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Research Paper

Electromagnetic Instability in Plasma with Kappa Distribution Function in the Presence of Coulomb Collision

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Abstract. In this research, an analytical expression of the Weibel electromagnetic instability growth rate is investigated for strongly coupled plasma in the presence of Coulomb collisions in the beam-plasma interaction under a low-frequency wave condition. In this regard, the distribution function governing the relativistic beam and plasma particles has been considered as semi-Maxwellian and Kappa distribution functions respectively. The effect of the temperature anisotropy parameter, the spectral index, quantum and relativistic parameters on Weibel electromagnetic instability growth rate have been investigated in collisional and non-collisional states.The obtained results show that the Coulomb collision frequency of particles plays an important role in suppressing the unstable modes in isotropic plasmas due to increase in the free energy of the plasma. Therefore, it was concluded that the Weibel instability growth rate in collision state has a more stable situation than in the non-collisional state in the strongly coupled plasma with Kappa distribution function.

Keywords: Weibel electromagnetic instability, Coulomb collision, Kappa distribution function, Temperature anisotropy

1 Introduction

Classical plasma is usually characterized by a relatively small de-Broglie wavelength in the range of low density and high temperatures. The quantum effects arise if the wavelength of the two particles is comparable to the distance between the particles. Quantum effects play

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an important role in the dynamics of quantum plasma [1]. Quantum plasma is a system of charged particles in which collective and quantum effects are dominant due to duality and quantum statistics [2]. In recent years, the effect of quantum properties of plasma on physical diversity in macroscopic and microscopic samples of plasma has drawn interest towards quantum plasma. Discussions about quantum plasma in the range of high density and low temperature were first raised by Pines in the 1960s [3]. Quantum plasma can be seen in various fields such as laser-solid dense experiments like in neodymium lasers, Spintronics semiconducting nanostructures, white dwarfs and most importantly inertial confinement fusion [4–8]. The Fast Ignition Scheme (FIS) is a variant of the confinement fusion process in which a super-pulsed laser actuator $10^{19}\ {}^{w}\!/_{cm^2}$ is used to ignite a preconcentrated target. Weibel instability is one of the electromagnetic instabilities of plasma, which prevents the deposition of beam energy in the fuel pellet, as a result, the energy loss in the fusion process increases. In the following years, many research studies have included the study of Weibel instability with non-Maxwellian distribution function for different distribution functions including the Kappa distribution function, generalized distribution function, etc. Growth rate changes with different distribution function parameters were also investigated [9]. Moreover, the analysis of the quantum effect in the dense quantum plasma shows that the quantum effect tends to stabilize the Weibel instability in the hydrodynamic regime [10]. The study of the effect of the dense gradient on the growth rate of the relativistic Weibel instability indicates that the growth rate of the Weibel instability increases with the increase in gradient density. Also, the growth rate decreases with the increase of the relativistic parameter and relativistic mass of the electron beam [11]. The effect of ion-electron collision on the Weibel instability for dense non-magnetic anisotropic plasma with bi- Maxwellian distribution function shows that, the growth rate of the instability increases due to the reduction of the Coulomb collision frequency [12]. In 2007, Hans found that the quantum effects decrease when growth rate increases; in other words, the quantum effects play a stabilizing role [13]. Similarly, other studies were conducted on the growth rate of Weibel instability [14–18]. Strongly coupled plasmas (or non-ideal plasmas) are charged multicomponent systems in which the average potential energy of the system is equal to or even higher than the average kinetic energy. Strong coupled plasma with high energy density is necessary in projects such as magnetic flux generators, plasma generators, powerful sources of light radiation and pulsed thermal reactors with confinement Inertia and etc. The quantum coupling parameter is the ratio of the interaction energy to the average Fermi energy ($g_Q = \frac{E_{\text{int}}}{E_F} \sim \left(\frac{\hbar w_p}{E_F}\right)^2$). The distribution functions that are seen a lot in natural and laboratory plasmas are non-Maxwellian distribution functions [19]. Non-Maxwellian distribution, which is the high-energy sequence in the velocity distribution and is caused by the presence of superheated electrons, is the Kappa distribution. In addition to the mentioned articles, Weibel instability for non-Maxwellian distributions has also been studied and analyzed in recent years. Among other things, in 2021, the Weibel instability beyond the semi-Maxwell anisotropy was investigated by T. Silva and N. Rubab in 2016 [20]. Weibel instability in space plasmas was analyzed in comparison of three-particle distributions with generalized kappa and bi-kappa distributions [21]. Also, other researches were done with non-Maxwellian functions [22,23]. Until recently, research on plasma instabilities was based on kinetic theory, relying on Maxwellian distribution functions. The existence of non-thermal particle distributions at high altitudes in the solar wind and many space plasmas has been widely confirmed by spacecraft [24]. The results obtained show that the particle distribution function differs from the Maxwellian distribution. The velocities of space plasma particles often follow kappa distribution functions with high-energy tails. The kappa index is an important parameter in understanding plasma dynamics [25]. On the other hand, the accurate determination of kappa distribution functions and a wide range of energies is very important for understanding the physical mechanisms. However, it is very important to quantify the uncertainty of the obtained plasma mass parameters, which determines the level of scientific results [26]. The generation of kappa distributions near the solar wind plasma at 1 AU was investigated. A specific relationship between the properties of solar wind plasmas, the interplanetary magnetic field and the kappa index is revealed [27]. Solar wind modeling the energy exchange between plasma particles and low-frequency Alfvén waves was considered. For such plasmas, the kappa distribution function is very useful. Also, the electron velocity distribution in solar winds was investigated using the Maxolin distribution and the kappa distribution [28]. As a result, in theoretical models to describe waves and instabilities for natural plasmas around the Earth such as the magnetosphere around the Earth, space plasmas, and solar winds, the best distribution function is the kappa distribution. In this paper, we have used the semi-Maxwell distribution function [14] for the relativistic beam and the Kappa distribution [29] for the plasma in the beam-plasma coupled model. We investigated the importance of the Coulomb collision effect on the growth rate of electromagnetic Weibel instability for the beam-plasma model.

