

Research Paper

A Study on the Lateral Distribution of Cherenkov Radiation of Extensive Air Showers at High Observation Levels

Davoud Purmohammad

Physics Department, Imam Khomeini International University, Qazvin, Postal Code 3414896818, Iran;
email: purmohammad@sci.ikiu.ac.ir

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Abstract. By using CORSIKA code, for simulation of extensive air showers for observation levels at different altitudes, the variation of the lateral distribution of Cherenkov radiation of showers with altitude is studied. The lateral distribution function introduced by Tunka experiment group has been examined for 13 observation levels. The possibility of estimating the maximum depth of showers, using the steepness of the lateral distribution of Cherenkov radiation at different observation levels is investigated. It is shown that the relationship between the steepness of the lateral distribution of Cherenkov radiation and the maximum depth of showers, which was introduced in previous researches, should be calibrated for each observation level separately. An evaluation for errors in shower maximum depths estimated by this method, for an observation level at 4000 meters above the sea level is provided as an example. The errors in the estimated shower maximums are larger than the corresponding errors in the atmospheric fluorescence technique.

Keywords: Extensive air showers, Cosmic rays, Cherenkov radiation, CORSIKA

1 Introduction

The detection of cosmic rays and gamma rays at very high energies is done through the detection of extensive air showers caused by these particles. When an energetic photon or charged particle enters the earth's atmosphere, it creates an avalanche of interactions that result in a multitude of secondary particles, mainly electrons¹ and muons, Cherenkov radiation, atmospheric fluorescence radiation, and radio waves. This swarm of secondary particles is called extensive air shower (EAS). Various technologies and methods have been used to measure the distribution of these radiations during the evolution of the extensive air shower or on the surface of the earth. The detection and study of Cherenkov radiation from extensive air showers was one of the first technologies used in cosmic ray physics [1]. This method is still used in some cosmic ray observatories [2]. The distribution of Cherenkov radiation of an EAS on the earth's surface depends on factors such as the size of the shower, which is the number of secondary particles, its maximum height, i.e. the atmospheric altitude at which the shower size is maximum, energy, and the type of primary particle. Previous researches have shown that it is possible to use the surface distribution of Cherenkov radiation of an EAS to estimate the energy and the type of its primary particle

¹In this paper we use this word for both positrons and electrons.

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[3]. In some experiments, the determination of the primary particle type is based on the estimation of the maximum depth of the EAS. Usually, the location of the maximum depth of an extensive air shower is determined by observing the longitudinal trajectory of the shower using atmospheric fluorescence radiation detectors, such as those used in the Fly's Eye experiment [4], or the Telescope Array experiment [5]. It has been shown that a quantity called the steepness of the lateral distribution of the Cherenkov radiation, in a shower can be used to estimate the maximum depth of it [6]. In this method, without relying on the observations of the evolution of the shower along its path in the atmosphere, the maximum depth of the shower is estimated simply by measuring the surface density of Cherenkov radiation that has reached the earth's surface (observation level). This method has been used in Tunka experiment at the TAIGA Observatory [7]. Considering that the lateral distribution of Cherenkov radiation of an extensive air shower depends on the maximum depth of that shower, the question arises that if observatories located at higher altitudes (lower atmospheric depth) want to measure the lateral distribution of Cherenkov radiation of extensive air showers, will they get different results from those obtained by Tunka experiment at an altitude close to sea level (675 meters above sea level)? In this case, will it be possible to estimate the maximum depth of showers using the steepness of the lateral distribution of Cherenkov radiation? Will there be a correlation between the maximum depth of the shower and the steepness parameter of the lateral distribution of Cherenkov radiation as suggested in the reference [3]? In order to answer these questions in the present study, simulations have been made to produce Cherenkov radiation of different extensive air showers to be observed at 13 different heights above the sea level. The lowest altitude is at sea level, and the highest altitude is 6000 meters above sea level. The results of these simulations, which are presented in the following sections, show that the lateral distribution of Cherenkov radiation of extensive air showers have greater lateral steepness at higher observation levels. Therefore, the proposed method by Rasekh and Purmohammad [3] should be recalibrated for higher observation levels. Here, while showing how the distribution of Cherenkov radiation of extensive air showers changes at different heights, we have investigated the possibility of estimating the maximum depth of showers at some high observation levels using the steepness of the lateral distribution of Cherenkov radiation.

