Iranian Journal of Astronomy and Astrophysics
Vol. 10, No. 3, Summer 2023, 279–286
©Available online at http://ijaa.du.ac.ir
DOI: 10.22128/ijaa.2023.728.1166
Online ISSN: 2383–403X

Iranian Journal of Astronomy and Astrophysics

Research Paper

On the Nature of Kink MHD Waves, Vorticity and Compressibility Versus Restoring Forces

Karam Bahari
Physics Department , Faculty of Science, Razi University, Kermanshah, Iran;
email: karam.bahari@gmail.com

Received: 3 September 2023; **Accepted:** 19 November 2023; **Published:** 27 November 2023

Abstract. In order to study the nature of kink MHD waves, zero beta plasma in a thin twisted magnetic-flux tube is considered. We use two parameters, the ratio of restoring forces and the ratio of the parallel vorticity to the compressibility to study the effect of magnetic twist on the nature of kink waves. Our aim is to invstigate whether the nature of the wave obtained from studying these two parameters are the same or not. The two parameters give two different twist parameters in which the wave becomes purely Alfvénic. The first parameter indicates that both in the internal and external regions of the tube, the wave can become purely magnetoacostic but the second parameter indicates that the wave can become magnetoacostic only in the external region of the tube. Our conclusion is that the two parameters are not equivalent for determining the nature of the wave.

Keywords: Corona, Magnetic Fields, Oscillations

1 Introduction

In the last years, many examples of magnetohydrodynamic (MHD) waves have been detected in the solar atmosphere using instruments with high resolution. Now, we know that MHD waves are present always and everywhere in the solar atmosphere. Much attempts have been made to study and classify these waves both theoretically and numerically in the inhomogeneous solar atmosphere. In an unbounded and infinite plasma, MHD oscillations are classified into three fundamental types: fast and slow magneto-acoustic waves and Alfvén waves, but in the highly inhomogeneous solar atmosphere a rich variety of modes can exist in the magnetic loops, filaments, *etc.* Some authors argue that the physical differences between these waves are the restoring forces acting on plasma during the oscillation. Van Doorsselaere *et al.* (2008) have discussed the observational evidences of kink MHD waves and Alfvén waves.

Aschwanden *et al.* (1999) and Nakariakov *et al.* (1999) for the first time reported transverse perturbations in coronal flux loops, triggered by solar flares and argued that these are fast kink waves. De Pontieu *et al.* (2007) investigated Hinode data and found Alfvén perturbations in the solar atmosphere; they found that Alfvén oscillations can permeate the solar corona and have significant energy to supply the solar corona with the required energy for solar wind and probably heating of the quiet corona. Also, Okamoto *et al.*

(2007) studied Hinode data and found propagating Alfvén waves in the solar corona and without introducing any damping mechanism they concluded that Alfvén waves may have a significant role in coronal heating. Resonant absorption is just the mechanism that causes the rapid damping of coronal waves. In the presence of magnetic twist, spatial damping of propagating MHD kink waves has been investigated by Bahari (2018) and temporal damping of propagating MHD kink waves has been studied by Ebrahimi and Bahari (2019) and Bahari, Petrukhin and Ruderman (2020). In order to determine the nature of the kink waves, Goossens *et al.* (2009) investigated the restoring forces in kink oscillations and found that in the inhomogeneous layer the gradient of the magnetic pressure is negligible and concluded that the suitable adjective for the kink waves is Alfvénic. Later, based on the investigation of restoring forces, the nature of MHD waves under various circumstances has been studied studied by Bahari and Khalvandi (2017) (hereafter paper I), Bahari (2018), Bahari and Ebrahimi (2020) and Bahari (2021). Goossens Arregui and Van Doorsselaere (2019) studied the nature of MHD waves based on the comparison of the parallel vorticity and compressibility of the environment.