2 Observations of the Theoretical Model

The governing equation of strongly coupled plasma, considering the effect of Coulomb collision between particles and using Maxwell's equations is the following

$$\left(\frac{\partial f_1}{\partial t}\right) + \vec{v}.\frac{\vec{\nabla} f_1}{\partial \overrightarrow{r}} + q\left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right).\frac{\partial f_0}{\partial \overrightarrow{p}} = -v_{ei}(f - f_0), \tag{1}$$

$$\vec{\nabla} \times \vec{E} = -\tfrac{1}{c}\left(\tfrac{\partial \vec{B}}{\partial t}\right), \tag{2}$$

$$\vec{\nabla} \times \vec{B} = -\tfrac{4\pi}{c}\vec{J} + \tfrac{1}{c}\left(\tfrac{\partial \vec{E}}{\partial t}\right). \tag{3}$$

In the above equations, the total electrons distribution function (f) is the sum of the equilibrium and perturbed distribution function at position $\overrightarrow{r}$, $f = f_0 + f_1$, $\overrightarrow{P} = m\overrightarrow{V}$ is the momentum. and v_{ei} is ion – electron Coulomb collision frequency. The quantities c, $\overrightarrow{J}$, $\overrightarrow{E}$, $\overrightarrow{B}$ and $\overrightarrow{v}$ are the velocity of light, the current density, perturbed electric, magnetic fields and the velocity of particles respectively.

By using the Vlasov equation with the collision term for the relativistic beam and combining it with the Wigner-Maxwell function for the quantum plasma, the general linear dispersion relation will be obtained as follows,

$$\begin{aligned}
&\omega^2 - c^2k^2 - \omega_{be}^2 - \pi\omega_{be}^2\left(\frac{k}{m}\right)\int_{-\infty}^{\infty}\int_0^{\infty}\frac{p_\perp^3}{(\omega' - kv_\parallel)}\frac{\partial f_0^b}{\partial p_\parallel}dp_\perp dp_\parallel - \omega_{pe}^2 \\
&+ \frac{\omega_{pe}^2}{2n_0\hbar m}\int_{-\infty}^{+\infty}\int_0^{+\infty}\frac{p_\perp^2}{(\omega' - kv_\parallel)}\left[f_0^p\left(p_\perp, p_\parallel + \frac{\hbar k}{2}\right) - f_0^p\left(p_\perp, p_\parallel - \frac{\hbar k}{2}\right)\right]dp_\perp dp_\parallel \\
&= 0.
\end{aligned} \tag{4}$$