2 Simulation of extensive air showers

To generate extensive air showers, we used CORSIKA version 7.56 [8]. All showers were generated with zenith direction for arrival of primary particles. The primary particles of the shower were defined as equal numbers of gamma rays, protons, alpha particles, and iron nuclei. The energy of these particles was determined to be 10^3 , 10^4 , 10^5 , and 10^6 GeV. For every primary particle type-energy combinations, simulations were performed for 13 different observation levels, from sea level to 6000 m above sea level, with 500 m intervals. For the energy of 10^6 GeV, 10 showers, and for the other energies, 30 showers were produced in each run. A total of 5,200 showers were produced. In the output file of each run, the number, and the location of Cherenkov photons whose wavelength was between 300 and 450 nm and reached the ground up to a distance of 300 meters from the center of the shower were recorded. Also, the longitudinal distribution of shower particles during its evolution in the atmosphere, from the top to the ground surface, at intervals of atmospheric thickness $\Delta X = 10\text{g/cm}^2$, were recorded in the longitudinal distribution file of each shower. In this way, it was possible to determine the maximum depth of each shower, X_{max} , with an accuracy of 10 g/cm^2 . In the simulations, GHEISHA hadronic model [9] was used for low energy hadronic interactions, and QGSJETII hadronic model [10] was used for

high energy hadronic interactions. Since the simulations had to produce the Cherenkov radiation of each shower, the electromagnetic interactions were generated with the EGS4 model [11]. Simulations were performed without thinning. In the simulations, the absorption of Cherenkov photons in the atmosphere was considered. The size of the Cherenkov photon bunch was taken as its default value (CERSIZ=0.). In this research, the quantum efficiency of the detectors, and the response of detector arrays were not taken into account, and it was assumed that all Cherenkov photons that reach the ground are detected and recorded at all points in the observation level. The output data of each simulation includes the number and location of Cherenkov photons at the observation surface, and the number of secondary particles of each shower in 10 g/cm² depth steps from the top of the atmosphere to the observation level. Using this information, the lateral distribution of Cherenkov radiation of each shower at the observation level, and the maximum depth of that shower were obtained. To determine the surface distribution of Cherenkov radiation, the observation surface was divided into circular strips of one meter width, centered on the shower core. By counting the number of photons that hit each strip, the surface density of Cherenkov photons for each area was obtained.

3 The lateral distribution of Cherenkov radiation of extensive air showers at different observation levels

By extracting data from the output of simulations, the transverse distribution of Cherenkov radiation of simulated showers was obtained. An example of these distributions is presented in Figure 1. In this graph, the surface distribution of Cherenkov radiation of showers initiated by 10⁶ GeV gamma rays or protons, at different observation levels are presented. Each point in this graph is the average value for 10 showers. From the distributions shown in Figure 1, it is clear that the higher the altitude of the observation level above sea level, the steeper the lateral distribution of Cherenkov radiation. Researchers in Tunka experiment have used the following model for the lateral distribution of Cherenkov radiation of extensive air showers [12]

$$Q(r) = Q_{kn} \begin{cases} \exp \left[\frac{(R_{kn}-r)(1+\frac{3}{r+3})}{R_0} \right], & r \leq R_{kn}, \\ \left(\frac{R_{kn}}{r} \right)^{2.2}, & R_{kn} \leq r < 200m, \\ \left(\frac{R_{kn}}{200} \right)^{2.2} \left(\frac{1}{2} + \frac{r}{400} \right)^{-b}, & 200m \leq r, \end{cases} \quad (1)$$

where in which

$$b = \begin{cases} 4.84 - 1.23 \ln(6.5 - P), & P \leq 6, \\ 3.43, & P > 6, \end{cases}$$

$$R_{kn} = 207 - 24.5P, \quad R_0 = \exp(6.79 - 0.564P), \quad Q_{kn} = Q_{175} \left(\frac{R_{kn}}{175} \right)^{-2.2},$$

and $P = \frac{Q(100)}{Q(200)}$ is the ratio of the surface densities of Cherenkov radiation at 100 m and 200 m from the shower core. Q_{175} is in fact, the density at a distance of 175 m from the shower core. R_0 and R_{kn} are other auxiliary parameters in meter unit. The surface density of photons, $Q(r)$ is in cm⁻², and r , the distance from the shower core is in meters. This empirical model is based on simulation data. The model, which we call Tunka Lateral Distribution Function (LDF), ultimately, has two parameters, P and Q_{175} which can be