Kink MHD waves in a magnetically twisted flux tube have been studied by some authors e.g. Bennett, Roberts, and Narain (1999), Carter and Erdély (2007), and Karami and Bahari (2010). Ruderman (2007) studied the kink oscillations of a stratified and twisted magnetic-flux tube. He found that the magnetic twist does not affect kink waves. Karami and Bahari (2012) used the the method suggested by Ruderman (2007) and concluded that in the twisted coronal loops the period ratio of the fundamental to first harmonic kink wave decreases; later Bahari (2017) and Bahari and Jahan (2020) studied the oscillation properties of standing kink waves of a flux tube in the presence of both magnetic twist and flow and concluded that the eigenfunctions can be essentially modified by plasma flow and magnetic twist. Lopin and Nagorny (2017) studied the propagation of kink oscillations in a stratified, non-isothermal and magnetically twisted tube. They found that the magnetic twist increases the vertical energy flux of the kink waves which has positive azimuthal mode number while decreases it for the kink oscillations which has negative azimuthal mode number.

In this article, we compare the results of studying the nature of the kink MHD waves based on the investigation of restoring forces and parallel vorticity and compressiblity of the environment. We examine whether the results obtained from two methods are the same or not. To do this, we consider the model studied in paper I and study the nature of the kink wave based on the investigation of the parallel vorticity and compressibility of the environment, and compare our results with the results determined in paper I. In the next section, we introduce the loop model, in Section 3, we derive the plasma displacement and restoring forces and study the force ratio, parallel vorticity and compressibility of the wave for various tube parameters, and Section 4 is devoted to conclusions.

2 The model of the tube and governing equations

We model a magnetic flux tube as a thin straight tube with circular cross section in a pressureless medium. Circular coordinate system is used for studying the oscillations of the coronal loop. The z-axis has been assumed to coincide with the axis of the loop and the boundary of the tube is denoted by $r = a$. The plasma densities of the tube (internal region) ($\rho_{\rm i}$) and the environment (external region) ($\rho_{\rm e}$) are assumed to be constant and plasma density in the internal region is larger than the plasma density in external region. For simplicity, and because we are not interested in wave damping, we consider a step-function profile for the plasma density. In addition to z component of the magnetic field, we consider a small azimuthal component too both in internal and external regions of the tube. For simplicity, we consider an equilibrium magnetic field which is called uniformly twisted magnetic field and the azimuthal component of the background magnetic field is proportional to r

$$\boldsymbol{B} = \big(0, Ar, B_{\mathrm{z}}(r)\big). \tag{1}$$

Here, A is the twist parameter and $B_{\mathrm{z}}(r)$ must be determined from the equilibrium condition of the tube. This model has been investigated earlier paper I and also in an incompressible medium by Karami and Bahari (2010), with the difference that they assumed the longitudinal magnetic field B_{z} to independent of r.

The azimuthal magnetic field introduces a magnetic-tension force which is in the negative r-direction. In order to have a balance between radial forces in background model, the magnetic-tension force induced by magnetic twist is canceled by a gradient force of the magnetic pressure which is induced by the variation of magnetic field; hence, the magnitude of the magnetic field must be a decreasing function of r. From these considerations the equilibrium condition can be written which gives the axial component of the magnetic field as

$$B_{\mathrm{z}}^2(r) = A^2\big(a^2 - 2r^2\big) + B_0^2, \tag{2}$$

here B_0 is an integration constant.
In the linear regime the small perturbations of the tube are given by these equations

$$\rho\frac{\partial^2\xi}{\partial t^2} = \frac{1}{4\pi}\Big[(\boldsymbol{b}\cdot\nabla)\boldsymbol{B} + (\boldsymbol{B}\cdot\nabla)\boldsymbol{b}\Big] - \frac{1}{4\pi}\nabla(\boldsymbol{B}\cdot\boldsymbol{b}), \tag{3}$$

$$\frac{\partial\boldsymbol{b}}{\partial t} = \nabla\times(\frac{\partial\xi}{\partial t}\times\boldsymbol{B}). \tag{4}$$

In these equations, $\boldsymbol{b}$ denotes the magnetic field perturbations and ξ represents the plasma displacement. Since the background of the tube is independent of z, ϕ, and t, the dependence of all perturbations on these variables can be Fourier-analyzed, hence, they can be considered proportional to $\mathrm{e}^{\mathrm{i}(\mathrm{m}\phi+\mathrm{kz}-\omega\mathrm{t})}$. Here, m is the wave number associated with the ϕ coordinate, and can take $0, \pm1, \pm2, \ldots$. Here, in the case of kink waves, we consider the values ±1 for m. The axial wave number is denoted by (k). Our formalism here is relevant for the thin-tube approximation as a result we assume $ka = \frac{\pi}{100}$.

