected entropy and the modified Friedmann equations. In Section 4, we study the cosmological inflation within the context of zero-point length. In Section 5, we consider different potentials taking into account the zero-point length, and compare our numerical results with Planck2018 TT, TE, EE + lowE + lensing, Planck 2018 TT, TE, EE + lowE + lensing + BK15, and Planck 2018 TT, TE, EE + lowE + lensing + BK15 + BAO data at the %68 and %95 confidence levels (CL). Section 6 is devoted to a summary and conclusion.

2 A short look at the cosmological inflation

It is widely believed that there was an early era before the epoch of nucleosynthesis known as cosmological inflation. The simplest models of inflation involve a single scalar field ϕ which is responsible for inflation, called the inflaton, and its dynamics (minimally) coupled to gravity can be defined through this action [33,34]

$$S=\int d^4x\sqrt{-g}[\frac{1}{2}R+\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-V(\phi)]=S_{EH}+S_\phi, \tag{7}$$

where $V(\phi)$ is the potential of the scalar field, decreasing slowly with time as ϕ rolles slowly down the slope of $V(\phi)$, S_{EH} is the Einstein-Hilbert action, and S_ϕ is the action of the scalar field with a canonical kinetic term.

The energy-momentum tensor for the scalar field is derived by varying the field's action with respect to the metric as

$$T^{(\phi)}_{\mu\nu}=-\frac{2}{\sqrt{-g}}\frac{\delta S_\phi}{\delta g^{\mu\nu}}=-\partial_\mu\phi\partial_\nu\phi-g_{\mu\nu}(\frac{1}{2}\partial^\sigma\phi\partial_\sigma\phi+V(\phi)). \tag{8}$$

Assuming spatially flat FLRW space-time, the energy-momentum tensor reduces to the form of a perfect fluid with the following energy density and pressure.

$$\rho^{(\phi)}=\frac{1}{2}\dot{\phi}^2+V(\phi), \tag{9}$$

$$p^{(\phi)}=\frac{1}{2}\dot{\phi}^2-V(\phi), \tag{10}$$

where the dot signifies the derivative concerning the cosmic time. The equation of state parameter obtains from the above relations as follows

$$w^{(\phi)}=\frac{p^{(\phi)}}{\rho^{(\phi)}}=\frac{\dot{\phi}^2-2V(\phi)}{\dot{\phi}^2+2V(\phi)}. \tag{11}$$

The condition for accelerated expansion is $\ddot{a}>0$ or equivalently $w<-\frac{1}{3}$. When potential energy exceeds kinetic energy, the pressure turns negative, leading to the inequality $(1+3w)<0$ from a slowly rolling scalar field potential.

The Friedman equation for a flat universe in the presence of a scalar field can be derived from the above equations as follows

$$H^2=\frac{1}{3{M_p}^2}(\frac{1}{2}\dot{\phi}^2+V(\phi)), \tag{12}$$

where $H(t) = \frac{\ddot{a}}{a}$ is the Hubble parameter. Varying the action with respect to the homogeneous field $\phi(t)$ leads to the Klein-Gordon equation for the scalar field

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0, \tag{13}$$

where $V'(\phi) = \frac{dV}{d\phi}$. The acceleration equation derives from the effective energy density and pressure as follows

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}G(\rho + 3p) = H^2(1 - \epsilon), \tag{14}$$

where $\epsilon = -\frac{\dot{H}}{H^2}$ called the first Hubble slow-roll parameter linked to the evolution of the Hubble parameter. Accelerated expansion happens when $\epsilon < 1$ and to achieve sufficient inflation, ϵ must not evolve much, so that the condition $\epsilon = 1$, which ends inflation, happens sufficiently late. This is equivalent to the following condition: $\eta < 1$, where by definition $\eta = -\frac{\ddot{\phi}}{H\dot{\phi}}$. The slow-roll conditions can explain an inflationary scenario in the early Universe. The de Sitter limit $p^{(\phi)} \to -\rho^{(\phi)}$ occurs when $\epsilon \to 0$ ($w_{(\phi)} \to -1$), leading to the dominance of potential energy over kinetic energy,

$$\dot{\phi}^2 \ll V(\phi). \tag{15}$$

The following conditions must be met for the accelerated expansion to continue for an adequate time

$$\ddot{\phi} \ll |3H\dot{\phi}|, |V'(\phi)|. \tag{16}$$

In this case, the Klein-Gordon equation for the scalar field becomes as follows in the presence of the slow-roll conditions

$$3H\dot{\phi} = -V'(\phi). \tag{17}$$

The slow-roll parameters can also be written in terms of the inflationary potential used

$$\epsilon_V = \frac{M_p^2}{2}\left(\frac{V'(\phi)}{V}\right)^2, \tag{18}$$

$$\eta_V = M_p^2 \frac{V''(\phi)}{V}. \tag{19}$$

these relations are known as the Potential slow roll parameters, and these conditions $\epsilon_V \ll 1$ and $\eta_V \ll 1$ are the slow-roll conditions for inflation. The relationship between Hubble and Potential slow-roll parameters in the slow-roll approximation is as follows [34]

$$\epsilon \approx \epsilon_V, \qquad \eta \approx \eta_V - \epsilon_V. \tag{20}$$

Inflation ends when one of the slow-roll parameters reaches unity,

$$\epsilon(\phi_{end}) \equiv 1, \quad \text{or} \quad \eta(\phi_{end}) \equiv 1. \tag{21}$$

The number of e-folds describes the rate of the expansion of the universe from the beginning of inflation to the end of inflation

$$N = \int_{t_i}^{t_{end}} H\ dt = \int_{\phi_i}^{\phi_{end}} \frac{H}{\dot{\phi}}\ d\phi \approx \int_{\phi_{end}}^{\phi} \frac{V}{V'}\ d\phi. \tag{22}$$

Investigations show that the total number of e-folds needs to be at least 50-60 to solve the standard hot Big Bang model problems depending on the inflation's potential.

