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Research Paper

Analytical Expression of the Beam-Plasma Particles Distribution Function Effect on the Electromagnetic Instability Growth Rate in Strongly Coupled Plasmas

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Abstract. The beam-plasma particles distribution function is one of the parameters which plays an important role in the energy-traveling mechanism of the relativistic electrons generated by the laser-plasma interaction in the Inertial Confinement Fusion Plasma. This paper investigates an analytical expression of the beam-plasma particles distribution function effect such as the Kappa, Semi-relativistic Maxwellian and bi-Maxwell distributions on the Weibel electromagnetic instability growth rate in strongly coupled plasmas under the low-frequency wave condition. The obtained results show that the maximum growth rate of the beam- plasma particles with semi-Maxwell distribution function is based on the temperature anisotropy parameter, density gradient, quantum and relativistic parameters has the highest possible value compared to the other two beam-plasma particles distribution functions. Also, the bi-Maxwellian distribution function has a more stable growth rate than the Kappa and the semi-Maxwell distribution functions.

Keywords: Weibel Instability, Coupled Plasmas, Temperature Anisotropy, Beam-Plasma Particles distribution Function, Low-Frequency Wave, Strongly Coupled Plasmas

1 Introduction

One of the methods of nuclear fusion is the inertial confinement fusion. In this process, high power lasers are used to ignite the target. One of the most important issues in this

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plan is the homogeneous compression the fusion fuel target and finally creating stable fuel ignition conditions in order to achieve the appropriate energy gain. In the fusion process, high power lasers are used to ignite the target. If the laser field is strong enough and has a suitable geometrical structure, the target is ignited and energy is released. The appearance of various electromagnetic instabilities such as the Weibel electromagnetic instability and stranding instability, prevent an ideal ignition [1-3]. The Weibel electromagnetic instability is one of the most important electromagnetic instabilities, which was introduced by Weibel as the cause of temperature anisotropy in fuel [4]. It was shown that the Weibel instability can be responsible for the production of the magnetic field in various objects such as gamma ray bursts, nuclear jets, and galaxy clusters [5]. The Weibel instability mechanism in dense magnetic plasma was investigated in 2016, which due to electron velocity distribution, ion-electron collision effect and relativistic properties plays an important role in this plasma [6]. In 2018, propagation of waves impinging and growing modes of Weibel instability of plasma density gradient with shear stress flow have been investigated [7]. In recent years the numerical Weibel instability was studied in a fusion plasma [8, 9]. The Weibel instability growth rate with generalized distribution was calculated in the following years. The study of the growth rate of Weibel instability caused by plasma particles distribution functions such as bi-Maxwellian, Kappa distribution, and generalized distribution function are among other studies that have been carried out in the description of nuclear fusion plasmas [10-12]. Also, the effect of the quantum parameter in energy transfer in fusion plasma and the growth rate of electromagnetic instabilities have been discussed in strongly coupled plasma with high density and high temperature [13, 14]. The beam-plasma particles distribution function is an important parameter in the calculation of the electromagnetic instability growth rate and the energy transfer of the projectile beam in the fusion plasma environment. We have decided to analyze the effect of the beam-plasma particles distribution function on the Weibel instability growth rate. In this research, the plasma distribution function is considered Maxwell-Boltzmann, and the electron beam that penetrates into the strongly coupled plasma is considered relativity and in three modes of Kappa, semi-Maxwell and bi-Maxwellian distribution functions for different strongly coupled plasma parameters separately. First, in section 2, the computational model of the growth rate is analyzed analytically for each individual beam distribution function. In section 3, the results of the calculations along with the growth rate graph for the basis of various parameters are presented. The obtained results are discussed and concluded in section 4.

2 Calculation of Theoretical Model

The general linear dispersion relation for the study of transverse waves in strongly coupled plasma is obtained by combining the kinetic theory of Vlasov relativistic equation with the Maxwell equation follows [15]

$$\begin{aligned}&\omega^2 - c^2k^2 - \omega_{be}^2 + \pi\omega_{be}^2 \int_{-\infty}^{+\infty}\int_{0}^{+\infty} \frac{kp_\perp^3}{m\gamma\left(\omega - kv_\parallel\right)}\frac{\partial f_0^b}{\partial p_\parallel} dp_\perp dp_\parallel - \omega_{pe}^2 \\ &+ \frac{m\omega_{pe}^2}{2n_p\hbar}\int_{-\infty}^{+\infty}\int_{0}^{+\infty} \frac{p_\perp^2}{\left(\omega - kv_\parallel\right)}\left[f_0^p\left(p_\perp, p_\parallel + \frac{\hbar k}{2}\right) - f_0^p\left(p_\perp, p_\parallel - \frac{\hbar k}{2}\right)\right] dp_\perp dp_\parallel = 0.\end{aligned} \tag{1}$$

In this equation, ω and k are the frequency and wave number of wave instability, respectively, f_0 is the equilibrium dispersion function, ω_{pe} and ω_{be} are the plasma and beam frequencies respectively, m, n_p and p are the rest mas, plasma particles density and particles momentum respectively. In order to analyze the effect of the beam-plasma particles distribution function on the growth rate of Weibel electromagnetic instability, we consider a model where the electromagnetic wave propagates in the direction $\vec{k} = ke_z$,the governing distribution function on plasma particles is considering as the Maxwell-Boltzmann distribution and the governing distribution function on beam particles is considering three different distribution function such as, Kappa, Semi-relativistic Maxwellian, and Maxwell distributions.

2.1 Beam particles with Kappa distribution function

To analyze the effect of the beam-plasma particles distribution function on the growth rate of Weibel electromagnetic instability, we consider the plasma and beam particles distribution function as the Maxwell-Boltzmann distribution (f_0^p) as the Kappa distribution (f_0^b) respectively as follows [16, 17]

$$f_{0\kappa}^b(p, p_\parallel) = \frac{1}{\pi^{\frac{3}{2}}\theta_b^2\theta_{\parallel b}} \frac{\Gamma(\kappa+1)}{\kappa^{\frac{3}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)} \left[1 + \frac{\left(p_\parallel - p_d^b\right)^2}{\kappa m^2\gamma^2\theta_{\parallel b}^2} + \frac{p_\perp^2}{\kappa m^2\gamma^2\theta_{\perp b}^2}\right]^{-\kappa-1}, \tag{2}$$

$$f_0^p(p_\perp, p_\parallel) = \frac{\sqrt{\eta}n_0}{\theta_{\perp p}^2\theta_{\parallel p}} \left(\frac{1}{\pi K_B}\right)^{3/2} exp\left(-\frac{p_\perp^2}{K_B m^2\theta_{\perp p}^2} - \frac{\eta\left(p_\parallel + p_d^p\right)^2}{K_B m^2\theta_{\parallel p}^2}\right), \tag{3}$$

which in the above equations K_B,η,n_0 and Γ represent the Boltzmann's constant, gradient density, plasma particles density and the gamma function, respectively. p_d^b and p_d^p are drift moment of beam and plasma respectively. θ represents the temperature velocity and it is related to the particle temperature, T, by

$$\theta_{\perp b}^2 = (\frac{2\kappa-3}{\kappa})v_{T_\perp}^2, \qquad \theta_{\parallel b}^2 = \left(\frac{2\kappa-3}{\kappa}\right)v_{T_\parallel}^2, \qquad \theta_{\perp p,\parallel p}^2 = \frac{2T}{m}.$$