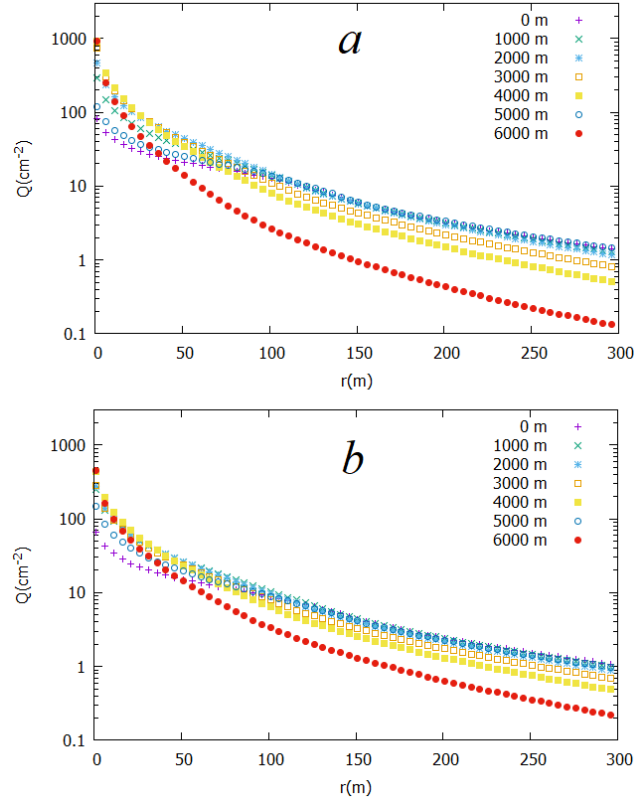


Figure 1: Distribution of the surface density of Cherenkov radiation versus distance from the center of the shower, for 10^6 GeV showers generated by, *a*: gamma rays and *b*: protons at different altitudes above sea level. Each point is the mean value for 10 showers. Error bars are not shown.

estimated by fitting the data. Researchers in Tunka experiment have used P which is known as the steepness parameter of the lateral distribution, for estimation of the altitude of shower maximum, H_{max} [12]. Based on the investigation of simulated air showers, Rasekh and Purmohammad [3] have shown that an empirical model can be used to relate the depth of shower maximum X_{max} with the parameter P . The model is expressed as

$$X_{max} = aP^2 + bP + c, \quad (2)$$

in which X_{max} is maximum depth in g/cm^2 , and a , b , and c are the fitting parameters. They showed that at the observation level of 675 meters above sea level, i.e. the altitude of Tunka experiment, using the measured value for P of each shower in equation 2, can give the X_{max} with acceptable accuracy. For the showers produced in the present study, which have different observation levels, the lateral distribution model 1 still can be used. Figure 2 shows the fitting of this model to the distribution of Cherenkov radiation at the observation levels of 1000 meters and 4000 meters above sea level. Since, the steepness parameter of the lateral distribution of Cherenkov shower radiation, has been used to estimate the maximum depth of the shower, and ultimately to estimate the mass of the initial particle of each shower, the question arises that due to the increase P in at higher observation levels, will the mentioned

method be applicable to other observatories located at high altitudes? For example, if in the AS-Gamma high energy observatory [13], which is located at an altitude of 4300 meters above the sea level in the Tibetan Plateau, the Cherenkov radiation front of showers can be sampled, can X_{max} be estimated by using measured P in equation 2?