3 Modified Friedmann equations via zero-point length and entropic corrections

Building on Verlinde's seminal work, and incorporating quantum effects on entropy stemming from loop quantum gravity, Sheykhi and Jusufi formulated an entropic-corrected version of Newton's law of gravity, consequently giving rise to modified Friedmann equations for the FRW universe [35–37] as follows modified

$$S = \frac{A}{4} + \mathcal{S}(A), \tag{23}$$

$$F = -\frac{Mm}{R^2}\left[1 + 4\frac{\partial \mathcal{S}}{\partial A}\right]. \tag{24}$$

Predicting a minimum measurable length is an intriguing aspect of quantum gravity, stemming from frameworks such as black hole physics and string theory. It was demonstrated in T-duality in string theory that the gravitational potential in the presence of the zero-point length is modified as [31,38]

$$\Phi(r) = -\frac{Mm}{\sqrt{r^2 + {L_0}^2}}, \tag{25}$$

where L_0 denotes the zero-point length and is anticipated to be on the order of the Planck length. One achieves the modified gravitational Newton's law using this relation

$$F = -\nabla\Phi(r)\Big|_{r=R}, \tag{26}$$

as

$$F = -\frac{Mm}{R^2}\left[1 + \frac{L_0^2}{R^2}\right]^{-\frac{3}{2}}. \tag{27}$$

Within the framework of the standard FRW cosmology for an isotropic and homogeneous spacetime, the line element is defined as follows [39],

$$ds^2 = h_{\mu\nu}dx^{\mu}dx^{\nu} + R^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{28}$$

where

$$R = a(t)r, \qquad h_{\mu\nu} = diag\left(-1, \frac{a^2}{(1 - Kr^2)}\right), \tag{29}$$

where $a(t)$ is the cosmological scale factor as a function of the cosmic time t and the term proportional to the spatial curvature K is an exceedingly small term tightly constrained by observation. The relation describing the dynamic apparent horizon is given as [40]

$$h^{\mu\nu}\partial_{\mu}R\partial_{\nu}R = 0, \tag{30}$$

where the vector ∇R is null on the apparent horizon surface. The direct assessment of the apparent horizon for the FRW universe yields the radius of the apparent horizon as

$$R = ar = \frac{1}{\sqrt{H^2 + \frac{K}{a^2}}}. \tag{31}$$

For a perfect fluid, the energy-momentum tensor is given by

$$T^{\mu}_{\nu} = (\rho + p)u^{\mu}u_{\nu} - p\delta^{\mu}_{\nu}, \tag{32}$$

in which ρ and p are the energy density and pressure of the fluid in the fluid's rest frame and u^μ is its 4-velocity. When the energy-momentum tensor is applied in the FRW universe, the conservation of energy for the metric inevitably results in

$$\dot{\rho} + 3H(\rho + p) = 0, \tag{33}$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter. In what follows, to derive the modified acceleration equation for the dynamical evolution of the FRW universe, we combine Newton's second law for a test particle m on a spherical surface of radius R, equation (31) with the modified Newton's law of gravity, equation (27),

$$F = m\ddot{R} = m\ddot{a}r = -\frac{Mm}{R^2}\left[1 + \frac{L_0^2}{R^2}\right]^{-\frac{3}{2}}. \tag{34}$$

The energy density of matter in volume $V = \frac{4}{3}\pi r^3$ is presented as $\rho = \frac{M}{V}$, then, the last equation can be rewritten as follows

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}\rho\left[1 + \frac{L_0^2}{R^2}\right]^{-\frac{3}{2}}. \tag{35}$$

By employing the entropic correction terms through the zero-point length, we can utilize the active gravitational mass $\mathbf{M}$ instead of the total mass M in achieving the Friedmann equation. Then, using equation (35), it follows

$$\mathbf{M} = -\ddot{a}a^2r^3\left[1 + \frac{L_0^2}{R^2}\right]^{\frac{3}{2}}. \tag{36}$$

Moreover, to determine the active gravitational mass, we can utilize the given definition

$$\mathbf{M} = \int_V dV(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu})u^\mu u^\nu, \tag{37}$$

and then, it can obtained:

$$\mathbf{M} = (\rho + 3p)\frac{4\pi}{3}a^3r^3. \tag{38}$$

From equations (36) and (38), the modified acceleration equation is obtained as

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}(\rho + 3p)\left[1 + \frac{L_0^2}{R^2}\right]^{-\frac{3}{2}}. \tag{39}$$

Multiplying both sides of the equation by $2a\dot{a}$ and applying the conservation of energy equation, results in

$$\dot{a}^2 + K = \frac{8\pi}{3}\int d(\rho a^2)\left[1 + \frac{L_0^2}{R^2}\right]^{-\frac{3}{2}}, \tag{40}$$

where K is the spatial curvature. By expanding the series around L_0 and substituting the density as $\rho = \rho_0 a^{-3(1+w)}$ one be obtains

$$H^2 + \frac{K}{a^2} = \frac{8\pi}{3}\rho\left[1 - \frac{L_0^2}{2}(H^2 + \frac{K}{a^2})\frac{1+3w}{1+w} + \cdots\right]. \tag{41}$$

In the $\lim_{L_0 \to 0}$, the standard Friedmann equation is recovered. To simply equation (41), we define

$$\Gamma \equiv \frac{4L_0^2\pi}{3}\left(\frac{1+3w}{1+w}\right), \tag{42}$$

and therefore it can be rewritten as

$$H^2 + \frac{K}{a^2} = \frac{8\pi}{3}\rho[1 - \Gamma\rho]. \tag{43}$$

From the modified Friedmann equations, in the next section we study the cosmological inflation within the context of zero-point length.