In this equation, ω and k are the frequency and wave number of wave instability, respectively; f_0 is the equilibrium dispersion function, ω_{pe} and ω_{be} are the beam and plasma frequency respectively and $w' = w + iv_{ei} = (w_r + i\delta_k) + iv_{ei}$. We consider the electromagnetic wave propagating in the direction $\overrightarrow{k} = k\widehat{e}_z$. The beam with semi-Maxwell distribution function, f_0^b enters the strongly coupled plasma with the Kappa three-dimensional equilibrium distribution $f_0^{p,k}$, as follows [14, 24].

$$f_0^b\left(p_\perp, p_\parallel\right) = \frac{1}{\pi^{\frac{3}{2}}}\left(\frac{1}{m^2\gamma^2\theta_{\perp b}^2}\right)\left(\frac{1}{m\gamma\theta_{\parallel b}}\right)\left(1+\frac{2c^2}{\theta_{\perp b}^2}\right)^{-1} \exp\left(-\frac{\left(p_\parallel - p_d^b\right)^2}{m^2\gamma^2\theta_{\parallel b}^2} - \frac{2c^2}{\theta_{\perp b}^2}\sqrt{1+\frac{p_\perp^2}{m^2\gamma^2c^2}} - 1\right), \quad (5)$$

$$f_0^{p,K}\left(p_\perp, p_\parallel\right) = \frac{u_k}{\pi^{\frac{3}{2}}\theta_{\perp p}^2\theta_{\parallel p}}\left(1+\frac{\left(p_\parallel - p_d^b\right)^2}{km^2\theta_{\parallel p}^2} + \frac{p_\perp^2}{km^2\theta_{\perp p}^2}\right). \quad (6)$$

Here, $u_k = \frac{\Gamma(k+1)}{k^{\frac{3}{2}}\Gamma\left(k-\frac{1}{2}\right)}$ and k is the spectral index. Where $\gamma = \left(1-\frac{v_b^2}{c^2}\right)^{-\frac{1}{2}}$ is relativistic mass factor of beam electrons. In equations (5-6), p_d^b and p_d^p represent the momentum of beam and plasma respectively. Γ is the gamma function, and θ is the thermal velocity of the particles, which is defined as

$$\theta_{\perp,\parallel p}^2 = \left(\frac{2k-3}{k}\right)\left(\frac{T_{\perp,\parallel}}{m}\right),$$

and

$$\theta_{\perp b,\parallel b}^2 = \frac{2T}{m}.$$

In order to calculate equation (4), it is necessary to calculate the necessary derivatives and integrals in equation (4). for the beam and plasma distribution functions by introducing the following scattering functions

$$\begin{aligned}
&\omega^2 - c^2k^2 - \omega_{be}^2 \\
&-\pi\omega_{be}^2\int_{-\infty}^{+\infty}\int_0^{+\infty}\frac{kp_\perp^3}{m\gamma\left(\omega' - kv_\parallel\right)}\left(\frac{2\left(p_\parallel - p_d^b\right)}{\pi^{\frac{3}{2}}}\left(\frac{1}{m^2\gamma^2\theta_{\perp b}^2}\right)\left(\frac{1}{m^3\gamma^3\theta_{\parallel b}^3}\right)\left(1+\frac{2c^2}{\theta_{\perp b}^2}\right)^{-1}\right)dp_\perp dp_\parallel \\
&-\pi\omega_{be}^2\int_{-\infty}^{+\infty}\int_0^{+\infty}\frac{kp_\perp^3}{m\gamma\left(\omega' - kv_\parallel\right)}\left(\exp\left(-\frac{\left(p_\parallel - p_d^b\right)^2}{m^2\gamma^2\theta_{\parallel b}^2} - \frac{2c^2}{\theta_{\perp b}^2}\sqrt{1+\frac{p_\perp^2}{m^2\gamma^2c^2}} - 1\right)\right)dp_\perp dp_\parallel \\
&-\omega_{pe}^2 + \frac{\omega_{pe}^2}{2n_0\hbar m}\int_{-\infty}^{+\infty}\int_0^{+\infty}\frac{p_\perp^2}{\left(\omega' - kv_\parallel\right)}\left(\frac{u_k}{\theta_{\perp p}^2\theta_{\parallel p}\pi^{\frac{3}{2}}}\right)\left(1+\frac{\left(p_\parallel + p_d^p + \frac{\hbar k}{2}\right)^2}{km^2\theta_{\parallel p}^2} + \frac{p_\perp^2}{km^2\theta_{\perp p}^2}\right)^{-k-1}dp_\perp dp_\parallel \\
&+\frac{\omega_{pe}^2}{2n_0\hbar m}\int_{-\infty}^{+\infty}\int_0^{+\infty}\frac{p_\perp^2}{\left(\omega' - kv_\parallel\right)}\left(1+\frac{\left(p_\parallel + p_d^p - \frac{\hbar k}{2}\right)^2}{km^2\theta_{\parallel p}^2} + \frac{p_\perp^2}{km^2\theta_{\perp p}^2}\right)^{-k-1}dp_\perp dp_\parallel = 0. \quad (7)
\end{aligned}$$