Here, $v_{T_{\perp,\parallel}}^2 = \frac{2T_{\perp,\parallel}}{m}$, and m is the mass of the particles. The derivative of the parallel beam distribution function in equation (1) is calculated as follows

$$\frac{\partial f_{0\kappa}^b}{\partial p_\parallel} = \frac{-2\left(p_\parallel - p_d^b\right)}{\pi^{\frac{3}{2}}\theta_{\perp b}^2\theta_{\parallel b}^3 m^2\gamma^2} \frac{\Gamma(\kappa+2)}{\kappa^{5/2}\Gamma\left(\kappa-\frac{1}{2}\right)} \left[1 + \frac{\left(p_\parallel - p_d^b\right)^2}{\kappa m^2\gamma^2\theta_{\parallel b}^2} + \frac{p_\perp^2}{\kappa m^2\gamma^2\theta_{\perp b}^2}\right]^{-\kappa-2} \tag{4}$$

By replacing equations (3) and (4) in equation (1), equation (1) is rewritten as follows

$$\begin{aligned}
&\omega^2 - c^2k^2 - \omega_{be}^2 - \pi\omega_{be}^2\left(\frac{k}{m}\right)\int_{-\infty}^{\infty}\int_0^{\infty} \frac{p_\perp^3}{(\omega - kv_\parallel)} \frac{2\left(p_\parallel - p_d^b\right)}{\pi^{\frac{3}{2}}\theta_{\perp b}^2\theta_{\parallel b}^3 m^2\gamma^2} \frac{\Gamma(\kappa+2)}{\kappa^{\frac{5}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)} \\
&\left[1 + \frac{\left(p_\parallel - p_d^p\right)^2}{\kappa m^2\theta_{\parallel p}^2} + \frac{p_\perp^2}{\kappa m^2\theta_{\perp p}^2}\right]^{-\kappa-2} dp_\perp dp_\parallel - \omega_{pe}^2 \\
&+ \frac{\omega_{pe}^2}{2n_0\hbar m}\int_{-\infty}^{+\infty}\int_0^{+\infty} \frac{p_\perp^2}{(\omega - kv_\parallel)} \frac{\sqrt{\eta}n_0}{\theta_{\perp p}^2\theta_{\parallel p}} \left(\frac{1}{\pi K_B}\right)^{\frac{3}{2}} exp\Bigg[-\frac{p_\perp^2}{K_B m\theta_{\perp p}^2} - \frac{\eta(p_\parallel + p_d^b + \frac{\hbar k}{2})^2}{K_B m\theta_{\parallel p}^2} \\
&- \left(-\frac{p_\perp^2}{K_B m\theta_{\perp p}^2} - \frac{\eta\left(p_\parallel + p_d^b - \frac{\hbar k}{2}\right)^2}{K_B m\theta_{\parallel p}^2}\right)\Bigg] dp_\perp dp_\parallel = 0.
\end{aligned} \tag{5}$$

By calculating the parallel and perpendicular integrals of equation (5) as follows

$$\int_0^{+\infty} p_\perp^3 \left[1+\frac{\left(p_\parallel - p_d^b\right)^2}{\kappa m^2\gamma^2\theta_{\parallel b}^2}+\frac{p_\perp^2}{m^2\gamma^2\theta_{\perp b}^2}\right]^{-\kappa-2} dp_\perp = \frac{\theta_{\perp b}^4 m^4\gamma^4}{2}, \tag{6}$$

$$\int_{-\infty}^{+\infty} \frac{k\left(p_\parallel - p_d^b\right)}{m\gamma\left(\omega' - kv_\parallel\right)}\left[1+\frac{\left(p_\parallel - p_d^b\right)^2}{\kappa m^2\gamma^2\theta_{\parallel b}^2}\right]^{-\kappa} dp_\parallel = -\frac{\omega'}{k}\int_{-\infty}^{+\infty}\frac{\left[1+\frac{x^2}{\kappa}\right]^{-\kappa}}{(x-\varepsilon)}dx - m\gamma\theta_{\parallel b}\sqrt{\pi}$$
$$-\frac{p_d^b}{m\gamma}\int_{-\infty}^{+\infty}\frac{\left[1+\frac{x^2}{k}\right]^{-\kappa}}{(x-\varepsilon)}dx, \tag{7}$$

where $\varepsilon = \frac{1}{m\gamma\theta_{\parallel b}}\left[\frac{\omega}{k} - p_d^b\right]$, $x = \frac{1}{m\gamma\theta_{\parallel b}}\left[p_\parallel - p_d^b\right]$, and

$$\int_0^{+\infty} p_\perp^2 exp\left(-\frac{p_\perp^2}{K_B m\theta_{\perp p}^2}\right)dp_\perp = \frac{\sqrt{\pi}}{4}\theta_{\perp p}^3\left(K_B m\right)^{\frac{3}{2}}, \tag{8}$$

$$\int_{-\infty}^{+\infty}\frac{1}{m\gamma(\omega' - kv_\parallel)}exp\left(-\frac{\eta(p_\parallel + p_d^b\frac{\hbar k}{2})^2}{K_B m\theta_{\parallel p}^2}\right)dp_\parallel = -\frac{1}{k}\int_{-\infty}^{+\infty}\frac{exp\left(-y^2\right)}{(y-\zeta)}dy, \tag{9}$$

where $y = \sqrt{\frac{\eta}{K_B m}}\frac{1}{\theta_{\parallel p}}(p_\parallel + p_d^b\frac{\hbar k}{2})$ and $\zeta = \sqrt{\frac{\eta}{K_B m}}\frac{1}{\theta_{\parallel p}}\left(\frac{\omega}{k} + p_d^p \pm \frac{\hbar k}{2}\right)$. Equation (5) is rewritten as below

$$\omega^2 - c^2k^2 - \omega_{be}^2 + \omega_{be}^2\frac{\theta_{\perp b}^2}{\theta_{\parallel b}^2}\frac{1}{\theta_{\parallel b}}m^2\gamma^2\left[\frac{1}{\sqrt{\pi}}\frac{\Gamma(\kappa)}{\kappa^{\frac{1}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)}\frac{\omega}{k}\int_{-\infty}^{+\infty}\frac{\left[1+\frac{x^2}{k}\right]^{-\kappa}}{(x-\varepsilon)}dx\right] + \omega_{be}\frac{\theta_{\perp b}^2}{\theta_{\parallel b}^2}$$
$$-\omega_{be}^2\frac{\theta_{\perp b}^2}{\theta_{\parallel b}^2}\frac{p_d^b}{\theta_{\parallel b}}m\gamma\left[\frac{1}{\sqrt{\pi}}\frac{\Gamma(\kappa)}{\kappa^{\frac{1}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)}\int_{-\infty}^{+\infty}\frac{\left[1+\frac{x^2}{\kappa}\right]^{-\kappa}}{(x-\varepsilon)}dx\right] - \omega_{pe}^2$$
$$-\frac{\omega_{pe}^2}{8\hbar k\pi}\sqrt{\eta}\sqrt{m}\frac{\theta_{\perp p}}{\theta_{\parallel p}}\int_{-\infty}^{+\infty}\frac{exp\left(-y^2\right)}{y-\zeta}dy = 0. \tag{10}$$

By introducing the dispersion function $Z_\kappa^*(\varepsilon)$ and $Z(\zeta)$ ([15, 16]) as follows

$$Z^*(\varepsilon) = \frac{1}{\sqrt{\pi}}\left[\frac{\Gamma(\kappa)}{\kappa^{\frac{1}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)}\int_{-\infty}^{+\infty}\frac{\left(1+\frac{x^2}{k}\right)^{-\kappa}}{(x-\varepsilon)}dx\right], \tag{11}$$

$$Z(\zeta) = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{+\infty}\frac{exp\left(-y^2\right)}{y-\zeta}dy. \tag{12}$$