4 Cosmic inflation in the presence of zero-point length

So far, we have discussed cosmological inflation briefly and examined the derivation of the modified Friedmann equations as the main equations for the inflation dynamics. The behavior of the inflaton field is determined by the potential $V(\phi)$ and according to the shape of the potential, the inflation models are classified. In the following, we analyze slow-roll inflation and calculate observable quantities of an inflationary model in the framework of zero-point length. Considering a flat universe with $K = 0$ and also, $8\pi G = \frac{1}{M_p^2}$, we rewrite the modified Friedmann equation with zero-point length as

$$H^2 = \frac{1}{3M_p^2}\rho[1 - \Gamma\rho]. \tag{44}$$

According to the energy density definition

$$\rho(\phi) = \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{45}$$

and satisfying the slow-roll condition

$$\dot{\phi}^2 \ll |V(\phi)|, \tag{46}$$

one finds $\rho(\phi) \simeq V(\phi)$ and then, equation (27) can be rewritten as follows

$$H^2 \simeq \frac{1}{3M_p^2}V(\phi)[1 - \Gamma V(\phi)]. \tag{47}$$

To discuss a specific model of inflation, we firstly consider a power-law potential as a toy model for the scalar potential as $V(\phi) = V_0\phi^n$, where $V_0 = \frac{1}{2}m^2$ and m is the mass of the scalar field. The latest observational data prefer models with $n \approx \mathcal{O}(10^{-1})$ or $n \approx \mathcal{O}(1)$ [41]. We continue with $n \approx \mathcal{O}(1)$ and obtain slow-roll parameters which are defined as Eqs. (18) and (19) as follows

$$\epsilon_V = \frac{M_p^2}{2}(n\phi^{-1} - 2n\Gamma V_0\phi^{n-1})^2, \tag{48}$$

$$\eta_V = M_p^2[n(n-1)\phi^{-2} - 2n(2n-1)\Gamma V_0\phi^{n-2}]. \tag{49}$$

The most intriguing aspect of the inflationary scenario is that cosmological inflation yields predictions that align consistently with observational data. To achieve this, one must scrutinize the theoretical predictions of observable quantities in inflationary models as follows, see for instance [34,42,43]. At this stage, it is crucial to note that the effect of the zero-point length shows itself essentially in the modification of the Hubble parameters as is seen in equation (47). The other point is that presence of the zero-point length modifies the geometric part of the theory and NOT the matter part of the field equations (here the inflaton field and its potential). So, we can use the definition of quantities derived in the standard inflation setup with a minimally coupled scalar field in the absence of the zero-point length, but with this point in mind that the Hubble parameter is now modified via equation (47). The power spectrum of the scalar and tensor fluctuations are respectively as follows

$$P_s(k) = \Delta_s^2(k) = \frac{H^2}{8\pi^2 M_p^2 \epsilon}, \tag{50}$$

$$P_t(k) = \Delta_t^2(k) = \frac{2H^2}{\pi^2 M_p^2}. \tag{51}$$

Substituting equation (47) and power-law potential in the above equations results

$$P_s(k) = \Delta_s^2(k) = \frac{1}{24\pi^2 M_p^4 \epsilon} V_0 \phi^n [1 - \Gamma V_0 \phi^n], \tag{52}$$

$$P_t(k) = \Delta_t^2(k) = \frac{2}{3\pi^2 M_p^4} V_0 \phi^n [1 - \Gamma V_0 \phi^n]. \tag{53}$$

The tensor-to-scalar ratio is given by the relation

$$r = \frac{\Delta_t^2(k)}{\Delta_s^2(k)} = 16\epsilon, \qquad \epsilon \approx \epsilon_V. \tag{54}$$

By considering the zero-point length, we obtain

$$r = 8M_p^2 (n\phi^{-1} - 2n\Gamma V_0 \phi^{n-1})^2. \tag{55}$$

The scalar spectral index is defined as

$$n_s = 1 + \frac{\mathrm{dln}(\Delta_s^2)}{\mathrm{dln}k} = 1 - 6\epsilon_V + 2\eta_V, \tag{56}$$

using the relations (48) and (49), we find finally

$$n_s = 1 - 3M_p^2 (n\phi^{-1} - 2n\Gamma V_0 \phi^{n-1})^2 + 2M_p^2 [n(n-1)\phi^{-2} - 2n(2n-1)\Gamma V_0 \phi^{n-2}]. \tag{57}$$

The tensor spectral index is defined as

$$n_t = \frac{\mathrm{dln}(\Delta_t^2)}{\mathrm{dln}k} = -2\epsilon_V, \tag{58}$$

whereby using relation (53) and the slow-roll parameters, is written as follows

$$n_t = -M_p^2 (n\phi^{-1} - 2n\Gamma V_0 \phi^{n-1})^2. \tag{59}$$

The number of e-folds, indicated by N, is expressed as follows

$$N = \int_{t_i}^{t_{end}} H dt = \int_{\phi_{end}}^{\phi_i} \frac{V}{V'} d\phi. \tag{60}$$

By considering the power-law potential, we obtain

$$N = \int_{\phi_{end}}^{\phi_i} \frac{V_0\phi^n - \Gamma V_0^2\phi^{2n}}{nV_0\phi^{n-1} - 2n\Gamma V_0^2\phi^{2n-1}} d\phi. \tag{61}$$

Up to this point, we have obtained the general form of the main background equations and also slow-roll and perturbation parameters in the presence of the zero-point length. In the next section, we compare this model with Planck's 2018 results for several inflation potentials to see the observational viability of this model for different potentials and also for constraining the value of the zero-point length. We note that since we set $L_{Planck} = 1$ in what follows, the values of the zero-point length will be stated in units of L_{Planck}.

For the sake of completeness and before starting our numerical analysis, as we have mentioned it is important to note that the effects of the zero-point length shows itself in the modification of the geometric part of the Einstein field equations and therefore Friedmann equation via the Hubble parameter. This can be seen in equation (47) where the effect of the Zero-point length is encoded in the second term on the right hand side with Γ defined as in equation (42). This modification propagates through subsequent equations including equations (50) and (51) along with (54) and (56). In another words, existence of a zero-point length has nothing to do with the scalar field and its potential; it affects the geometric part of the theory which in our context is encoded in the modification of the Hubble parameter as equation (47). Nevertheless, we can proceed in another fashion to see the effect of zero-point length as follows (see Refs. [42,44,45]).