Using the variables

$$\xi = \frac{1}{m\gamma\theta_{\parallel b}}\left[\frac{w'}{k} - p_d^b\right], \qquad x = \frac{1}{m\gamma\theta_{\parallel b}}\left[p_\parallel - p_d^b\right],$$

$$y = \frac{1}{m\theta_{\parallel p}}\left[p_\parallel + p_d^p \pm \frac{\hbar k}{2m}\right], \qquad \eta = \frac{1}{m\theta_{\parallel p}}\left[\frac{w'}{k} + p_d^p \pm \frac{\hbar k}{2m}\right],$$

in equation (7), the equation is corrected as follows

$$\omega^2 - c^2k^2 - \omega_{be}^2$$
$$-\omega_{be}^2 \frac{2}{\pi^{\frac{3}{2}}} \left(\frac{1}{m^2\gamma^2\theta_{\perp b}^2}\right) \left(\frac{1}{m^3\gamma^3\theta_{\| b}^3}\right) \left(1 + \frac{2c^2}{\theta_{\perp b}^2}\right)^{-1} \left(\frac{3\theta_{\perp b}^6 m^4\gamma^4}{2c^2} \left[\frac{\theta_{\perp b}^2}{4c^2} + \frac{c^2}{3\theta_{\perp b}^2} + \frac{1}{2}\right]\right)$$
$$\times \left[-\frac{w'}{k} \int_{-\infty}^{+\infty} \frac{\exp\left(-x^2\right)}{(x-\xi)} dx - m\gamma\theta_{\| b}\sqrt{\pi} + \frac{p_d^b}{m\gamma} \int_{-\infty}^{+\infty} \frac{\exp\left(-x^2\right)}{(x-\xi)} dx\right] - w_{pe}^2$$
$$-\frac{mw_{pe}^2\theta_{\perp p}}{8n_0\hbar k\theta_{\| p}} \left[\frac{1}{\sqrt{\pi}} \frac{\Gamma(k)}{k^{\frac{1}{2}}\Gamma\left(k-\frac{1}{2}\right)} \int_{-\infty}^{+\infty} \frac{\left[1+\frac{y^2}{k}\right]}{(y-\eta)} dy\right] = 0. \tag{8}$$

By introducing the following scattering function as follows,

$$Z(\xi) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} \frac{\exp\left(-x^2\right)}{(x-\xi)} dx, \tag{9}$$

and

$$Z_k^*(\eta) = \frac{1}{\sqrt{\pi}} \left[\frac{\Gamma(k)}{k^{\frac{1}{2}}\Gamma\left(k-\frac{1}{2}\right)} \int_{-\infty}^{+\infty} \frac{\left[1+\frac{y^2}{k}\right]^{-k}}{(y-\eta)} dy\right], \tag{10}$$

and placing it in equation (8), the equation is rewritten

$$\omega^2 - c^2k^2 - \omega_{be}^2$$
$$-\frac{\omega_{be}^2}{m\gamma\theta_{\| b}} \frac{\theta_{\perp b}^2}{\theta_{\| b}^2} \frac{w'}{k} \left(1 + \frac{2c^2}{\theta_{\perp b}^2}\right)^{-1} \left(\frac{3\theta_{\perp b}^2 m^4\gamma^4}{c^2} \left[\frac{\theta_{\perp b}^2}{4c^2} + \frac{c^2}{3\theta_{\perp b}^2} + \frac{1}{2}\right]\right) Z(\xi)$$
$$+w_{be}^2 \frac{\theta_{\perp b}^2}{\theta_{\| b}^2} \left(1 + \frac{2c^2}{\theta_{\perp b}^2}\right)^{-1} \left(\frac{3\theta_{\perp b}^2 m^4\gamma^4}{c^2} \left[\frac{\theta_{\perp b}^2}{4c^2} + \frac{c^2}{3\theta_{\perp b}^2} + \frac{1}{2}\right]\right)$$
$$-\frac{\omega_{be}^2}{\theta_{\| b}} \frac{\theta_{\perp b}^2}{\theta_{\| b}^2} \left(1 + \frac{2c^2}{\theta_{\perp b}^2}\right)^{-1} \left(\frac{3\theta_{\perp b}^2 m^4\gamma^4}{c^2} \left[\frac{\theta_{\perp b}^2}{4c^2} + \frac{c^2}{3\theta_{\perp b}^2} + \frac{1}{2}\right]\right) \frac{p_d^b}{m^2\gamma^2} Z(\xi)$$
$$-\omega_{pe}^2 - \frac{m\omega_{pe}^2}{8n_0\hbar k} \frac{\theta_{\perp b}}{\theta_{\| b}} Z_k^*(\eta) = 0. \tag{11}$$