Equation (10) is rewritten as follows

$$\omega^2 - c^2k^2 - \omega_{be}^2 + \omega_{be}^2\frac{\theta_{\perp b}^2}{\theta_{\parallel b}^2}\frac{1}{\theta_{\parallel b}}m^2\gamma^2\frac{\omega}{k}Z_\kappa^*(\varepsilon) + \omega_{be}^2 m^3\gamma^3\frac{\theta_{\perp b}^2}{\theta_{\parallel b}^2} - \omega_{be}^2\frac{\theta_{\perp b}^2}{\theta_{\parallel b}^2}\frac{p_d^b}{\theta_{\parallel b}}m\gamma Z_\kappa^*(\varepsilon)$$
$$-\omega_{pe}^2 - \frac{\left(\omega_{pe}^2\sqrt{\pi}\right)}{8\hbar k\pi}\sqrt{\eta}\sqrt{m}\frac{\theta_{\perp p}}{\theta_{\parallel p}}Z(\zeta) = 0. \tag{13}$$

By substituting the value of ζ in equation (13), it is rewritten as follows

$$\omega^2 - c^2k^2 - \omega_{be}^2 + \omega_{be}^2 \frac{\theta_{\perp b}^2}{\theta_{\parallel b}^2} \frac{1}{\theta_{\parallel b}} m^2\gamma^2 \frac{\omega}{k} Z^*(\varepsilon) + \omega_{be}^2 m^3\gamma^3 \frac{\theta_{\perp b}^2}{\theta_{\parallel b}^2} - \omega_{be}^2 \frac{\theta_{\perp b}^2}{\theta_{\parallel b}^2} \frac{p_d^b}{\theta_{\parallel b}} m\gamma Z_\kappa^*(\varepsilon) - \omega_{pe}^2$$
$$- \frac{\omega_{pe}^2\sqrt{\pi}}{8\hbar k\pi} \sqrt{\eta}\sqrt{m} \frac{\theta_{\perp p}}{\theta_{\parallel p}} Z\left(\sqrt{\frac{\eta}{K_Bm}} \frac{1}{\theta_{\parallel p}} \left(\frac{\omega}{k} + p_d^p + \frac{\hbar k}{2}\right) - \sqrt{\frac{\eta}{K_{Bm}}} \frac{1}{\theta_{\parallel p}} \left(\frac{\omega}{k} + p_d^p - \frac{\hbar k}{2}\right)\right) = 0. \tag{14}$$

By definition the quantum parameter as; $R = \frac{\hbar k}{2}$, for small wavelengths ($\zeta \ll 1$) in the limit of small quantum effects ($R \ll 1$), equation (14) is corrected as follows

$$\omega^2 - c^2k^2 - \omega_{be}^2 + \omega_{be}^2 \frac{\theta_{\perp b}^2}{\theta_{\parallel b}^2} \frac{1}{\theta_{\parallel b}} m^2\gamma^2 \frac{\omega}{k} Z^*(\varepsilon) + \omega_{be}^2 m^3\gamma^3 \frac{\theta_{\perp b}^2}{\theta_{\parallel b}^2} - \omega_{be}^2 \frac{\theta_{\perp b}^2}{\theta_{\parallel b}^2} \frac{p_d^b}{\theta_{\parallel b}} m\gamma Z_\kappa^*(\varepsilon)$$
$$- \omega_{pe}^2 \left[1 + \frac{\theta_{\perp p}}{\theta_{\parallel p}} \sqrt{\eta} \left(-1 - \zeta Z(\zeta) + \frac{R^2}{6}\right)\right] = 0. \tag{15}$$

For $\kappa = 3$ and ($\varepsilon \ll 1$), the dispersion relation $Z_\kappa^*(\varepsilon)$ is written as follows

$$Z_3^*(\varepsilon) = \varepsilon\left(-1.66 - 0.370^2 - \cdots\right) + i\left(1.539 - 1.539\varepsilon^2 + \cdots\right). \tag{16}$$

Also, for ($\zeta \ll 1$), the dispersion relation of $Z(\zeta)$ is written as follows

$$Z(\zeta) = -2\zeta + \cdots + i\sqrt{\pi} exp\left(-\zeta^2\right). \tag{17}$$

For low-frequency wave, ($\omega^2 \ll c^2k^2$), and by replacing $\omega = \omega_r + i\delta$, the growth rate is obtained as follows

$$\delta_\kappa = \frac{k\left[-\frac{c^2k^2}{\omega_{pe}^2} - \frac{\omega_{be}^2}{\omega_{pe}^2} + \frac{\omega_{be}^2}{\omega_{pe}^2}\alpha m^3\gamma^3 - 1 + \beta\sqrt{\eta}\left(1 - \frac{R^2}{6}\right)\right]}{1.539\left[\frac{\omega_{be}^2}{\omega_{pe}^2}\frac{\alpha}{\theta_{\parallel b}} m^2\gamma^2 + \beta\sqrt{\pi}(1.539)\right]}, \tag{18}$$

where $\alpha = \frac{\theta_{\perp b}^2}{\theta_{\parallel b}^2}$ and $\beta = \frac{\theta_{\perp p}}{\theta_{\parallel p}}$. Using the following variables

$$u = \frac{\frac{\omega_{be}^2}{\omega_{pe}^2}\left[-1 + \alpha m^3\gamma^3\right]}{\left(\frac{\omega_{be}^2}{\omega_{pe}^2}\frac{\alpha}{\theta_{\parallel b}} m^2\gamma^2 + \beta\sqrt{\pi}(1.539)\right)}, \tag{19}$$

$$n = \frac{\omega_{be}^2}{\omega_{pe}^2}\frac{\alpha}{\theta_b} m^2\gamma^2 + \beta\sqrt{\pi}(1.539). \tag{20}$$

The Weibel electromagnetic instability growth rate for the beam particles with Kappa distribution function is calculated as follows

$$\delta_\kappa = 0.650k\left[u - \frac{\left(1 + \frac{c^2k^2}{\omega_{pe}^2} + \beta\sqrt{\eta}\left(1 - \frac{R^2}{6}\right)\right)}{n}\right]. \tag{21}$$

2.2 Beam particles with Semi-relativistic Maxwellian distribution function

By considering the Semi-relativistic Maxwellian distribution function of the beam as follows [12]

$$f_0^b(p_\perp, p_\parallel) = \frac{1}{\pi^{\frac{3}{2}}}\left(\frac{1}{m^2\gamma^2\theta_{\perp b}^2}\right)\left(\frac{1}{m\gamma\theta_{\parallel b}}\right)\left(1+\frac{2c^2}{\theta_{\perp b}^2}\right)^{-1} \times exp\left(-\frac{\left(p_\parallel - p_d^b\right)^2}{m^2\gamma^2\theta_{\parallel b}^2} - \frac{2c^2}{\theta_{\perp b}^2}\sqrt{1+\frac{p_\perp^2}{m^2\gamma^2c^2}} - 1\right). \tag{22}$$

The derivative of the beam function to the parallel momentum is calculated as follows.

$$\frac{\partial f_0^b}{\partial p_\parallel} = \frac{-2\left(p_\parallel - p_d^b\right)}{\pi^{\frac{3}{2}}}\left(\frac{1}{m^2\gamma^2\theta_{\perp b}^2}\right)\left(\frac{1}{m^3\gamma^3\theta_{\parallel b}^3}\right)\left(1+\frac{2c^2}{\theta_{\perp b}^2}\right)^{-1} \times exp\left(-\frac{\left(p_\parallel - p_d^b\right)^2}{m^2\gamma^2\theta_{\parallel b}^2} - \frac{2c^2}{\theta_{\perp b}^2}\sqrt{1+\frac{p_\perp^2}{m^2\gamma^2c^2}} - 1\right). \tag{23}$$