Using the perturbation theory suggested by Ruderman (2007) and later used by Karami and Bahari (2012), equations (3) and (4) can be solved to obtain the perturbations of magnetic pressure $p = \boldsymbol{b}\cdot\boldsymbol{B}/4\pi$ and the radial displacement in both the internal and external regions of the tube.

$$\xi_r(r) = \begin{cases}\beta r^{|m|-1}, & r < a,\\ \alpha r^{-|m|-1}, & r > a,\end{cases} \tag{5}$$

$$p(r) = \begin{cases}\Big(\frac{1}{|m|}(\rho_{\mathrm{i}}\omega^2 - \frac{f^2}{4\pi}) + \frac{Af}{2\pi m}\Big)\beta r^{|m|}, & r < a,\\ -\Big(\frac{1}{|m|}(\rho_{\mathrm{e}}\omega^2 - \frac{f^2}{4\pi}) - \frac{Af}{2\pi m}\Big)\alpha r^{-|m|}, & r > a.\end{cases} \tag{6}$$

In these equations, β and α are constants which their ratio can be obtained from the relevant boundary conditions, and $f = mA + kB_0$. Using the relevant boundary conditions of the tube oscillations, *i.e.* the continuity of pressure perturbations $p(r)$ and $\xi_r(r)$, the oscillation frequency of kink waves is determined

$$\omega = \frac{|f|}{\sqrt{2\pi(\rho_\mathrm{i} + \rho_\mathrm{e})}}. \tag{7}$$

We have assumed the coronal-loop model here to investigate the nature of the kink oscillations. Here, we are not interested in studying resonant absorption of the waves. Resonant absorption of the kink waves has been studied by Goossens *et al.* (2009), Ruderman and Roberts (2002), Ruderman (2015) and Ebrahimi and Karami (2016). In the model considered in this paper, the Alfvén frequency ($\omega_\mathrm{A} = f/\sqrt{4\pi\rho}$) is piecewise constant as a result we have no Alfvén continuum, hence, resonant absorption does not occur; anf the oscillation frequency obtaine in equation (7), is a real quantity.

3 Restoring forces, parallel vorticity and compressibilty

The nature of kink oscillations is studied by the restoring forces of the oscillations. These are given by equation (3). It has two terms, the gradient of magnetic pressure force,

$$\boldsymbol{F} = -\nabla_\perp p = -\nabla p + \hat{\boldsymbol{l}}_t \frac{\mathrm{d}p}{\mathrm{d}s} = -\nabla p + \frac{\mathrm{i}kp(\hat{z} + Ar/B_0\hat{\phi})}{\sqrt{1 + (Ar/B_0)^2}}, \tag{8}$$

and the magnetic-tension force

$$\begin{aligned} \boldsymbol{\Pi} &= \frac{1}{4\pi}\Big[(\boldsymbol{B}\cdot\nabla)\boldsymbol{b} + (\boldsymbol{b}\cdot\nabla)\boldsymbol{B}\Big] - \hat{\boldsymbol{l}}_t \frac{\mathrm{d}p}{\mathrm{d}s} \\ &= \frac{1}{4\pi}\Big[(\boldsymbol{B}\cdot\nabla)\boldsymbol{b} + (\boldsymbol{b}\cdot\nabla)\boldsymbol{B}\Big] - \frac{\mathrm{i}kp(\hat{z} + Ar/B_0\hat{\phi})}{\sqrt{1 + (Ar/B_0)^2}} \end{aligned} \tag{9}$$

.