For $(\xi \ll 1, \eta \ll 1)$ the dispersion function $Z(\xi)$ and $(Z_k^*(\eta)$ for k=3,4,5)

$$Z(\xi) = -2\xi + \cdots + i\sqrt{\pi}\exp\left(-\xi^2\right), \tag{12}$$
$$Z_3^*(\eta) = \eta\left(-1.66 - 0.370\eta^2 - \cdots\right) + i\left(1.539 - 1.539\eta^2 + \cdots\right), \tag{13}$$
$$Z_4^*(\eta) = \eta\left(-1.75 - 0.437\eta^2 - \cdots\right) + i\left(1.6 - 1.6\eta^2 + \cdots\right), \tag{14}$$
$$Z_5^*(\eta) = \eta\left(-1.8 - 0.487\eta^2 - \cdots\right) + i\left(1.635 - 1.635\eta^2 + \cdots\right), \tag{15}$$

which we denote $H = \frac{\hbar k}{2}$ by the non-dimensional parameter. Quantum effects only appear through non-dimensional parameters and can generally be said to be longitudinal quantity, k, dependent. When the quantum parameter tends to zero, the dispersion relation (16) reduces to the classical dispersion relation. In the range of small quantum effects, $(H \ll 1)$, and the small wavelengths, $(\xi \ll 1, \eta \ll 1)$, the dispersion relation of equation (11), becomes

$$\omega^2 - c^2k^2 - \omega_{be}^2$$

$$+i\frac{\omega_{be}^2}{m\gamma\theta_{\|b}}\frac{\theta_{\perp b}^2}{\theta_{\|b}^2}\frac{w}{k}D\sqrt{\pi}\omega_{be}^2\frac{\theta_{\perp b}^2}{\theta_{\|b}^2}D-i\frac{\omega_{be}^2}{\theta_{\|b}}\frac{\theta_{\perp b}^2}{\theta_{\|b}^2}\frac{p_d^b}{m^2\gamma^2}D\sqrt{\pi}+\omega_{pe}^2$$
$$+\omega_{pe}^2\frac{\theta_{\perp p}}{\theta_{\|p}}+\omega_{pe}^2\frac{\theta_{\perp p}}{\theta_{\|p}}(i1.539)\frac{1}{\theta_{\|p}}\left[\left(\frac{w}{k}+p_d^p\pm H\right)\right]-\omega_{pe}^2\frac{\theta_{\perp p}}{\theta_{\|p}}\frac{H^6}{6}=0, \qquad (16)$$

where D represents the relativistic effects,

$$D=\left(\frac{\theta_{\perp b}^2}{4c^2}+\frac{c^2}{3\theta_{\perp b}^2}+\frac{1}{2}\right)\Big/\left(1+\frac{2c^2}{\theta_{\perp b}^2}\right)\left(\frac{c^2}{3\theta_{\perp b}^2}\right).$$