By replacing equations (3) and (23) in equation (1), equation (1) is rewritten as follows

$$\omega^2 - c^2k^2 - \omega_{be}^2 - \pi\omega_{be}^2 \int_{-\infty}^{+\infty}\int_0^{+\infty} \frac{kp_\perp^3}{m\gamma\left(\omega - kv_\parallel\right)} \frac{2\left(p_\parallel - p_d^b\right)}{\pi^{\frac{3}{2}}}\left(\frac{1}{m^2\gamma^2\theta_{\perp b}^2}\right)\left(\frac{1}{m^3\gamma^3\theta_{\parallel b}^3}\right) \times \left(1+\frac{2c^2}{\theta_{\perp b}^2}\right)^{-1} exp\left(-\frac{\left(p_\parallel - p_d^b\right)^2}{m^2\gamma^2\theta_{\parallel b}^2} - \frac{2c^2}{\theta_{\perp b}^2}\sqrt{1+\frac{p^2}{m^2\gamma^2c^2}} - 1\right) dp_\perp dp_\parallel - \omega_{pe}^2 + \frac{\omega_{pe}^2}{2n_0\hbar m}\int_{-\infty}^{+\infty}\int_0^{+\infty} \frac{p_\perp^2}{\left(\omega - kv_\parallel\right)} \frac{\sqrt{\eta}n_0}{\theta_{\perp p}^2\theta_{\parallel p}}\left(\frac{1}{\pi K_B}\right)^{\frac{3}{2}} exp\left(-\frac{p_\perp^2}{K_Bm\theta_{\perp p}^2} - \frac{\eta\left(p_\parallel + p_d^b + \frac{\hbar k}{2}\right)^2}{K_Bm\theta_{\parallel p}^2}\right) \times exp\left(\frac{p_\perp^2}{K_Bm\theta_{\perp p}^2} + \frac{\eta\left(p_\parallel + p_d^b - \frac{\hbar k2}{2}\right)^2}{K_Bm\theta_{\parallel p}^2}\right) dp_\perp dp_\parallel = 0. \tag{24}$$

By calculating the vertical and parallel integral of the beam as follows

$$\int_0^\infty p_\perp^3\left(-\frac{2c^2}{\theta_{\perp b}^2}\sqrt{1+\frac{p_\perp^2}{m^2\gamma^2c^2}} - 1\right) dp_\perp = \frac{3\theta_{\perp p}^6 m^4\gamma^4}{2c^2}\left[\frac{\theta_{\perp b}^2}{4c^2} + \frac{c^2}{3\theta_b^2} + \frac{1}{2}\right], \tag{25}$$

$$\int_{-\infty}^{+\infty} \frac{k\left(p_\parallel - p_d^b\right)}{m\gamma\left(\omega - kv_\parallel\right)} exp\left(-\frac{\left(p_\parallel - p_d^b\right)^2}{m^2\gamma^2\theta_{\parallel b}^2}\right) dp_\parallel = -\frac{\omega}{k}\int_{-\infty}^{+\infty}\frac{exp\left(-x^2\right)}{x-\varepsilon}dx - m\gamma\theta_{\parallel b}\sqrt{\pi} + \frac{p_d^b}{m\gamma}\int_{-\infty}^{+\infty}\frac{exp\left(-x^2\right)}{x-\varepsilon}dx, \tag{26}$$

where $\xi = \frac{1}{m\gamma\theta_{\| b}}\left[\frac{\omega}{k} - p_d^b\right]$ and $x = \frac{1}{m\gamma\theta_{\| b}}\left[\frac{p_\|}{m\gamma} - p_d^b\right]$. Equation (24) is rewritten as fallows

$$\omega^2 - c^2k^2 - \omega_{be}^2 - \omega_{be}^2 \frac{2}{\pi^{\frac{3}{2}}}\left(\frac{1}{m^2\gamma^2\theta_{\perp b}^2}\right)\left(\frac{1}{m^3\gamma^3\theta_{\| b}^3}\right)\left(1 + \frac{2c^2}{\theta_b^2}\right)^{-1}$$
$$\left(\frac{3\theta_{\perp b}^6 m^4\gamma^4}{2c^2}\left[\frac{\theta_{\perp b}^2}{4c^2} + \frac{c^2}{3\theta_{\perp b}^2} + \frac{1}{2}\right]\right)$$
$$\left[-\frac{\omega}{k}\int_{-\infty}^{+\infty}\frac{exp\left(-x^2\right)}{(x-\varepsilon)}dx - m\gamma\theta_{\| b}\sqrt{\pi} + \frac{p_d^b}{m\gamma}\int_{-\infty}^{+\infty}\frac{exp\left(-x^2\right)}{(x-\xi)}dx\right]$$
$$-\omega_{pe}^2 - \frac{\omega_{pe}^2}{8\hbar k\pi}\sqrt{\eta}\sqrt{m}\frac{\theta_{\perp p}}{\theta_{\| p}}\int_{-\infty}^{+\infty}\frac{exp\left(-y^2\right)}{(y-\zeta)}dy = 0. \quad (27)$$

For the scatter function as follows

$$Z(\xi) = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{+\infty}\frac{exp\left(-y^2\right)}{(y-\xi)}dy, \quad (28)$$

equation (27) is rewritten as follows

$$\omega^2 - c^2k^2 - \omega_{be}^2 + \frac{\omega_{be}^2}{\theta_{\| b}m\gamma}\frac{\theta_{\perp b}^2}{\theta_{\| b}^2}\frac{\omega}{k}(1 + \frac{2c^2}{\theta_{\perp b}^2})^{-1}\left(\frac{3\theta_{\perp b}^2}{c^2}\left(\frac{\theta_{\perp b}^2}{4c^2} + \frac{c^2}{3\theta_{\perp b}^2} + \frac{1}{2}\right)\right)Z(\xi)$$
$$+\omega_{be}^2\frac{\theta_{\perp b}^2}{\theta_{\| b}^2}\left(1 + \frac{2c^2}{\theta_{\perp b}^2}\right)^{-1}\left(\frac{3\theta_{\perp b}^2}{c^2}\left(\frac{\theta_{\perp b}^2}{4c^2} + \frac{c^2}{3\theta_{\perp b}^2} + \frac{1}{2}\right)\right) - \frac{\omega_{be}^2}{\theta_{\| b}}\frac{\theta_{\perp b}^2}{\theta_{\| b}^2}\left(1 + \frac{2c^2}{\theta_{\perp b}^2}\right)^{-1}$$
$$\left(\frac{3\theta_{\perp b}^2}{c^2}(\frac{\theta_{\perp b}^2}{4c^2} + \frac{c^2}{3\theta_{\perp b}^2} + \frac{1}{2})\right)\frac{p_d^b}{m^2\gamma^2}Z(\xi) - \omega_{pe}^2 - \frac{\omega_{pe}^2\sqrt{2}}{8\hbar k\sqrt{\pi}}\sqrt{\eta}\frac{\theta_{\perp p}}{\theta_{\| p}}Z(\zeta) = 0. \quad (29)$$