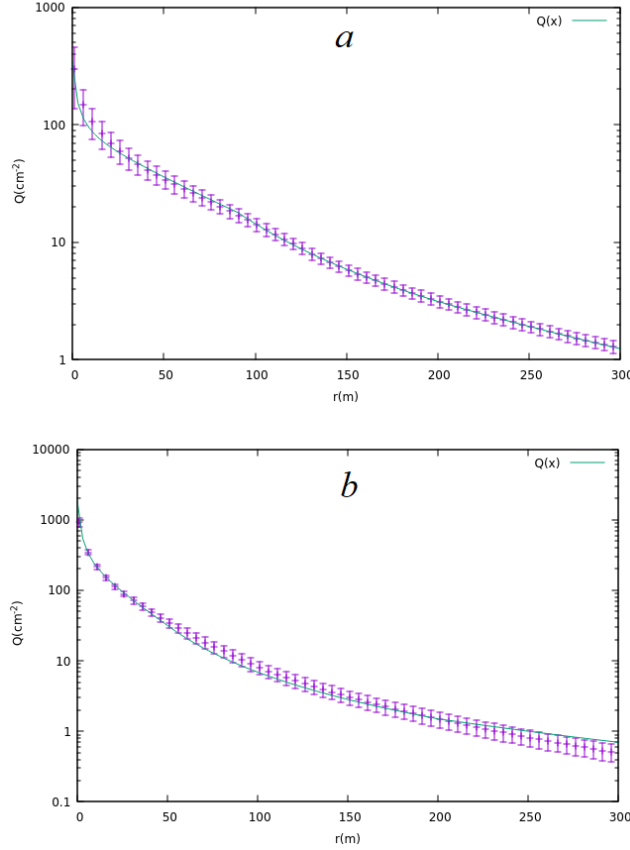


Figure 2: Fitting the Tunka LDF to the lateral distribution of Cherenkov radiation of showers generated by 10^6 GeV gamma rays for two observation levels: (a) 1000 meters above sea level, and (b) 4000 meters above sea level.

4 Examining the relationship between X_{max} and P in simulated showers for different observation levels

For each shower produced in the simulations, the steepness of the lateral distribution of Cherenkov radiation at the observation level, and the maximum depth was obtained. In Figure 3, the distribution of the values of these quantities for all showers is displayed. Each point in this diagram corresponds to an extensive air shower. All showers generated from primary particles of different types and energies, and for different observation levels are presented in this diagram. The general shape of this distribution is compatible with what was found in reference [3].

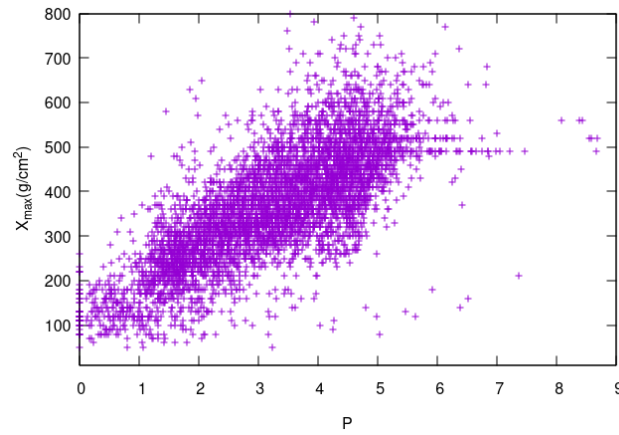


Figure 3: Distribution of X_{max} versus P for all showers generated in the simulations of this work.

Now, if we limit the data, to two observation levels, for example at sea level and 4000 meters above it, and in order to better show the effect of the difference in altitude, consider only the showers generated from gamma rays and protons, the diagram which is presented in Figure 4 is obtained. Figure 5 shows the average values of X_{max} and P for showers produced for the same two observation levels. In this diagram, each point represents the average value of all showers produced from the same type of primary particle and the same energy.

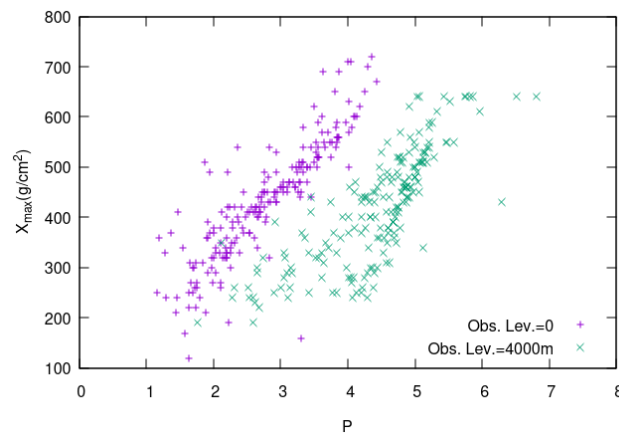


Figure 4: Distribution of X_{max} versus P for showers generated by protons and gamma rays at two observation levels; sea level (plus symbol) and 4000 meters above sea level (cross symbol).