In these equations, the operator $\nabla_\perp$ denotes the gradient operator in the direction perpendicular to the background magnetic-field lines and $\hat{\boldsymbol{l}}_t$ and s are the unit vector and length along the magnetic-field lines respectively.

We can use equation (4) to obtain all the three components of the magnetic field perturbation in terms of the plasma displacement components

$$\begin{aligned} b_r &= \mathrm{i}f\xi_r \\ b_\phi &= \mathrm{i}f\xi_\phi - Ar\nabla\cdot\xi \\ b_z &= \mathrm{i}f\xi_z - B_z(r)\nabla\cdot\xi. \end{aligned} \tag{10}$$

This result can be Substituted in equation (3) which after some manipulation both ξ_ϕ and ξ_z are determined in terms of ξ_r and $p(r)$

$$\xi_\phi = \frac{1}{\mathrm{i}(\frac{f^2}{4\pi} - \rho\omega^2)}\Big((\frac{m}{r} - \frac{fAr}{B(r)^2})p(r) + \frac{fA}{2\pi}(\frac{A^2r^2}{B(r)^2} - 1)\xi_r\Big), \tag{11}$$

$$\xi_z = \frac{1}{\mathrm{i}(\frac{f^2}{4\pi} - \rho\omega^2)}\Big((k_z - \frac{fB_z(r)}{B(r)^2})p(r) + \frac{fB_z(r)A^2r}{2\pi B(r)^2}\xi_r\Big), \tag{12}$$

then, substituting the perturbations to the magnetic field from equation (10) into equation (9), the magnetic-tension force are obtained in terms of radial displacement and perturbed

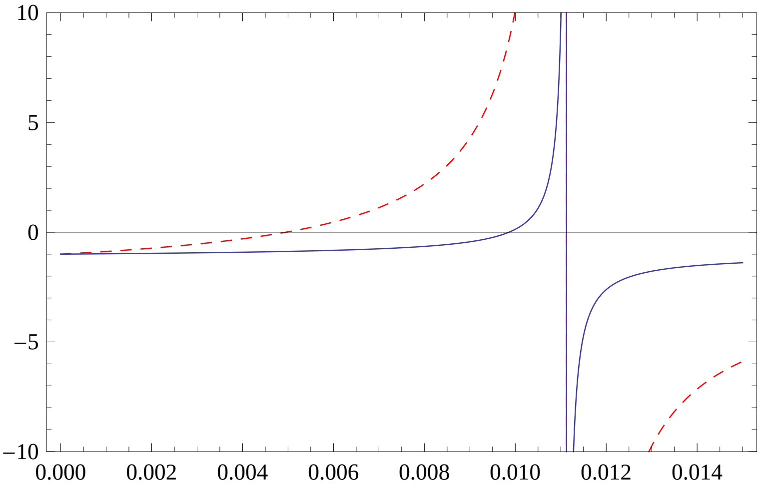

Figure 1: The ratio of the parallel vorticity to the compressibility of the medum i.e. $i(\nabla \times \xi)_{\|}/\nabla \cdot \xi$ as a function of the dimensionless twist parameter Aa/B_0. The red-dashed line corespond to the internal region of the tube and is calculated in $r = 0.5a$ while the blue-solid is for the external region of the tube and is calculated in $r = 1.5a$. The parameters of the tube are $m = -1$, $\rho_{\rm e} = \rho_{\rm i}/5$, $L = 100a$ and $k = \pi/L$

magnetic pressure,

$$\Pi_r = \frac{1}{4\pi}(-f^2\xi_r - \mathrm{i}Af\xi_\phi + A^2r\nabla\cdot\xi), \tag{13}$$

$$\Pi_\phi = \frac{1}{4\pi}(\mathrm{i}Af\xi_r - f^2\xi_\phi - \mathrm{i}Afr\nabla\cdot\xi) - \frac{\mathrm{i}kpAr/B_0}{\sqrt{1+(Ar/B_0)^2}}. \tag{14}$$

With the use of equations (11) and (12) the magnetic-tension force can be written in terms of the perturbed magnetic pressure and radial component of displacement, and finally using equations (5) and 6 it can be obtained as a function of r.