By substituting the value of ζ in equation (29), it is corrected as follows

$$\omega^2 - c^2k^2 - \omega_{be}^2 + \frac{\omega_{be}^2}{\theta_{\| b}m\gamma}\frac{\theta_{\perp b}^2}{\theta_{\| b}^2}\frac{\omega}{k}\left(1 + \frac{2c^2}{\theta_{\perp b}^2}\right)^{-1}\left(\frac{3\theta_{\perp b}^2}{c^2}\left(\frac{\theta_{\perp b}^2}{4c^2} + \frac{c^2}{3\theta_{\perp b}^2} + \frac{1}{2}\right)\right)Z(\xi)$$
$$+\omega_{be}^2\frac{\theta_{\perp b}^2}{\theta_{\| b}^2}\left(1 + \frac{2c^2}{\theta_{\perp b}^2}\right)^{-1}\left(\frac{3\theta_{\perp b}^2}{c^2}\left(\frac{\theta_{\perp b}^2}{4c^2} + \frac{c^2}{3\theta_{\perp b}^2} + \frac{1}{2}\right)\right)$$
$$-\frac{\omega_{be}^2}{\theta_{\| b}}\frac{\theta_{\perp b}^2}{\theta_{\| b}^2}\left(1 + \frac{2c^2}{\theta_{\perp b}^2}\right)^{-1}\left(\frac{3\theta_{\perp b}^2}{c^2}\left(\frac{\theta_{\perp b}^2}{4c^2} + \frac{c^2}{3\theta_{\perp b}^2} + \frac{1}{2}\right)\right)\frac{p_d^b}{m^2\gamma^2}Z(\xi) - \omega_{pe}^2$$
$$-\frac{\omega_{pe}^2\sqrt{\pi}}{8\hbar k\pi}\sqrt{\eta}\sqrt{m}\frac{\theta_{\perp p}}{\theta_{\| p}}Z\left(\sqrt{\frac{\eta}{K_B m}}\frac{1}{\theta_{\| p}}\left(\frac{\omega}{k} + p_d^p + \frac{\hbar k}{2}\right) - \sqrt{\frac{\eta}{K_B m}}\frac{1}{\theta_{\| p}}\left(\frac{\omega}{k} + p_d^p - \frac{\hbar k}{2}\right)\right) = 0. \quad (30)$$

For small wavelengths ($\zeta \ll 1$) in the limit of small quantum effects ($R \ll 1$), equation (30) is approximated as follows

$$\omega^2 - c^2k^2 - \omega_{be}^2 + \frac{\omega_{be}^2}{\theta_{\parallel b} m\gamma}\frac{\theta_{\perp b}^2}{\theta_{\parallel b}^2}\frac{\omega}{k}\left(1+\frac{2c^2}{\theta_{\perp b}^2}\right)^{-1}\left(\frac{3\theta_{\perp b}^2}{c^2}\left(\frac{\theta_{\perp b}^2}{4c^2}+\frac{c^2}{3\theta_{\perp b}^2}+\frac{1}{2}\right)\right)Z(\xi)$$
$$+\omega_{be}^2\frac{\theta_{\perp b}^2}{\theta_{\parallel b}^2}\left(1+\frac{2c^2}{\theta_{\perp b}^2}\right)^{-1}\left(\frac{3\theta_{\perp b}^2}{c^2}\left(\frac{\theta_{\perp b}^2}{4c^2}+\frac{c^2}{3\theta_{\perp b}^2}+\frac{1}{2}\right)\right)$$
$$-\frac{\omega_{be}^2}{\theta_{\parallel b}}\frac{\theta_{\perp b}^2}{\theta_{\parallel b}^2}\left(1+\frac{2c^2}{\theta_{\perp b}^2}\right)^{-1}\left(\frac{3\theta_{\perp b}^2}{c^2}\left(\frac{\theta_{\perp b}^2}{4c^2}+\frac{c^2}{3\theta_{\perp b}^2}+\frac{1}{2}\right)\right)\frac{p_d^b}{m^2\gamma^2}Z(\xi)$$
$$-\omega_{pe}^2\left[1+\frac{\theta_{\perp p}}{\theta_{\parallel p}}\sqrt{\eta}\left(-1-\zeta Z(\zeta)+\frac{R^2}{6}\right)\right]=0. \tag{31}$$

By applying the variable χ which is defined as follows

$$\chi = \frac{\left(\frac{\theta_{\perp b}^2}{4c^2}+\frac{c^2}{3\theta_{\perp b}^2}+\frac{1}{2}\right)}{\left(1+\frac{2c^2}{\theta_{\perp b}^2}\right)\left(\frac{c^2}{3\theta_{\perp b}^2}\right)}. \tag{32}$$

Also, for ($\zeta \ll 1$), the dispersion relation $Z(\zeta)$ is written as follows

$$Z(\xi) = -2\xi + \cdots + i\sqrt{\pi}exp\left(-\xi^2\right). \tag{33}$$

Equation (31) is corrected as follows

$$\omega^2 - c^2k^2 - \omega_{be}^2 + i\frac{\omega_{be^2}}{\theta_{\parallel b} m\gamma}\frac{\theta_{\perp b}^2}{\theta_{\parallel b}^2}\frac{\omega}{k}\chi\sqrt{\pi} + \omega_{be}^2\frac{\theta_{\perp b}^2}{\theta_{\parallel b}^2}\chi - i\frac{\omega_{be}^2}{\theta_{\parallel b}}\frac{\theta_{\perp b}^2}{\theta_{\parallel b}^2}\frac{p_d^b}{m^2\gamma^2}\chi\sqrt{\pi}$$
$$-\omega_{pe}^2\left[1+\frac{\theta_{\perp p}}{\theta_{\parallel p}}\sqrt{\eta}\left(-1-\zeta\left(i\sqrt{\pi}\right)+\frac{R^2}{6}\right)\right]=0. \tag{34}$$

For low-frequency wave, ($\omega^2 \ll c^2k^2$), and by replacing $\omega = \omega_r + i\delta$, the growth rate is obtained as follows

$$\delta^* = \frac{k\left[-\frac{c^2k^2}{\omega_{pe}^2}-\frac{\omega_{be}^2}{\omega_{pe}^2}+\frac{\omega_{be}^2}{\omega_{pe}^2}\alpha\chi - 1 + \beta\sqrt{\eta}\left(1-\frac{R^2}{6}\right)\right]}{\chi\left[\frac{\omega_{be}^2\alpha}{\omega_{pe}^2 m\gamma\theta_{\parallel b}}+\frac{\beta\sqrt{\pi}}{\chi}\right]}. \tag{35}$$

By using the following variables

$$u^* = \frac{\frac{\omega_{be}^2}{\omega_{pe}^2}\left[-1+\alpha\chi\right]}{\left(\frac{\omega_{be}^2\alpha}{\omega_{pe}^2 m\gamma\theta_b}+\frac{\beta\sqrt{\pi}}{\chi}\right)}, \tag{36}$$

$$n^* = \frac{\omega_{be}^2\alpha}{\omega_{pe}^2 m\gamma\theta_{\parallel b}}+\frac{\beta\sqrt{\pi}}{\chi}. \tag{37}$$

The Weibel electromagnetic instability growth rate for the beam particles with Semi-relativistic Maxwellian distribution function is calculated as follows

$$\delta^* = \frac{k}{\chi}\left(u^* - \frac{\left(1+\frac{c^2k^2}{\omega_{pe}^2}+\beta\sqrt{\eta}\left(1-\frac{R^2}{6}\right)\right)}{n^*}\right). \tag{38}$$