In this diagram, we see that no single relation like equation 2 can be properly fitted to the data of the two different observation levels. This makes us expect the relationship between X_{max} and P , i.e. equation 2, should be recalibrated for each observation level. It seems that the shape of the lateral distribution of Cherenkov radiation of showers, and consequently the steepness parameter P , depends on the distance between the shower maximum and

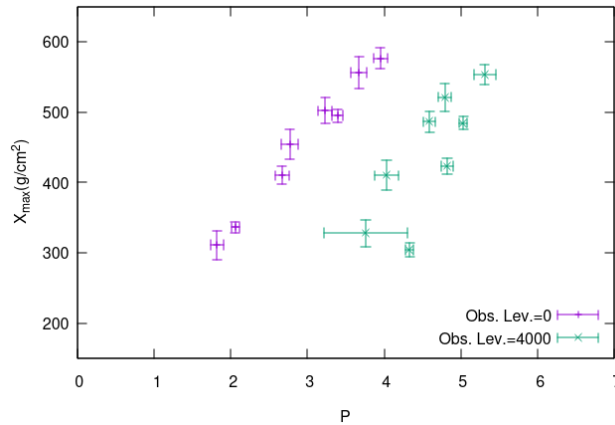


Figure 5: Distribution of average values of X_{max} versus P for showers generated by protons and gamma rays at two observation levels; sea level (plus symbol) and 4000 meters above sea level (cross symbol).

the observation level rather than the depth of the shower maximum itself. In Figure 6, $X_{obs} - X_{max}$, that is, the difference between the depth of the observation level and the maximum depth, versus P of each shower is displayed. In this diagram, for brevity, only the results related to showers generated by protons and gamma rays are presented. In this diagram, different symbols are used for each energy to observe the influence of the shower energy. From the data presented in Figure 6, it seems that for a fixed observation level, there is an acceptable one-to-one correlation between P and X_{max} , though due to the random nature of the mechanisms that create extensive air showers, the fluctuations in X_{max} values are high, especially in hadronic showers. But it can be seen that the higher the shower energy, the more accurate correlation between X_{max} and P . In Figure 6, the data with $X_{obs} - X_{max}$ close to zero are for the showers whose maximums have occurred at altitudes close to the observation levels. In such a situation, due to the fact that the charged particles of the shower and the Cherenkov radiation caused by them have not yet laterally expanded, the steepness values are much larger than usual values, and are more random. To find out if it is possible to establish a relationship between them, graphs of average values of $X_{obs} - X_{max}$ versus P for showers produced from the same primary particles types and energies are presented in the Figure 7. Such a relationship can be used for estimation of $X_{obs} - X_{max}$ from steepness parameter. It can be seen in these graphs that the variation of $X_{obs} - X_{max}$ with P at a constant energy depends on the mass of the initial particle. At the highest energy presented, it may be possible to define a relation between the relative maximum and the steepness, independent of the initial particle mass. But such a relation cannot be used to estimate the mass of the initial shower particle, which is the final goal. According to what can be seen in the graphs of Figure 7, using the measured P cannot provide an accurate estimate of $X_{obs} - X_{max}$. However, the data presented in Figures 4 and 5 showed that the relationship between P and X_{max} can be calibrated for each observation level separately. It will be an interesting test to see how much the actual X_{max} values of showers differ from those obtained using the equation 2 with fit parameters obtained for the observation level of the Tunka experiment in reference [3]. In Figure 8, the values of the maximum depth estimated in this way, are compared with the actual values of the maximum depth of showers for three different observation levels. The estimated values in