In Figure 1, the ratio of the parallel vorticity to the compressibility of the medum i.e. $i(\nabla\times\xi)_{\|}/\nabla\cdot\xi$ is plotted as a function of the dimensionless twist parameter Aa/B_0. Since this quantity is almost independent of r, we have not ploted it versus the radial coordinate. In this figure, the red-dashed line corespond to the internal region of the tube and is calculated in $r = 0.5a$ while the blue-solid is for the external region of the tube and is calculated in $r = 1.5a$. In Figure 2, the ratio of radial forces $|\Pi_r/F_r|$ is plotted as a function of the dimensionless twist parameter Aa/B_0. Also this quantity is almost independent of r, we have not ploted it versus the radial coordinate. In this figure too, the red-dashed line corespond to the internal region of the tube and is calculated in $r = 0.5a$ while the blue-solid is for the external region of the tube and is calculated in $r = 1.5a$.

The quantities represented in Figures 1 and 2 are different quantities and in general it is not necessary that the diagrams in these two figures be the same. But since these two figure represent the nature of the same wave expect some similarities in these figures. In particular, if the results of one of the diagrams conclude that under some conditions the wave is purely Alfvénic (magnetoacoustic) then, the other figure too must give the same conclusion. But surprisingly this is not the case! For example Figure 1 shows that for $Aa/B_0 = 0.011$ the wave becomes purely Alfvénic both inside and out side the tube, but Figure 2 shows that

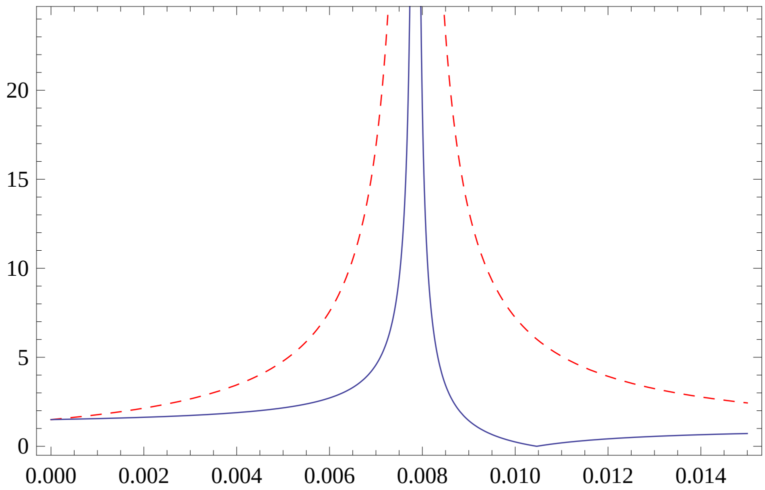

Figure 2: The ratio of radial forces $|\Pi_r/F_r|$ is plotted as a function of the dimensionless twist parameter Aa/B_0. The red-dashed line correspond to the internal region of the tube and is calculated in $r = 0.5a$ while the blue-solid is for the external region of the tube and is calculated in $r = 1.5a$. The parameters of the tube are the same as those in Figre 1.

this happens for $Aa/B_0 = 0.008$. Also, it is clear from Figure 1 that for two values of the twist parameter 0.005 and 0.0095 the parallel vorticity becomes negligible relative to the compressibility, in the internal and external region respectively, hence the wave becomes purely magnetoacoustic under this conditions, while Figure 2 that this occurs only for the external region for the twist parameter 0.0105.

4 Conclusions

The zero beta plasma in a thin and magnetically twisted magnetic-flux tube is considered. The MHD equations in the linear regime have been solved to determine the oscillation frequency, the perturbations to magnetic pressure and plasma displacement of the loop. For different tube parameters, the restoring forces loop, which are the gradient of perturbed magnetic pressure and magnetic-tension force, the parallel vorticity and comprressibility have been obtained as a function of r.