2.3 Beam particles with bi-Maxwellian distribution function

In this part, the relativistic beam distribution function is considered as bi-Maxwell's distribution [11]

$$f_0^b\left(p_\perp, p_\parallel\right) = \frac{n_0}{\theta_{\perp p}^2 \theta_{\parallel p}} \left(\frac{1}{\pi}\right)^{\frac{3}{2}} exp\left(-\frac{p_\perp^2}{m^2\gamma^2\theta_{\perp p}^2} - \frac{\left(p_\parallel + p_d^b\right)^2}{m^2\gamma^2\theta_{\parallel p}^2}\right), \tag{39}$$

and $\frac{\partial f_0^b}{\partial p_\parallel}$ is calculated as follows

$$\frac{\partial f_0^b}{\partial p_\parallel} = \frac{-2\left(p_\parallel - p_d^b\right)}{\pi^{\frac{3}{2}}\theta_{\perp b}^2\theta_{\parallel b}^3 m^2\gamma^2} exp\left(-\frac{p_\perp^2}{m^2\gamma^2\theta_{\perp p}^2} - \frac{\left(p_\parallel + p_d^b\right)^2}{m^2\gamma^2\theta_{\parallel p}^2}\right). \tag{40}$$

By inserting equations (2) and (40) in equation (1), equation (1) becomes as follows

$$\begin{aligned}
&\omega^2 - c^2k^2 - \omega_{be}^2 + \pi\omega_{be}^2\frac{k}{m}\int_{-\infty}^{+\infty}\int_0^{+\infty}\frac{p_\perp^3}{\left(\omega - kv_\parallel\right)}\frac{-2\left(p_\parallel - p_d^b\right)}{\pi^{\frac{3}{2}}\theta_{\perp b}^2\theta_{\parallel b}^3 m^2\gamma^2}\\
&\times exp\left(-\frac{p_\perp^2}{m^2\gamma^2\theta_{\perp p}^2} - \frac{\left(p_\parallel + p_d^b\right)^2}{m^2\gamma^2\theta_{\parallel p}^2}\right) - \omega_{pe}^2\\
&+ \frac{\omega_{pe}^2}{2n_0\hbar m}\int_{-\infty}^{+\infty}\int_0^{+\infty}\frac{p_\perp^2}{\left(\omega - kv_\parallel\right)}\frac{\sqrt{\eta}n_0}{\theta_{\perp p}^2\theta_{\parallel p}}\left(\frac{1}{\pi K_B}\right)^{\frac{3}{2}}\\
&\times exp\left[\left(-\frac{p_\perp^2}{K_B m\theta_{\perp p}^2} - \frac{\eta\left(p_\parallel + p_d^b + \frac{\hbar k}{2}\right)^2}{K_B m\theta_{\parallel p}^2}\right) - \left(-\frac{p_\perp^2}{K_B m\theta_{\perp p}^2} - \frac{\eta\left(p_\parallel + p_d^b - \frac{\hbar k}{2}\right)^2}{K_B m\theta_{\parallel p}^2}\right)\right] dp_\perp dp_\parallel\\
&= 0.
\end{aligned} \tag{41}$$

By calculating the vertical and parallel integral of the beam as follows

$$\int_0^{+\infty} p_\perp^3 exp\left(-\frac{p_\perp^2}{m^2\gamma^2\theta_{\perp p}^2}\right) dp_\perp = \frac{\theta_{\perp b}^4 m^4\gamma^4}{2} \tag{42}$$

$$\begin{aligned}
&\int_{-\infty}^{+\infty}\frac{k\left(p_\parallel - p_d^b\right)}{m\gamma\left(\omega - kv_\parallel\right)} exp(\frac{-\left(p_\parallel - p_d^b\right)^2}{m^2\gamma^2\theta_{\parallel b}^2}) = -\frac{\omega}{k}\int_{-\infty}^{+\infty}\frac{exp\left(-x^2\right)}{(x-\varepsilon)}dx - m\gamma\theta_{\parallel b}\sqrt{\pi}\\
&+ \frac{p_d^b}{m\gamma}\int_{-\infty}^{+\infty}\frac{exp\left(-x^2\right)}{(x-\varepsilon)}dx.
\end{aligned} \tag{43}$$

Equation (41) is rewritten as below

$$\begin{aligned}
&\omega^2 - c^2k^2 - \omega_{be}^2 + \omega_{be}^2\frac{\theta_{\perp b}^2}{\theta_{\parallel b}^2}\frac{1}{\theta_{\parallel b}}m^2\gamma^2\left[\frac{1}{\sqrt{\pi}}\frac{\omega}{k}\int_{-\infty}^{+\infty}\frac{exp\left(-x^2\right)}{(x-\epsilon)}dx\right]\\
&- \omega_{be}^2\frac{\theta_{\perp b}^2}{\theta_{\parallel b}^2}m^3\gamma^3 + \omega_{be}^2\frac{\theta_{\perp b}^2}{\theta_{\parallel b}^2}\frac{p_d^b}{\theta_{\parallel b}}m\gamma\left[\frac{1}{\sqrt{\pi}}\int_{-\infty}^{+\infty}\frac{exp\left(-x^2\right)}{(x-\epsilon)}dx\right]\\
&- \omega_{pe}^2 - \frac{\omega_{pe}^2}{8\hbar k\pi}\sqrt{\eta}\sqrt{m}\frac{\theta_{\perp p}}{\theta_{\parallel p}}\int_{-\infty}^{+\infty}\frac{exp\left(-y^2\right)}{(y-\zeta)}dy = 0.
\end{aligned} \tag{44}$$

By introducing the dispersion function as follows

$$Z(\epsilon) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} \frac{exp\left(-y^2\right)}{(y-\epsilon)} dy, \tag{45}$$

equation (44) is corrected as follows

$$\begin{aligned} &\omega^2 - c^2k^2 - \omega_{be}^2 + \omega_{be}^2 \frac{\theta_{\perp b}^2}{\theta_{\parallel b}^2} \frac{1}{\theta_{\parallel b}} \frac{\omega}{k} m^2\gamma^2 Z(\epsilon) - \omega_{be}^2 \frac{\theta_{\perp b}^2}{\theta_{\parallel b}^2} m^3\gamma^3 + \omega_{be}^2 \frac{\theta_{\perp b}^2}{\theta_{\parallel b}^2} \frac{p_d^b}{\theta_{\parallel b}} m\gamma Z(\epsilon) \\ &- \omega_{pe}^2 - \frac{\omega_{pe}^2}{8\hbar k\pi} \sqrt{\eta}\sqrt{m} \frac{\theta_{\perp p}}{\theta_{\parallel p}} \int_{-\infty}^{+\infty} \frac{exp\left(-y^2\right)}{(y-\zeta)} dy = 0. \end{aligned} \tag{46}$$

By putting the value of ζ in equation (46), equation (47) is obtained

$$\begin{aligned} &\omega^2 - c^2k^2 - \omega_{be}^2 + \omega_{be}^2 \frac{\theta_{\perp b}^2}{\theta_{\parallel b}^2} \frac{1}{\theta_{\parallel b}} m^2\gamma^2 \frac{\omega}{k} Z(\epsilon) - \omega_{be}^2 \frac{\theta_{\perp b}^2}{\theta_{\parallel b}^2} m^3\gamma^3 \\ &+ \omega_{be}^2 \frac{\theta_{\perp b}^2}{\theta_{\parallel b}^2} \frac{p_d^b}{\theta_{\parallel b}} m\gamma Z(\epsilon) - \omega_{pe}^2 \\ &- \frac{\omega_{pe}^2\sqrt{\pi}}{8\hbar k\pi} \sqrt{\eta}\sqrt{m} \frac{\theta_{\perp p}}{\theta_{\parallel p}} Z\left(\sqrt{\frac{\eta}{K_B m}} \frac{1}{\theta_{\parallel p}} \left(\frac{\omega}{k} + p_d^p + \frac{\hbar k}{2}\right) - \sqrt{\frac{\eta}{K_B m}} \frac{1}{\theta_{\parallel p}} \left(\frac{\omega}{k} + p_d^p - \frac{\hbar k}{2}\right)\right) = 0. \end{aligned} \tag{47}$$