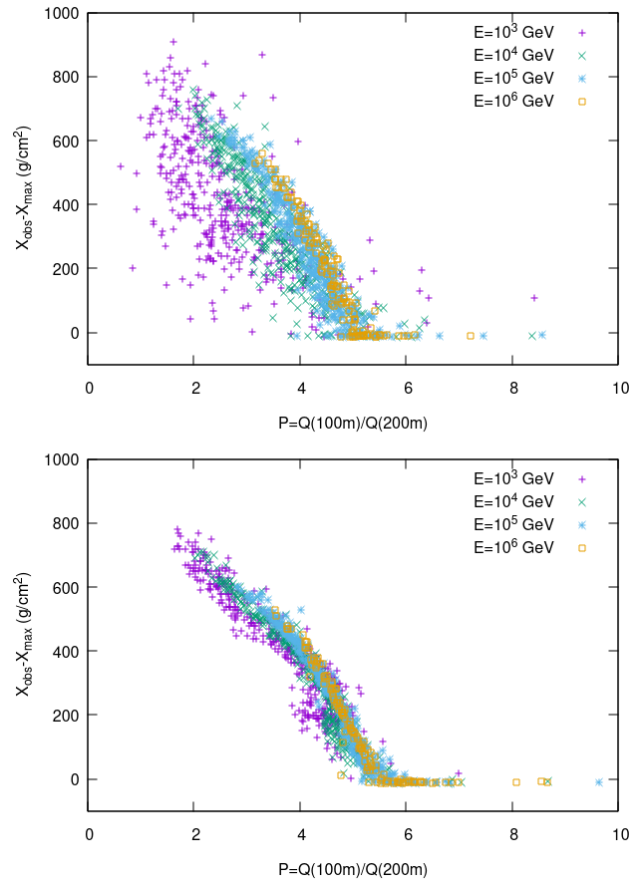


Figure 6: Distribution of the difference between the depth of the observation level and the maximum depth versus P for showers generated by protons in the upper diagram, and showers generated by gamma rays in the lower diagram. Shower energies are marked with different symbols. In fact, for most of the 10^6 GeV showers initiated by protons or gamma rays, H_{max} which is the height of shower maximum, is lower than 5 Km. That means for those showers $X_{max} > X_{obs}$ for observation levels 5000 m, 5500 m, and 6000 m above sea level. For almost half of 10^5 GeV showers generated by protons and gamma rays we have $H_{max} < 6Km$, which results in similar situation. These showers correspond to the data points near and below zero in relative depth.

the cases of 0 and 500 meters above sea level are in better agreement with the actual values of the maximum depth, but the estimates for the observation level of 4000 meters above sea level are very different from the actual values. To make this comparison more clear, the histograms of the differences between the estimated maximum depths and their actual values, $X_{est} - X_{max}$, for all showers observed at 0, 500 m, and 4000 m observation levels are presented in Figure 9

If the coefficients a , b , and c in equation 2 are obtained for a specific observation level, i.e. by fitting the observed showers data at that level, better estimations for X_{max} will be provided. To estimate the maximum depth values with relation 2, for the data represented in Figure 10, the coefficients obtained by fitting the observed showers at an altitude of 4000

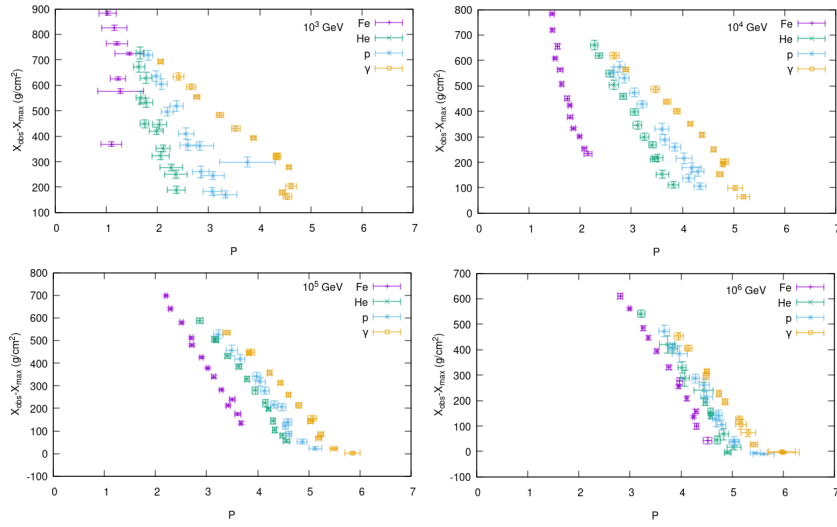


Figure 7: Average $X_{obs} - X_{max}$ versus P . Each box is for showers with the same energy. The energy of showers is given inside each box. Different symbols are used in for showers created from different primary particles.

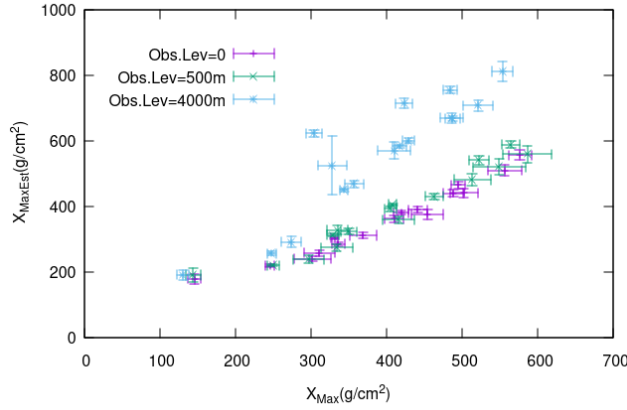


Figure 8: Comparison of the estimated values for the maximum depth of showers, $X_{MaxEst} = 13.7P^2 + 61P + 101$, using equation 2 with the parameters fitted for the Tunka observation level (675 m), with actual values of maximum depth X_{Max} . Each point represents the mean value for 10 or 30 showers. Different symbols which are used for different observation levels are given at upper left guide.