The Weibel unstable wave is a low-frequency wave ($|w|\ll kc$). Since, for a short wavelength mode ($\xi\ll 1,\eta\ll 1$), by ignoring the contribution of the second and higher orders sentences in equations (12-15) and considering $w'=w_r+i\left(\delta_k+v_{ei}\right)$ in the dispersion relation, the instability growth rate can be derived by solving the coupled equation (for example spectral index $k=3$) where δ_k represents the growth rate of the strongly coupled plasma instability with the kappa distribution function. Thus, Weibel instability growth rate in spectral index 3 is calculated as follows

$$\delta_3=\frac{k\left(-\frac{w_{be}^2}{w_{pe}^2}+\frac{w_{be}^2}{w_{pe}^2}\alpha D-\frac{c^2k^2}{w_{pe}^2}-1+\beta-\beta\frac{H^2}{6}\right)}{1.539D\left(\frac{w_{be}^2}{w_{pe}^2}\frac{\alpha}{\theta_{\|b}}m^2\gamma^2+\frac{\beta}{m\theta_{\|p}}\right)}-v_{ei}, \qquad (17)$$

where $\alpha=\frac{\theta_{\perp b}^2}{\theta_{\|b}^2}$ and $\beta=\frac{\theta_{\perp p}}{\theta_{\|p}}$.
The ion–electron Coulomb collision frequency, v_{ei} for the Kappa function is defined as follows [24]

$$v_{ei}=\frac{\Gamma(k+1)}{\sqrt{\pi}\Gamma\left(k-\frac{1}{2}\right)}\frac{\sqrt{k}}{(k+1)^2}\frac{n_i}{8}\frac{Ze^4}{4\pi\varepsilon_0^2}\frac{4\pi(m_e+m_i)}{m_im_e^2v_\perp^2v_\|}\ln\Lambda. \qquad (18)$$

Applying the variable χ is defined as follows

$$\chi=u_k-\frac{j_k}{k}\frac{\Gamma(k+1)}{\sqrt{\pi}\Gamma\left(k-\frac{1}{2}\right)}\frac{\sqrt{k}}{(k+1)^2}\frac{n_i}{8}\frac{Ze^4}{4\pi\varepsilon_0^2}\frac{4\pi(m_e+m_i)}{m_im_e^2v_\perp^2v_\|}\ln\Lambda, \qquad (19)$$

where j_k is equal 1.539, 1.600, and 1.635 respectively, also

$$u_k=\frac{\frac{w_{be}^2}{w_{pe}^2}\left[-1+\alpha D\right]+\beta}{\left(\frac{w_{be}^2\alpha\sqrt{\pi}}{w_{pe}^2m\gamma(1.539)\theta_{\|b}}+\frac{\beta}{\chi m\theta_{\|p}}\right)}. \qquad (20)$$

Then, the growth rate of Weibel instability is rewrite as

$$\delta_3=\frac{k}{(1.539)D}\left[\chi-\frac{\left(1+\frac{c^2k^2}{w_{pe}^2}+\beta\frac{H^2}{6}\right)}{n_k}\right], \qquad (21)$$

where

$$n_k=\frac{w_{be}^2\alpha\sqrt{\pi}}{w_{pe}^2m\gamma(1.539)\theta_{\|b}}+\frac{\beta}{\chi m\theta_{\|p}}. \qquad (22)$$

Also, δ_4 and δ_5 represent the growth rate of Weibel instability in spectral indices 4 and 5 $k = 4, 5$ are calculated as follows

$$\begin{aligned} \delta_4 &= \frac{k}{(1.600)\,D}\left[\chi - \frac{\left(1 + \frac{c^2k^2}{w_{pe}^2} + \beta\frac{H^2}{6}\right)}{n_k}\right], \\ \delta_4 &= \frac{k}{(1.635)\,D}\left[\chi - \frac{\left(1 + \frac{c^2k^2}{w_{pe}^2} + \beta\frac{H^2}{6}\right)}{n_k}\right]. \end{aligned} \tag{23}$$