For small wavelengths ($\zeta \ll 1$) in the limit of small quantum effects ($R \ll 1$), equation (47) is corrected as follows

$$\begin{aligned} &\omega^2 - c^2k^2 - \omega_{be}^2 + \omega_{be}^2 \frac{\theta_{\perp b}^2}{\theta_{\parallel b}^2} \frac{1}{\theta_{\parallel b}} m^2\gamma^2 \frac{\omega}{k} Z(\epsilon) - \omega_{be}^2 \frac{\theta_{\perp b}^2}{\theta_{\parallel b}^2} m^3\gamma^3 + \omega_{be}^2 \frac{\theta_{\perp b}^2}{\theta_{\parallel b}^2} \frac{p_d^b}{\theta_{\parallel b}} m\gamma Z(\epsilon) \\ &- \omega_{pe}^2 \left[1 + \frac{\theta_{\perp p}}{\theta_{\parallel p}} \sqrt{\eta} \left(-1 - \zeta Z(\zeta) + \frac{R^2}{6}\right)\right] = 0. \end{aligned} \tag{48}$$

For ($\epsilon \ll 1$) the dispersion relation, $Z(\epsilon)$, is written as follows

$$Z(\epsilon) = -2\epsilon + + i\sqrt{\pi} exp\left(-\epsilon^2\right). \tag{49}$$

For low-frequency wave, ($\omega^2 \ll c^2k^2$), and by replacing $\omega = \omega_r + i\delta$, the growth rate is obtained as follows

$$\delta^\circ = \frac{k\left(-\frac{c^2k^2}{\omega_{pe}^2} - \frac{\omega_{be}^2}{\omega_{pe}^2} + \frac{\omega_{be}^2}{\omega_{pe}^2}\alpha m^3\gamma^3 - 1 + \beta\sqrt{\eta}\left(1 - \frac{R^2}{6}\right)\right)}{\sqrt{\pi}\left(\frac{\omega_{be}^2}{\omega_{pe}^2}\frac{\alpha}{\theta_{\parallel b}} m^2\gamma^2 + \beta\right)}. \tag{50}$$

Using variables

$$u^\circ = \frac{\frac{\omega_{be}^2}{\omega_{pe}^2}\left[-1 + \alpha m^3\gamma^3\right]}{\left(\frac{\omega_{be}^2}{\omega_{pe}^2}\frac{\alpha}{\theta_{\parallel b}} m^2\gamma^2 + \beta\right)}, \tag{51}$$

$$n^\circ = \frac{\omega_{be}^2}{\omega_{pe}^2}\frac{\alpha}{\theta_{\parallel b}} m^2\gamma^2 + \beta. \tag{52}$$

The Weibel electromagnetic instability growth rate for the beam particles with bi-Maxwellian distribution function is calculated as follows

$$\delta^{\circ} = \frac{k}{\sqrt{\pi}} \left[u^{\circ} - \frac{\left(1 + \frac{c^2k^2}{\omega_{pe}^2} + \beta\sqrt{\eta}\left(1 - \frac{R^2}{6}\right)\right)}{n^{\circ}} \right]. \tag{53}$$

3 Results and Discussion

The general dispersion relation is derived by combining the relativistic Vlasov equation with the Wigner-Maxwell equation for the coupled beam-plasma. In this research, the plasma distribution function is considered to be Maxwell-Boltzmannian and the electron beam that penetrates into the strongly coupled plasma is considered to be in relativistic mode, and in three modes of Kappa, semi-Maxwell and bi-Maxwellian distribution functions separately. The growth rate of Weibel electromagnetic instability has been calculated as function of different parameters such as temperature anisotropy, gradient density, quantum parameter and relativistic parameter. The results of calculations show that the growth rate of Weibel electromagnetic instability increases with temperature anisotropy (Figures (1a, 1b, and 1c)). In Figure (1d) is shown that the electron beam with the semi-Maxwell distribution function has its highest value compared with the Kappa and the bi-Maxwellian beam distribution functions in temperature anisotropy, $\frac{\theta_{\perp p}}{\theta_{\parallel p}} = 2.2$. In order to study the effect of density gradient on the growth rate of the Weibel electromagnetic instability, the growth rate is plotted in Figures (2a, 2b, and 2c) for different values of the density gradient for three different distribution functions. It is shown that the growth rate decreases with increasing of the density gradient. In Figure (2d) it is shown that the instability growth rate of the semi-Maxwell distribution function has the highest value compared to the Kappa and the bi-Maxwellian beam distribution functions in density gradient, $\eta = 0.1$.

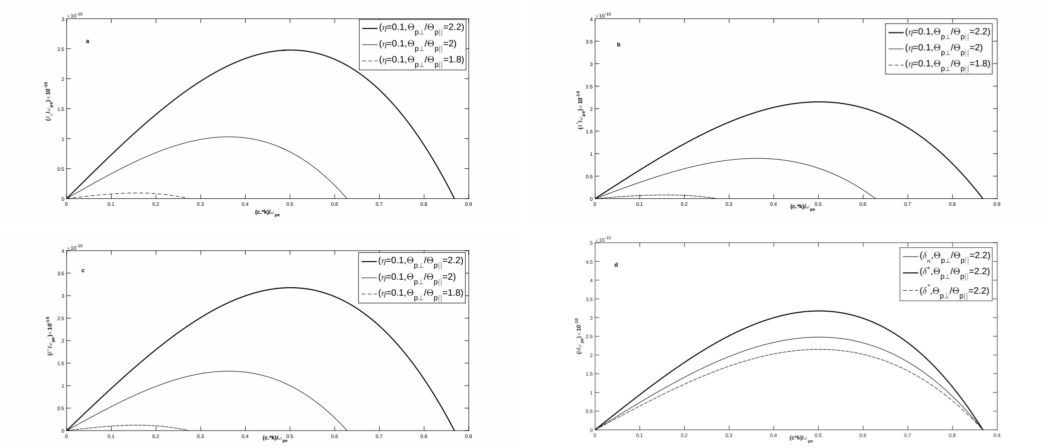

Figure 1: (a, b, c), the instability growth rate of three distribution functions normalized to ω_{pe}, $\frac{\delta}{\omega_{pe}}$, according to $\frac{ck}{pe}$ for different temperature anisotropy parameters, $\frac{\theta_{\perp p}}{\theta_{\parallel p}}$, and (d), comparison of three instability growth rate, (δ_{κ},δ^{*},δ°), for $\frac{\theta_{\perp p}}{\theta_{\parallel p}} = 2.2$.