meters are used. We can see that there is now a better match between the estimated and the actual values. Figure 11 shows the histogram of the differences between the estimated maximum depths and their actual values for showers observed at an altitude of 4000 meters above sea level. This histogram shows that the use of equation 2 to estimate the maximum depth of showers from the steepness parameter of Cherenkov radiation lateral distribution has a systematic error of 31 g/cm^2 , and a statistical error of 191 g/cm^2 . These errors

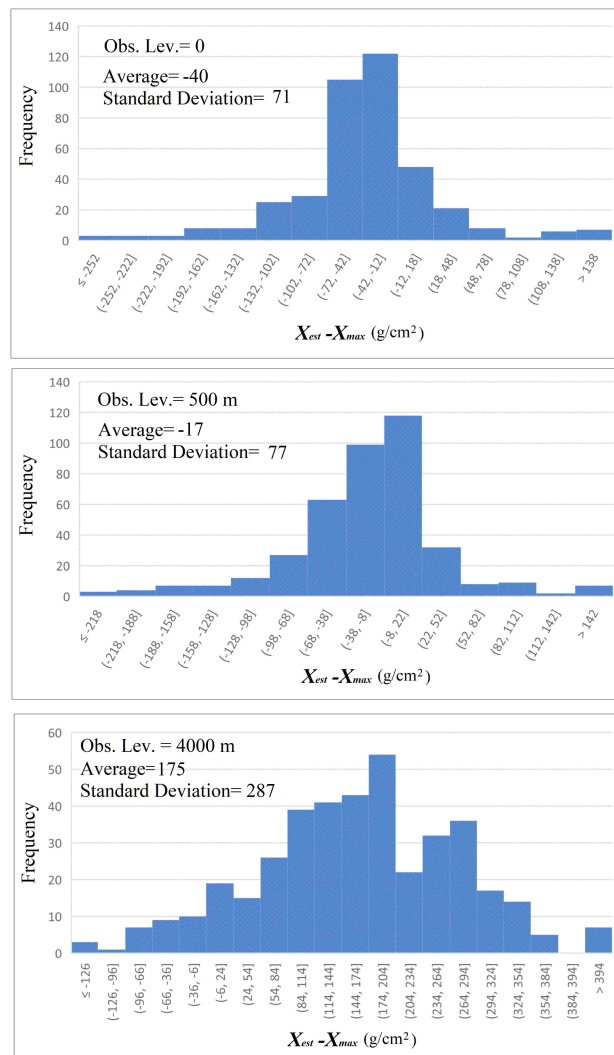


Figure 9: Histogram of the difference between the estimated maximum depth and its actual values for showers observed at three different altitudes. The altitude of the observation level, mean value, and standard deviation of each histogram is given in its agenda at upper left corner. The number of showers in each sample is 400. X_{est} is obtained by using equation 2 with fit parameters obtained for Tunka observation level (675 m) in the reference [3].

are much larger than the values reported in reference [3] for the observation level of 675 meters above sea level. Since the size of the data sample presented here (400 for each observation level) is much smaller than the size of data sample used by reference [3] (9900 showers for one observation level), higher statistical errors are expected. This range of errors for estimating the maximum depth of extensive air showers is greater than those reported for other methods that are used for maximum depth estimation, especially Fly's Eye type experiments which measure the atmospheric fluorescent radiation of showers. However, since Cherenkov radiation front measurement technique is easier and cheaper than the atmospheric fluorescent technique, it is worth to be employed despite lower accuracy.

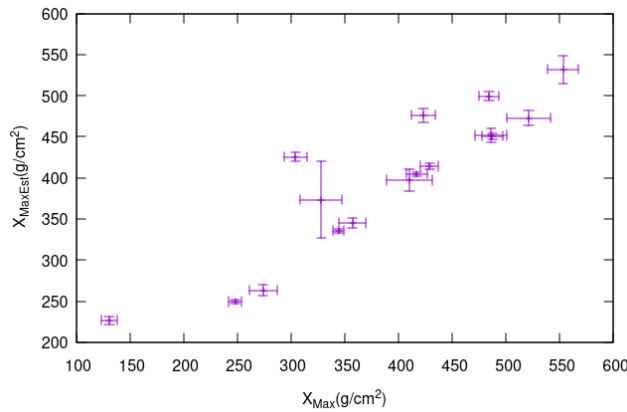


Figure 10: Comparison of the estimated values for the maximum depth of showers, $X_{EstMax} = 11P^2 + 2.4P + 209$, using equation 2 with the parameters fitted for 4000 meters observation level, with actual values of maximum depth X_{Max} . Each point represents the mean value for 10 or 30 showers.