We consider the effect of the existence of a zero-point length through the following Generalized Uncertainty Relation (Principle)

$$\Delta x \Delta p \geq \frac{\hbar}{2}(1 + \beta(\Delta p)^2), \tag{62}$$

which gives nontrivially a zero-point length as $L_0 = \hbar\sqrt{\beta}$. By taking quantum gravity into account, essentially there is a fundamental energy scale, Λ (Planck or String scale), that quantum gravity effects become important [46]. To include this fundamental scale into our analysis, we define a conformal time as

$$\tau = -\frac{1}{aH}. \tag{63}$$

Then, the physical momentum P and the comoving wave number k are related as usual as

$$k = aP = -\frac{P}{\tau H}. \tag{64}$$

In this respect, the conformal time that the new physics (due to dominance of quantum gravity) governs is given by

$$\tau_0 = -\frac{\Lambda}{kH}\,, \tag{65}$$

where Λ is the Planck energy scale. Now, the existence of a zero-point length requires the change in comoving wave number before the conformal time τ_0 as $k \to k(1 + \beta k^2)$. In this situation, the equation governing the evolution of the perturbations in the inflation period is as follows [42]

$$\mu_k'' + (k^2 - \frac{a''}{a})\mu_k = 0\,, \tag{66}$$

where a prime marks differentiation with respect to the conformal time and by definition $\mu = a\delta\phi$.

Now, the scalar spectral index in the presence of the zero-point length gets the following modified form

$$n_s - 1 = \frac{d\ln P_s}{d\ln k(1+\beta k^2)}, \tag{67}$$

where P_s is the amplitude of the scalar perturbation. Therefore we find for the scalar spectral index

$$n_s = \frac{1+\beta k^2}{1+3\beta k^2}\frac{d\ln P_s}{d\ln k} \simeq (1-2\beta k^2)\frac{d\ln P_s}{d\ln k} + 1. \tag{68}$$

As an interesting result, we see that the scalar spectral index is not scale invariant in the presence of the zero-point length. This is also the case when we proceed with the modified Hubble parameter as equation (47). Now, at the horizon crossing we have [42,46]

$$\frac{dH}{dk} = -\frac{\epsilon H}{k}\,, \tag{69}$$

where ϵ is the first slow-roll parameter. By setting $k \to k(1+\beta k^2)$, we find

$$H \simeq k^{-\epsilon} e^{-\beta\epsilon k^2}. \tag{70}$$

Since

$$P_t(k) = \frac{1}{a^2}\langle|\mu_k(\tau)|^2\rangle\,, \tag{71}$$

following Ref. [46] we find

$$P_t(k) = \left(\frac{H}{2\pi}\right)^2\left(1 - \frac{H}{\Lambda}\sin(\frac{2\Lambda}{H})\right). \tag{72}$$

The second term is directly from the contribution of the zero-point length. Similarly, for scalar density fluctuations, we have

$$P_s(k) = \left(\frac{H}{\dot{\phi}}\right)^2\left(\frac{H}{2\pi}\right)^2\left(1 - \frac{H}{\Lambda}\sin(\frac{2\Lambda}{H})\right). \tag{73}$$

So, the tensor-to-scalar ratio in this setup is given by

$$\frac{P_t}{P_s} = \left(\frac{\dot{\phi}}{H}\right)^2 = \left(\frac{16\pi\sqrt{\epsilon}V}{M_p k^{-\epsilon}e^{-\beta\epsilon k^2}}\right)^2. \tag{74}$$

With these theoretical inputs, we perform our numerical analysis on the parameter space of the model in the next section. The results are the same as the previous approach since the contribution of the zero-point length is essentially very tiny.

5 Comparision of some selected zero-point length inflation models with Planck's 2018 results

In an inflationary scenario, it is noteworthy how cosmic inflation aligns its predictions with observational data. Therefore, examining the theoretical forecasts of the suggested inflationary models for observables is essential. Building on the previous section, we performed a numerical analysis of the scalar spectral index and tensor-to-scalar ratio for a power-law potential, focusing on the 50 to 60 e-folds where inflation is expected to end. Our goal is to determine the zero-point length value that yields observable values for r and n_s. The outcome is displayed in Figure 1. To plot this figure, we have utilized the constraints $n_s = 0.9649 \pm 0.0042$ at %68 CL and $r < 0.035$ at %95 CL from Planck 2018 TT, TE, EE+lowE+lensing, Planck 2018 TT, TE, EE+lowE+lensing+BK15 and Planck 2018 TT, TE, EE+lowE+lensing+BK15+BAO [7] for the potential $V(\phi) = V_0\phi^n$ with $V_0 = (6 \times 10^{-4} M_p)^3$ and $n = 1$. Based on our analysis, as is seen in Figure 1, the impact of zero point length on power-law potential is intriguing. Without this minimum length, the model aligns solely with Planck 2018 TT, TE, EE+lowE+lensing at %95 CL. But, by taking the zero-point length into account and setting it $L_0 = 5000$ (in the units of the Planck Length, $L_{Planck} = 1$) the model is consistent with both Planck 2018 TT, TE, EE+lowE+lensing+ BK15, and Planck 2018 TT, TE, EE+lowE+lensing+ BK15 +BAO at %68 CL. We have also demonstrated the effect of values close to zero-point length, exclusively for this potential and also for the Chaotic Inflation model.

$$V(\phi) = V_0\phi^n. \tag{75}$$

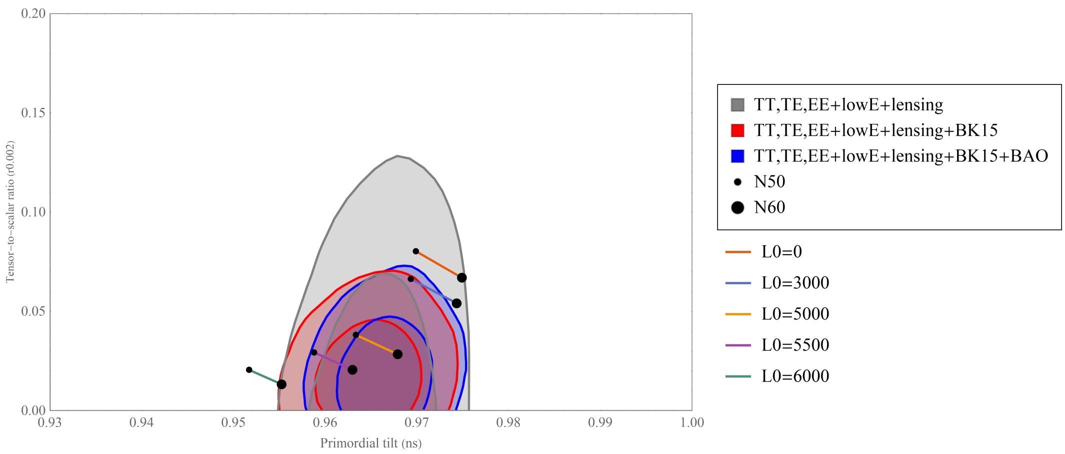

Figure 1: $r - n_s$ diagram for Power Law model with $V(\phi) = V_0\phi^n$ with $n = 1$ in the background of the several observational datasets.