The nature of the kink waves has been studied by investigating two parameters, the first parameter is the ratio of parallel vorticity to the compressibility and the second parameter is the ratio of restoring forces. Surprisingly, the results which are obtained from investigating the two parameters are different from each other considerably. The two parameters give two different twist parameters in which the wave becomes purely Alfvénic. The first parameter indicates that both in the internal and external regions of the tube the wave can become purely magnetoacostic but the second parameter indicates that the wave can become magnetoacostic only in the external region of the tube. Our conclusion is that the two parameters are not equivalent for determining the nature of the wave.

Authors' Contributions

The author contributed to data analysis, drafting, and revising of the paper and agreed to be responsible for all aspects of this work.

Data Availability

No data available.

Conflicts of Interest

The author declares that there is no conflict of interest.

Ethical Considerations

The author has diligently addressed ethical concerns, such as informed consent, plagiarism, data fabrication, misconduct, falsification, double publication, redundancy, submission, and other related matters.

Funding

This research did not receive any grant from funding agencies in the public, commercial, or non profit sectors.

References

[1] Aschwanden, M. J., Fletcher, L., Schrijver, C. J., & Alexander, D. 1999, Astrophys. J., 520, 880.

[2] Bahari, K. 2017, Solar Phys., 292, 110.

[3] Bahari, K. 2018, ApJ, 864, 2.

[4] Bahari, K. 2018, New Astronomy, 61, 30.

[5] Bahari, K. 2021, Solar Phys., 296, 126.

[6] Bahari, K., & Ebrahimi, Z. 2020, MNRAS, 497, 1135.

[7] Bahari, K., & Jahan, Z. 2020, ApJ, 901, 28.

[8] Bahari, K., & Khalvandi, M. R. 2017, Solar Phys., 292, 192.

[9] Bahari, K., Petrukhin, N. S., & Ruderman, M. S. 2020, MNRAS, 496, 67.

[10] Bennett, K., Roberts, B., & Narain, U. 1999, Solar Phys., 185, 41.

[11] Carter, B. K., & Erdélyi, R. 2007, A&A, 475, 323.

[12] De Pontieu, B., McIntosh, S. W., Carlsson, M., Hansteen, V. H., Tarbell, & et al. 2007, Science, 318, 1574.

[13] Ebrahimi, Z., & Bahari, K. 2019, MNRAS, 490, 1644.

[14] Ebrahimi, Z., & Karami, K. 2016, MNRAS, 462, 1002.

[15] Ebrahimi, Z., Karami, K., & Soler, R. 2017, Astrophys. J., 845, 86.

[16] Goossens, M. L., Arregui, I., & Van Doorsselaere, T. 2019, Front. Astron. Space Sci., 6, 20.

[17] Goossens, M., Terradas, J., Andries, J., Arregui, I., & Ballester, J. L. 2009, A&A, 503, 213.

[18] Karami, K., & Bahari, K. 2010, Solar Phys., 263, 87.

[19] Karami, K., & Bahari, K. 2012, Astrophys. J., 757, 186.

[20] Lopin, I., & Nagorny, I. 2017, Astrophys. J., 840, 26.

[21] Nakariakov, V. M., Ofman, L., DeLuca, E. E., Roberts, B. R., & Davila, J. M. 1999, Science, 285, 862.

[22] Okamoto, T. J., Tsuneta, S., Berger, T. E., Ichimoto, K., Katsukawa, Y., & et al. 2007, Science, 318, 1577.

[23] Ruderman, M. S. 2007, Solar Phys., 246, 119.

[24] Ruderman, M. S. 2015, A&A, 575, A130.

[25] Ruderman, M. S., & Roberts, B. 2002, Astrophys. J., 577, 475.

[26] Sakurai, T., Goossens, M., & Hollweg, J. V. 1991, Solar Phys., 133, 227.

[27] Van Doorsselaere, T., Nakariakov, V. M., & Verwichte, E. 2008, Astrophys. J., 676, L73.