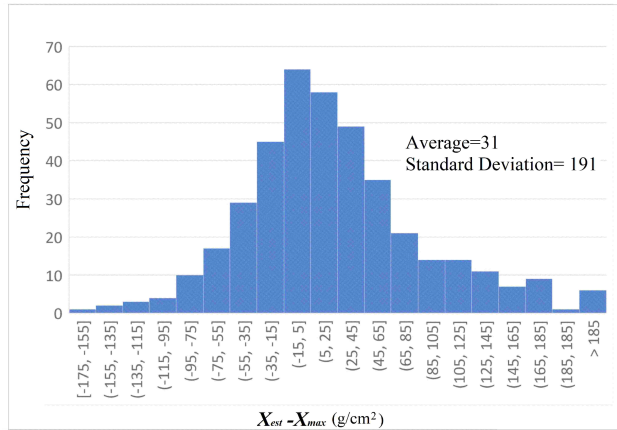


Figure 11: Histogram of the difference between the estimated maximum depth and its actual values for showers observed at an altitude of 4000 meters above sea level. The number of showers in this sample is 400. X_{est} is obtained by using equation 2 with fit parameters obtained for observation at 4000 m above sea level.

5 Conclusion

As what is done in the Tunka experiment at the TAIGA Observatory, it has been shown that it is possible to use the steepness of the lateral distribution of Cherenkov radiation of extensive air showers at observation levels near the sea level, to estimate the value of the maximum depth of extensive air showers. A question arises about the possibility of application of the technique in observatories installed at higher altitudes. In the present study, it has been shown that the Cherenkov radiation of extensive air showers at high altitudes, due to being close to location of the shower maximum, has a distribution with a greater lateral steepness than that seen near the sea level. So, it is necessary to have the

lateral distribution of Cherenkov radiation, and accordingly the steepness of this distribution, i.e. the parameter P in Tunka LDF, to be calibrated for high altitudes. The empirical relationship between P and H_{max} , the the height of shower maximum, utilized by the Tunka experiment, and the empirical relationship between P and X_{max} , proposed by Rasekh and Purmohammad, should be re-fitted for each observed level. In the present study, it has been shown that the direct application of the proposed $X_{max}(P)$ relation, leads to inaccurate estimates for X_{max} , but if we fit the relation to the data generated for high observation levels, it can produce more accurate estimations for of X_{max} . This could be useful for the application of the Cherenkov radiation front detection technique of extensive air showers in cosmic ray observatories installed at high altitudes, such as the one built as the AS-Gamma experiment on the Tibetan Plateau. Although the errors for estimation of maximum depth in this method is larger than the errors of the experiments based on measuring the atmospheric fluorescent radiation of showers, the installation and operation of the detectors of the Cherenkov radiation front of showers are easier than the atmospheric fluorescence experiments.

It has to be noted that, in simulations used for this work, the heights of the first interaction in each shower were not fixed. The technique which is presented here, claims that it can be used by extensive air shower observatories based on Cherenkov front measurements to estimate the shower maximum. So, our simulations have to be as near as possible to real conditions in observed EASs. Therefore, we avoided fixing the first interactions. In real situations we can sample the Cherenkov front of a shower on the ground. Thus, measuring the steepness parameter (P) is feasible for real showers. However, X_{max} is not a priori known. It worth mentioning that steepness and shower maximum values were calculated for each event separately. The average values were only used for fitting of whole data to the model given in Equation 2. In calculation of the average values, shower types or energies are not mixed. Only showers of the same primary type-energy were averaged to provide a single pair of (P, X_{max}) . Showers generated from the same primary particle type-energy are expected to have statistical fluctuations in their first interaction height. This is the main reason for fluctuations in X_{max} of showers of same type-energy. As we know, a more important (systematic) variation in the first interaction height happens when the primary particle's type or energy changes. Therefore, we didn't average shower fronts from different primary types or energies. Fixing the first interaction will induce artificially lower statistical error, which in turn, gives a false improvement in the accuracy of results.

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Authors' Contributions

The author contributed to data analysis, drafting, and revising of the paper and agreed to be responsible for all aspects of this work.

Data Availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The author declares no potential conflicts of interest.

Ethical Considerations

The author has diligently addressed ethical concerns, such as informed consent, plagiarism, data fabrication, misconduct, falsification, double publication, redundancy, submission, and other related matters.

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