In what follows, we explore various potentials (see Ref. [47]) in the presence of the zero-point length and present our findings in the figures and finally a table.
We have fully investigated the Starobinsky inflation model (SI) (1980)[7,48] with potential defined as

$$V(\phi) = M^4 \left(1 - \exp(-\sqrt{\frac{2}{3}}\frac{\phi}{M_P})\right)^2. \tag{76}$$

In our analysis, we have set the Planck mass as $M_p = 1$, while the parameter M is the mass scale, is fixed by the CMB normalization. Regardless of the zero-point length, the Starobinsky Inflation model aligns with the %68 CL of the Planck data. Considering the zero-point length, it is observed that the potential behavior remains consistent with the mentioned data and unchanged. This constancy indicates that no noticeable variations or fluctuations occur with any alterations made to the value of the zero-point length.

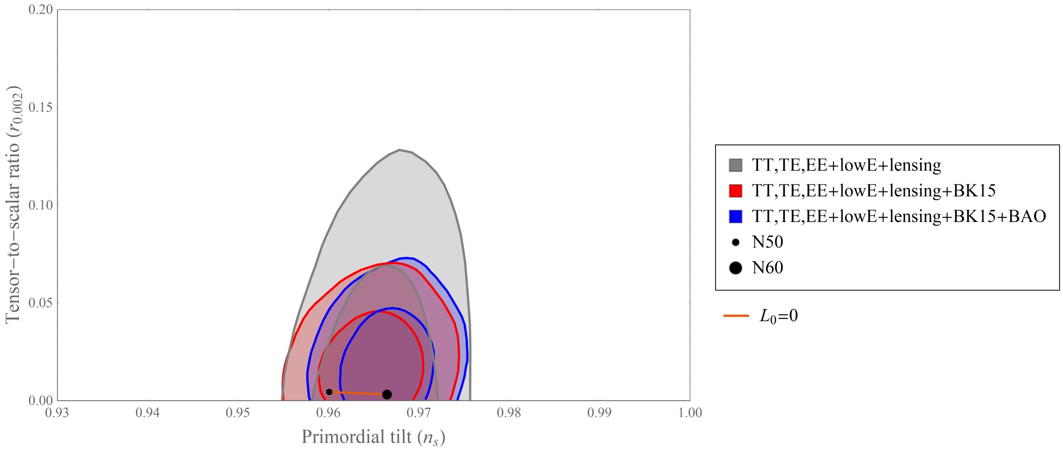

Figure 2: $r - n_s$ diagram for Starobinsky Inflation with $V(\phi) = M^4(1 - \exp(-\sqrt{\frac{2}{3}}\frac{\phi}{M_P}))^2$ in the background of the several observational datasets.

We have also achieved this result for the Kähler Moduli Inflation I (KMII) and Exponential SUSY Inflation (ESI).

$$V^{(KMII)}(\phi) = M^4(1 - \alpha\frac{\phi}{M_p}\exp(-\frac{\phi}{M_p})), \quad (77)$$

$$V^{(ESI)}(\phi) = M^4(1 - \exp(-\frac{q}{M_p}\phi)), \quad (78)$$

where α is a free positive coefficient and q is a free parameter [49]. Here we set $\alpha = 2.5$ and $q = 1$.
In our analysis, we employ the following models corresponded to the Large Field Inflation (LFI), Mixed Large Field Inflation (MLFI), Radion Gauge Inflation (RGI), and Radiatively Corrected Higgs Inflation (RCHI) with potentials defined respectively as

$$V^{(LFI)}(\phi) = M^4(\frac{\phi}{M_p})^p, \qquad \text{with} \quad p = 2, \quad (79)$$

$$V^{(MLFI)}(\phi) = M^4(\frac{\phi}{M_p})^2(1 + \alpha\frac{\phi^2}{M_p^2}), \qquad \text{with} \quad \alpha = 10^{-6}, \quad (80)$$

$$V^{(RGI)}(\phi) = M^4\frac{(\frac{\phi}{M_p})^2}{\alpha + (\frac{\phi}{M_p})^2}, \qquad \text{with} \quad \alpha = 0.5, \quad (81)$$

$$V^{(RCHI)}(\phi) = M^4(1 - 2\exp(-\frac{2}{\sqrt{6}}\frac{\phi}{M_p}) + \frac{A_1}{16\pi^2}\frac{\phi}{\sqrt{6}M_p}), \qquad \text{with} \quad A_1 = 100, \quad (82)$$

where p is the free parameter that can be constrained by observational data. Also, α is a positive dimensionless parameter, and A_1 is the inflationary anomalous scaling. These models are not consistent with observation without the presence of the zero-point length. But considering the zero-point length of $\mathcal{O}(10^4)$ for the first three models and $\mathcal{O}(10^6)$ for the last model, these models lie inside the contour plot of %95 confidence level of the Planck 2018 TT, TE, EE+lowE+lensing joint data as are shown in Figures 3 and 4.