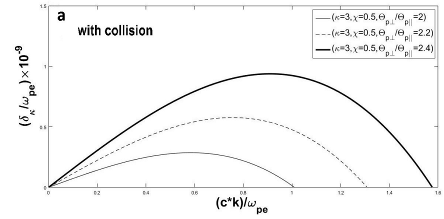
3 Observations

The purpose of this article was to calculate the growth rate of the Weibel instability in the presence of relativistic electron beam that has penetrated into the coupled plasma by considering the Coulomb collisions in the beam-plasma interaction. In Figure 1a, the Weibel instability growth rate is plotted for different values of temperature anisotropy for the constant collision term $x = 0.5$ in spectral index 3. It can be seen that the growth rate of instability will increase by increasing the temperature anisotropy fraction of plasma due to increase in the free energy of the plasma. The point that should be taken into account is that with the significant effect of the temperature anisotropy of the plasma on the maximum value of the instability growth rate, it will be possible to suppress the unstable modes in isotropic plasmas. In Figure 1b, the variation of Weibel instability growth rate is calculated with and without the presence of the Coulomb collision effect in the beam-plasma interaction for the anisotropy fraction $\left(\frac{\theta_{\perp p}}{\theta_{\parallel p}} = 1.8\right)$. Observations show that the Coulomb collision effect reduces growth rate significantly. The maximum value of the growth rate in the presence of the Coulomb collision effect is equal to $\left(0.2 \times 10^{-9}\right)$ while it is equal to $\left(0.8 \times 10^{-9}\right)$ without the Coulomb collision effect. It seems that this reduction in the effect of the maximum growth rate is due to the increase in plasma free energy (see Figure 1c).

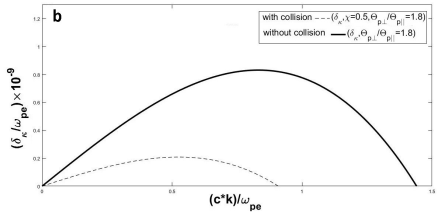
The variation of the growth rate according to the quantum parameter was calculated to study the effect of the quantum parameter on the growth rate. It has been found that the growth rate decreases with increase of the quantum parameter Figure 2a. The Weibel instability growth rate is plotted for quantum parameter value ($H = 1.2$) with collisional and non-collisional states, as in Figure 2b. The results show that the maximum growth rate corresponds to the absence of the Coulomb collision effect.

In Figures 3a and 3b, the graphs of the growth rate for various spectral indices have been drawn in the presence of collision and non-collision effect. Observations show that with the increase of the spectral index, the growth rate decreases due to the convergence of the beam spectrum in the plasma. Also, by comparing the maximum growth rate for the spectral index in Figures 3a and 3b, it can be seen that in the absence of the collision effect, the maximum growth rate of Weibel instability is lower than in the state where the collision effect is present.

Finally, the variations of the growth rate for the relativity parameter $\Gamma = 1.5$ in collisional and non-collisional state are show in Figure 4.
It is concluded that the growth rate in the collision state has a more stable situation than in the non-collision state in the relativity parameter ($\gamma = 15$).



(a)



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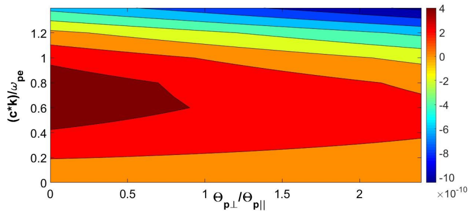
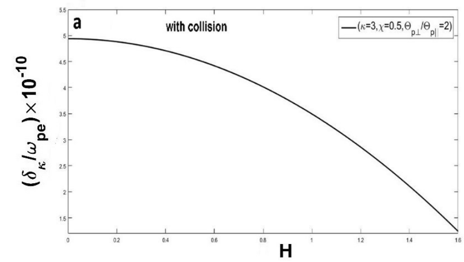
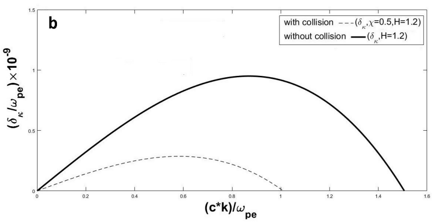


(c)

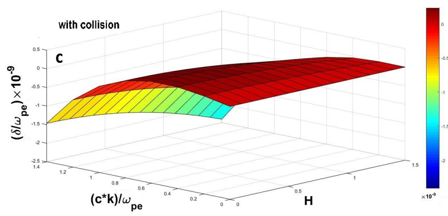
Figure 1: (a) Variation of the growth rate normalized to w_{pe}, $\frac{\delta}{w_{pe}}$, according to $\frac{ck}{w_{pe}}$ for different $\frac{\theta_{\perp p}}{\theta_{\parallel p}}$ for $k = 3$ (b) Variation of the growth rate normalized to w_{pe}, $\frac{\delta}{w_{pe}}$, according to $\frac{ck}{w_{pe}}$ in $\frac{\theta_{\perp p}}{\theta_{\parallel p}} = 1.8$ in the presence of collision and non-collision effect (c) 3D-graphvariation of $\frac{ck}{w_{pe}}$ according to $\frac{\theta_{\perp p}}{\theta_{\parallel p}}$ with collision.