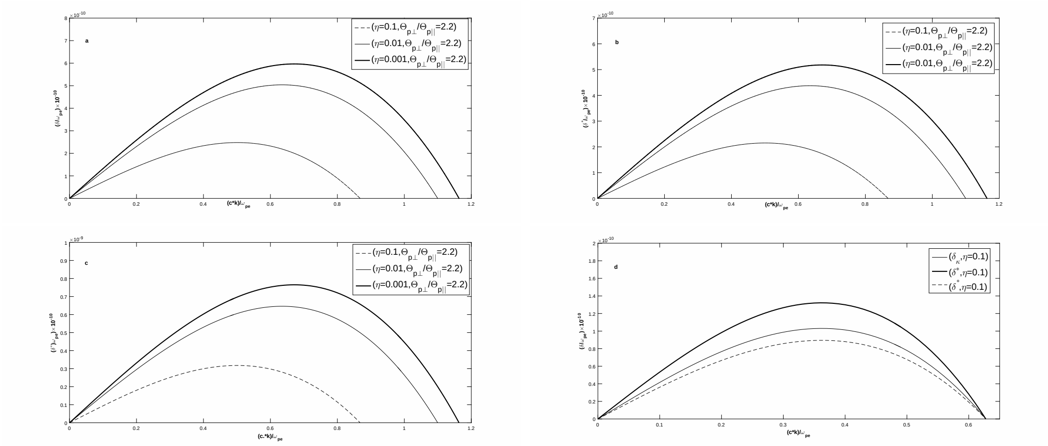

Figure 2: (a, b, c) the instability growth rate of three distribution functions normalized to ω_{pe}, $\frac{\delta}{\omega_{pe}}$, according to $\frac{ck}{\omega_{pe}}$ for different density gradients, η, (d) comparison of three instability growth rate, (δ_κ,δ^*,δ°), for $\eta = 0.1$.

The instability growth rate of three distribution functions normalized to ω_{pe}, $\frac{\delta}{\omega_{pe}}$, according to $\frac{ck}{\omega_{pe}}$ is shown in Figure (3a)) for quantum parameter of 1.5. Observations show that the maximum growth rate of Kappa, semi-Maxwell and bi-Maxwellian distribution function is respectively about 0.58, 0.8 and 0.48, whit the lowest maximum growth rate is corresponding to the bi-Maxwellian distribution function. In this way, the highest maximum growth rate is related to the semi-Maxwell function. In Figures (3b. 3c and 3d), the three-dimensional of the growth rate of the Weibel electromagnetic instability for three distribution functions plots based on the quantum parameter, which shows that the growth rate decreases with the increase of the quantum parameter in all three modes. In other words, the quantum parameter plays a stabilizing role in the growth rate. In the last step, the growth rate of the Weibel electromagnetic instability for three distribution functions drawn based on the relativity parameter in Figures (4a, 4b and 4c). It can be seen that with the increase in the relativistic parameter value, due to the increase in braking radiation, the energy loss in the plasma environment has increased, and this causes an increase in the Weibel instability growth rate. In Figure (4d), the Weibel electromagnetic instability for three distribution functions is plotted for the relativistic parameter value ($\gamma = 1.5$), temperature anisotropy value 2, and density gradient value 0.2. It is observed that the highest value of the growth rate is related to the semi-Maxwell distribution function.

4 Conclusions

In this research, the plasma distribution function is considered to be Maxwell-Boltzmannian and the electron beam that penetrates into the strongly coupled plasma is considered in three models of Kappa, semi-Maxwell, and bi-Maxwellian distribution functions separately. For this aim, an analytical expression is derived for imaginary parts of the dielectric constant as the instability Weibel growth rate for the beam particle with three different distribution function under the low-frequency wave condition. The growth rate of instability depends on the temperature anisotropy parameter, density gradient, quantum parameter and relativistic parameter. The obtained results show that the maximum growth rate of the beam with the semi-Maxwell distribution function is based on the temperature anisotropy parameters, density gradient, quantum effect and relativistic parameter have the highest possible value compared to the other two beam distribution functions. The bi-Maxwellian distribution function also has a more stable growth rate than the Kappa and the semi-Maxwell distribution functions.

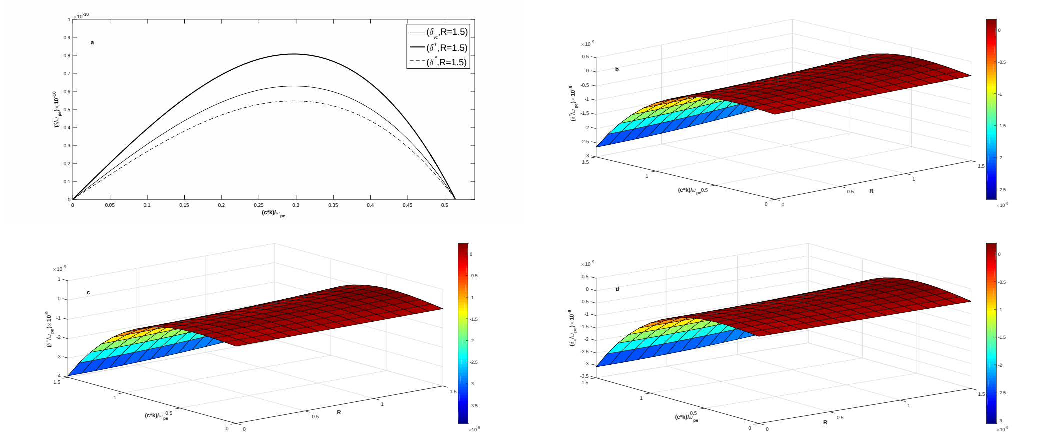

Figure 3: (a) Comparison of three instability growth rate, $(\delta_\kappa, \delta^*, \delta^\circ)$, for the quantum parameter, $R = 1.5$ and (b, c, d) 3D-graph variation of the three instability growth rate normalized to, ω_{pe}, according to according to $\frac{ck}{\omega_{pe}}$ and the quantum parameter, R.

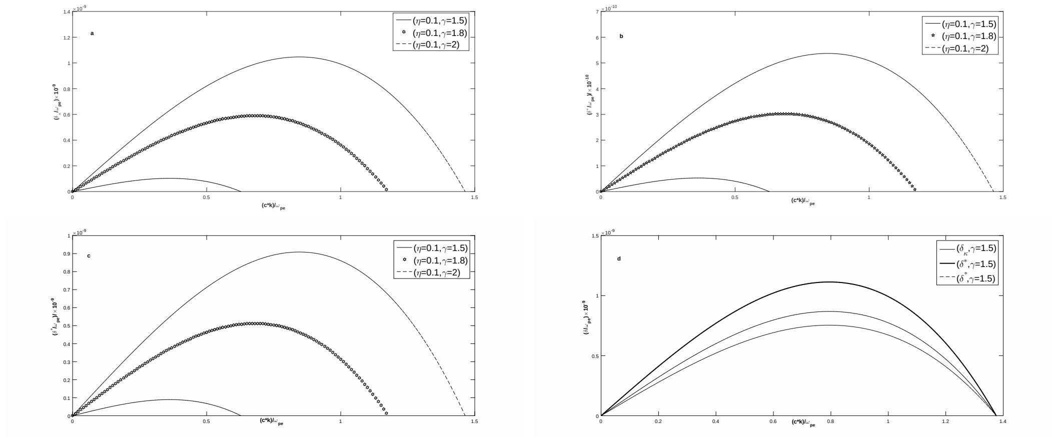

Figure 4: (a, b, c) the instability growth rate of three distribution functions normalized to ω_{pe}, $\frac{\delta}{\omega_{pe}}$, according to $\frac{ck}{\omega_{pe}}$ for different relativistic parameters, γ, and (d) comparison of three instability growth rate, $(\delta_\kappa, \delta^*, \delta^\circ)$, for $\gamma = 1.5$.

Authors' Contributions

All authors have the same contribution.

Data Availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare no potential conflicts of interest.

Ethical Considerations

The authors have diligently addressed ethical concerns, such as informed consent, plagiarism, data fabrication, misconduct, falsification, double publication, redundancy, submission, and other related matters.

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