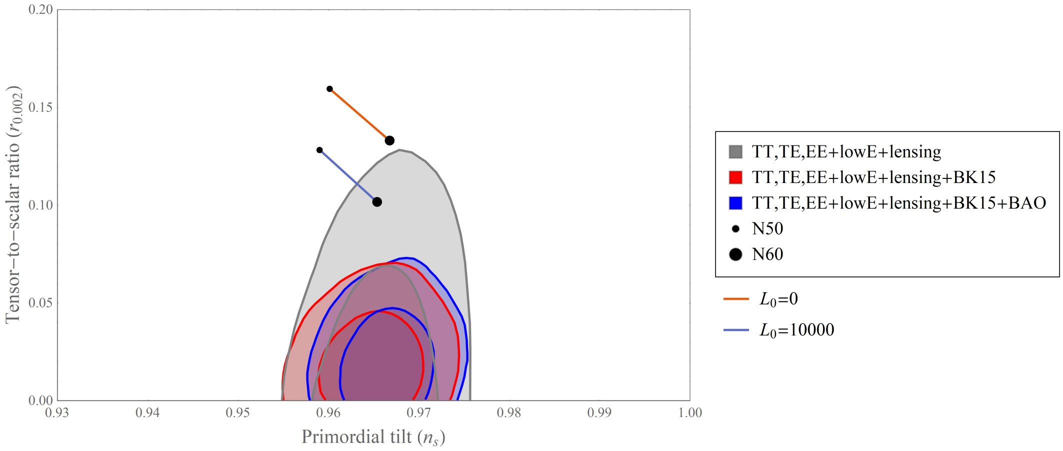

Figure 3: $r-n_s$ diagram for LFI model with $V(\phi)=M^4(\frac{\phi}{M_p})^p$, MLFI model with $V(\phi)=M^4(\frac{\phi}{M_p})^2(1+\alpha\frac{\phi^2}{M_p^2})$ and RGI model with $V(\phi)=M^4\frac{(\frac{\phi}{M_p})^2}{\alpha+(\frac{\phi}{M_p})^2}$ in the background of several observational datasets.

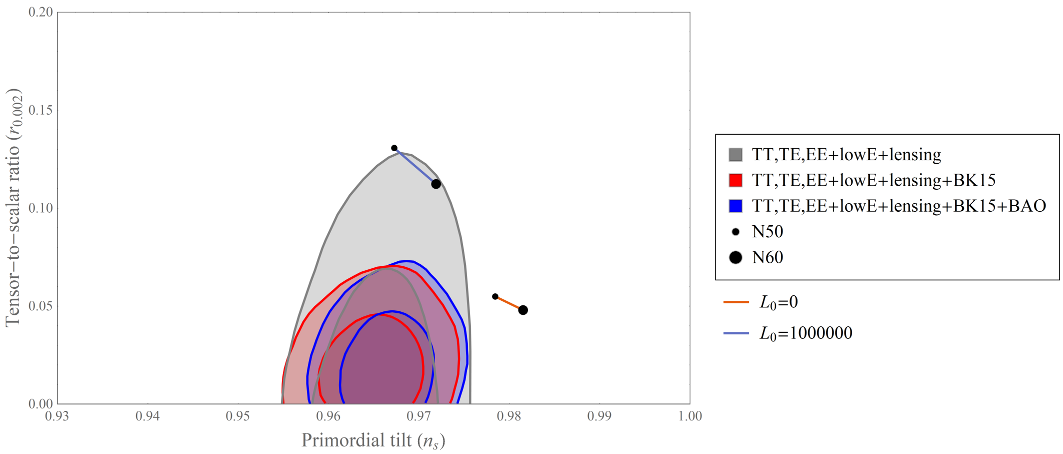

Figure 4: $r-n_s$ diagram for RCHI model with $V(\phi)=M^4(1-2\exp(-\frac{2}{\sqrt{6}}\frac{\phi}{M_p})+\frac{A1}{16\pi^2}\frac{\phi}{\sqrt{6}M_p})$ in the background of the several observational datasets

We have achieved this result for Chaotic Inflation, that is $n=2$, and $m=1.4\times10^{13}$ GeV, as well [50]. But by considering the zero-point length and setting it $L_0=17000$ (in the units of the Planck Length, $L_{Planck}=1$) the model lies inside the contour plot of %95 confidence level of the Planck 2018 TT, TE, EE+lowE+lensing joint data as is shown in Figure 5.

$$V(\phi)=\frac{1}{2}m^2\phi^2. \tag{83}$$

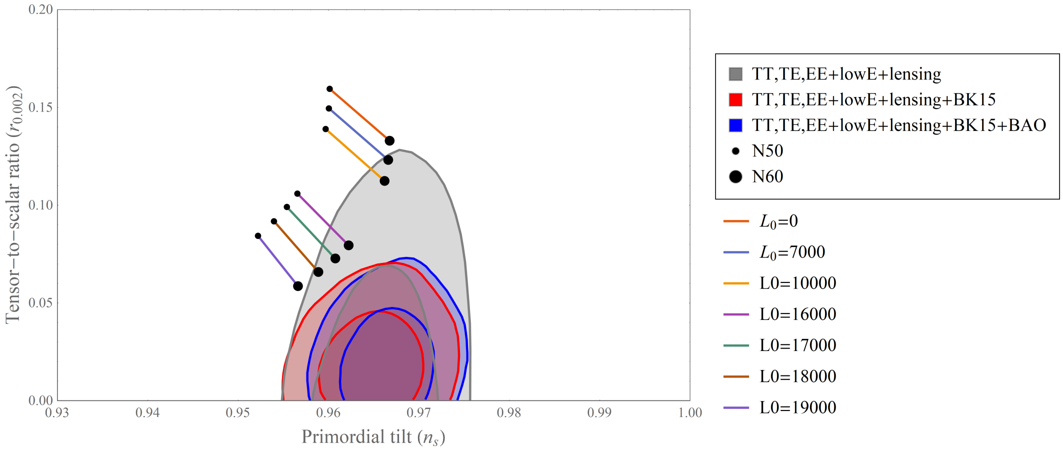

Figure 5: $r - n_s$ diagram for Chaotic Inflation model with $V(\phi) = \frac{1}{2}m^2\phi^2$ in the background of the several observational datasets