(a)

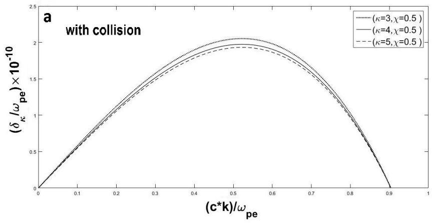


(b)

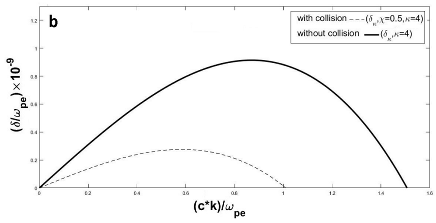


(c)

Figure 2: (a) Variation of the growth rate, $\frac{\delta}{w_{pe}}$, to different H for $k = 3$ (b) Variation of the growth rate normalized to w_{pe}, $\frac{\delta}{w_{pe}}$, according to $\frac{ck}{w_{pe}}$, $H = 1.2$ in the presence of collision and non-collision effect (c) 3D-graph variation of $\frac{\delta}{w_{pe}}$ according to $\frac{ck}{w_{pe}}$ and quantum parameters H with collision.



(a)



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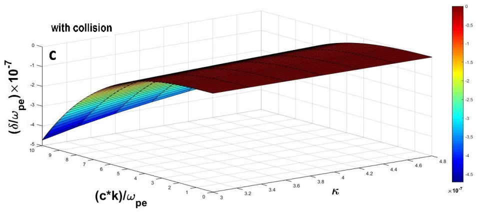

(c)

Figure 3: (a) Variation of the growth rate normalized to w_{pe}, $\frac{\delta}{w_{pe}}$, according to $\frac{ck}{w_{pe}}$ for different k with collision effect $x = 0.5$ (b) Variation of the growth rate normalized to w_{pe}, $\frac{\delta}{w_{pe}}$, according to $\frac{ck}{w_{pe}}$ for different k in non-collision effect (c) 3D-graphvariation of $\frac{\delta}{w_{pe}}$ according to $\frac{ck}{w_{pe}}$ and spectral index k.

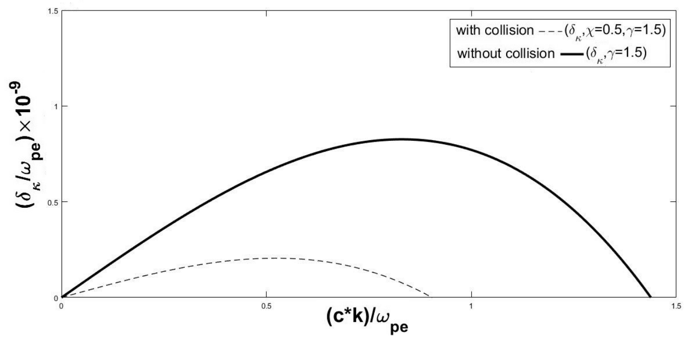

Figure 4: Variation of the growth rate normalized to w_{pe}, $\frac{\delta}{w_{pe}}$, according to $\frac{ck}{w_{pe}}$ for $\gamma = 1.5$ in the presence of collision and non-collision effect.

4 Conclusions

In this research, the growth rate of Weibel instability was investigated in strongly coupled plasma of inertial confinement fusion fuel with Kappa distribution function in the presence of Coulomb collision. In this regard, analytical expressions were derived for imaginary parts of dielectric constants as the Weibel instability of growth rate for the plasma particle Kappa distribution function under the low-frequency wave condition. The obtained results indicate that instability growth rate depends on the temperature anisotropy fraction, Coulomb collision frequency, quantum parameter, relativistic parameter and the values of the spectral index, k, of the distribution function. Results indicated that the instability growth rate increases by increasing the temperature anisotropy fraction and the relativistic parameter for a fixed special index ($k = 3$). Finally, the obtained results also show that the Coulomb collision frequency of particles plays an important role in suppressing the unstable modes in isotropic plasmas due to increase in the free energy of the plasma. Therefore, it is concluded that the Weibel instability growth rate in collision state has a more stable situation than in the non-collision state in the strongly coupled plasma with Kappa distribution function.

Authors' Contributions

All authors have the same contribution.

Data Availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare no potential conflicts of interest.

Ethical Considerations

The authors have diligently addressed ethical concerns, such as informed consent, plagiarism, data fabrication, misconduct, falsification, double publication, redundancy, submission, and other related matters.

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