Finally, we found models that were not consistent with the observations for any value of the zero-point length. Among these models, Natural Inflation (NI), Small Field Inflation (SFI), and Generalized Mixed Inflation (GMLFI) with the following potentials can be mentioned,

$$V^{(NI)}(\phi) = M^4[1 + cos[\frac{\phi}{f}]], \tag{84}$$

$$V^{(SFI)}(\phi) = M^4(1 - (\frac{\phi}{\mu})^p), \quad \text{with} \quad p = 2, \quad \mu = M_p, \tag{85}$$

$$V^{(GMLFI)}(\phi) = M^4(\frac{\phi}{M_p})^p)[1 + \alpha(\frac{\phi}{M_p})^q], \quad \text{with} \quad p = 3, \quad q = 2, \quad \alpha = 0.1, \tag{86}$$

where f is a free parameter, and its scale must be very close to M_P to avoid fine-tuning the initial conditions. α, p, and q are three dimensionless positive parameters and μ is the mass scale. The adopted values for these quantities in numerical analysis of the corresponding models are presented explicitly.
Table 1 presents the acceptable values of the inflationary observables for each inflation model based on the different values of L_0 as we have obtained in our numerical analysis. We note again that the values of L_0 are in the units of the Planck Length with $L_{Planck} = 1$.

6 Summery and conclusion

In this paper, we analyzed the consistency of some zero-point length inflationary models with observational data. We reviewed the basic theoretical framework of cosmological inflation within a flat FLRW spacetime. Then, we discussed how zero-point length modifies the Friedmann equations. This analysis focuses essentially on how azero-point length affects the entropy of the cosmological apparent horizon. By applying Verlinde's entropic force scenario and correcting the gravitational potential, we presented the modified Newton's law of gravity and the modified Friedmann equations. These modified Friedmann equations were utilized to see how a zero-point length affects the basic equations and observables of the inflationary cosmology. In this work, at the first step by considering power law potential

Table 1: Values of the parameter L_0 (in units of $L_{Planck} = 1$) in which the tensor-to-scalar ratio and scalar spectral index of various inflation models align with different observational datasets.

Inflationary model	Potential $V(\phi)$	L_0	$N = 50$	$N = 50$	$N = 60$	$N = 60$
PLI	$V = V_0\phi^n, \ (n = 1)$	5000	n_s= 0.9633	r= 0.0382	n_s= 0.9679	r= 0.0284
SI	$V(\phi) = M^4(1 - \exp(-\sqrt{\frac{2}{3}}\frac{\phi}{M_P}))^2$	0	n_s=0.9600	r=0.0045	n_s=0.9665	r=0.0032
KMII	$V(\phi) = M^4(1 - \alpha\frac{\phi}{M_p}\exp(-\frac{\phi}{M_p}))$	0	n_s=0.9600	r=0.0045	n_s=0.9665	r=0.0032
ESI	$V(\phi) = M^4(1 - \exp(-\frac{q}{M_p}\phi))$	0	n_s=0.9600	r=0.0045	n_s=0.9665	r=0.0032
LFI	$V(\phi) = M^4(\frac{\phi}{M_p})^p$	10^4	n_s= 0.9590	r=0.1282	n_s=0.9654	r=0.1016
MLFI	$V(\phi) = M^4(\frac{\phi}{M_p})^2(1 + \alpha\frac{\phi^2}{M_p^2})$	10^4	n_s=0.9590	r=0.1283	n_s=0.9654	r=0.1016
RGI	$V(\phi) = M^4\frac{(\frac{\phi}{M_p})^2}{\alpha+(\frac{\phi}{M_p})^2}$	10^4	n_s=0.9590	r=0.1283	n_s=0.9654	r=0.1016
RCHI	$V(\phi) = M^4(1 - 2\exp(-\frac{2}{\sqrt{6}}\frac{\phi}{M_p}) + \frac{A1}{16\pi^2}\frac{\phi}{\sqrt{6}M_p})$	10^6	n_s=0.9673	r=0.1306	n_s=0.9719	r=0.1123
CI	$V(\phi) = \frac{1}{2}m^2\phi^2$	17000	n_s= 0.9554	r= 0.0991	n_s= 0.9607	r= 0.0727
NI	$V(\phi) = M^4[1 + cos[\frac{\phi}{f}]]$	-	*Not consistent*	*Not consistent*	*Not consistent*	*Not consistent*
SFI	$V(\phi) = M^4(1 - (\frac{\phi}{\mu})^p)$	-	*Not consistent*	*Not consistent*	*Not consistent*	*Not consistent*
GMLFI	$V(\phi) = M^4(\frac{\phi}{M_p})^p)[1 + \alpha(\frac{\phi}{M_p})^q]$	-	*Not consistent*	*Not consistent*	*Not consistent*	*Not consistent*

as a toy model, we derived the main background equations, the slow-roll parameters and essential inflation observables such as the scalar spectral index (n_s) and the tensor-to-scalar ratio (r) in the presence of the zero-point length. Then, by analyzing $r - n_s$ behavior in the background of Planck 2018 TT, TE, EE +lowE +lensing +BK15+ BAO data, and assuming an inflation duration of $50 \leq N \leq 60$, we found that this model is consistent with Planck 2018 TT, TE, EE+lowE+lensing at %95 confidence level (CL) without zero-point length. Setting $L_0 = 5000$ (in units of the Planck Length with $L_{Planck} = 1$), we obtained n_s=0.9633, r=0.0382 for $N = 50$, and $n_s = 0.9679, r = 0.0284$ for $N = 60$ consistent with Planck 2018 TT, TE,EE+lowE+lensing+ BK15+BAO at %68 CL. Then, to extend our study, we examined various potentials regarding zero-point length, identified inflationary observables, and evaluated their consistency with observational data. We can categorize the results as follows:

Some potentials are consistent with Planck 2018 TT, TE, EE + lowE + lensing data at %95 CL without considering the zero-point length, but when the zero-point length is taken into account, they align with Planck 2018 TT, TE, EE + lowE + lensing + BK15 + BAO just at %68 CL. For instance, our modified toy model with $L_0 = 5000$ demonstrates this feature. Additionally, some potentials, such as Starobinsky inflation, Kähler Moduli inflation, and Exponential SUSY inflation, are consistent with Planck 2018 TT, TE, EE+lowE+lensing+ BK15+BAO at %68 CL without including L_0.The inflation models for these potentials unchanged for all values of L_0. Some models failed to fit observational data without zero-point length, but setting a zero-point length around 10^4 and 10^6 and also 17000 for Chaotic Inflation renders them viable with Planck 2018 TT, TE, EE +lowE +lensing at %95 CL. Notable examples include Large Field Inflation, Mixed Large Field Inflation, Radion Gauge Inflation, Radiatively Corrected Higgs Inflation, and Chaotic Inflation. Finally, we identified a group of models that did not align with observations without a zero-point length. Additionally, we could not find any value for L_0 to fit these last models to the observations data.

Authors' Contributions

All authors have the same contribution.

Data Availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare no potential conflicts of interest.

Ethical Considerations

The authors have diligently addressed ethical concerns, such as informed consent, plagiarism, data fabrication, misconduct, falsification, double publication, redundancy, submission, and other related matters.

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References

[1] Guth, A. H. 1981, Phys. Rev. D, 23, 347.

[2] Linde, A. D. 1982, Physics Letters B, 108, 389.

[3] Kolb, E. 2018, The early universe, CRC press.

[4] Lyth, D. H., & Liddle, A. R. 2009, The primordial density perturbation: Cosmology, inflation and the origin of structure, Cambridge university press.

[5] Baumann, D. 2022, Cosmology, Cambridge University Press.

[6] Lyth, D. H., & Riotto, A. 1999, Phys. Rep., 314, 1.

[7] Akrami, Y., et al. 2020, A&A, 641, A10.

[8] Fontanini, M., Spallucci, E., & Padmanabhan, T. 2006, Physics Letters B, 633, 627.

[9] Garay, L. J. 1995, International J. Modern Physics A, 10, 145.

[10] Maggiore, M. 1993, Physics Letters B, 304, 65.

[11] Monk, N. A. M. 1998, Endeavour, 22, 27.

[12] Padmanabhan, T. 2010, Reports on Progress in Physics, 73, 046901.

[13] Hawking, S. W. 1975, Commun. Math. Phys., 43, 199.

[14] Bekenstein, J. D. 1972, Commun. Math. Phys., 43, 199.

[15] Bekenstein, J. D. 1973, Phys. Rev. D, 7, 2333.

[16] Bekenstein, J. D. 1974, Phys. Rev. D, 9, 3292.

[17] Jacobson, T. 1995, Phys. Rev. Lett., 75, 1260.

[18] Verlinde, E. 2011, J. High Energy Physics, 2011, 1.

[19] Padmanabhan, T. 2012, [arXiv:1206.4916 [hep-th]].

[20] ’t Hooft, G. 1993, [arXiv:gr-qc/9310026].

[21] Susskind, L. 1995, J. Mathematical Physics, 36, 6377.

[22] Padmanabhan, T. 1985, Annals of Physics, 165, 38.

[23] Medved, A. J. M., & Vagenas, E. C. 2004, Phys. Rev. D-Particles, Fields, Gravitation, and Cosmology, 70, 124021.

[24] Nicolini, P., Spallucci, E., & Wondrak, M. F. 2019, Physics Letters B, 797, 134888.

[25] Adler, R. J., Chen, P., & Santiago, D. I. 2001, General Relativity and Gravitation, 33, 2101.

[26] Cavaglia, M., Das, S., & Maartens, R. 2003, Classical and Quantum Gravity, 20, L205.

[27] Cavaglia, M., & Das, S. 2004, Classical and Quantum Gravity, 21, 4511.

[28] Majumder, B. 2011, Physics Letters B, 703, 402.

[29] Ali, A. F. 2012, J. High Energy Physics, 2012, 1.

[30] Ali, A. F., Nafie, H., & Shalaby, M. 2012, Europhysics Letters, 100, 20004.

[31] Nicolini, P. 2022, General relativity and gravitation, 54, 106.

[32] Luciano, G. G., & Sheykhi, A. 2024, The European Physical Journal C, 84, 682.

[33] Riotto, A. 2002, [arXiv:hep-ph/0210162].

[34] Baumann, D. 2009, [arXiv:0907.5424v2 [hep-th]].

[35] Sheykhi, A. 2010, Phys. Rev. D-Particles, Fields, Gravitation, and Cosmology, 81, 104011.

[36] Sheykhi, A. 2021, Phys. Rev. D, 103, 123503.

[37] Jusufi, K., & Sheykhi, A. 2023, Physics Letters B, 836, 137621.

[38] Gaete, P., Jusufi, K., & Nicolini, P. 2022, Physics Letters B, 835, 137546.

[39] Akbar, M., & Cai, R.-G. 2007, Phys. Rev. D-Particles, Fields, Gravitation, and Cosmology, 75, 084003.

[40] Cai, R.-G., & Kim, S. P. 2005, J. High Energy Physics, 2005, 050.

[41] Luciano, G. G. 2023, The European Physical Journal C, 83, 1.

[42] Liddle, A. R., & Lyth, D. H. 2000, Cosmological inflation and large-scale structure, Cambridge University Press.

[43] Schwarz, D. J., Terrero-Escalante, C. A., & Garcıa, A. A. 2001, Physics Letters B, 517, 243.

[44] Ashoorioon, A., Kempf, A., & Mann, R. B. 2005, Phys. Rev. D-Particles, Fields, Gravitation, and Cosmology, 71, 023503.

[45] Heydarzadeh, M., Mohsenzadeh, M., Abbasiyan-Motlaq, M., & Yusofi, E. 2022, Modern Physics Letters A, 37, 2250191.

[46] Danielsson, U. H. 2002, Phys. Rev. D, 66, 023511.

[47] Martin, J., Ringeval, C., & Vennin, V. 2014, Physics of the Dark Universe, 5, 75.

[48] Ellis, J., Nanopoulos, D. V., & Olive, K. A. 2013, Phys. Rev. Lett., 111, 111301.

[49] Martin, J., Ringeval, C., Trotta, R., & Vennin, V. 2014, J. Cosmology Astropart. Phys., 2014, 039.

[50] Mantziris, A., Markkanen, T., & Rajantie, A. 2021, J. Cosmology Astropart. Phys., 2021, 077.