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Research Paper

Arbitrary Amplitude Dust Acoustic Solitary Waves in a Quantum Dusty Plasma with Arbitrary Dust Size Distribution

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Abstract. Arbitrary amplitude dust acoustic waves (DAWs) in a quantum dusty plasma including the effect of the dust size distribution (DSD) are presented here. By using the Sagdeev pseudopotential method for large amplitude waves, the energy integral is derived which includes Sagdeev potential. The upper and lower limits of Mach number are presented, by applying the conditions in which a solitary solution can exist. Two cases are studied, a mono-sized dust grains case and dust grains possessing power-law size distribution case. The result shows that, the allowed Mach number's range is increased for mono-sized dust grains case. Sagdeev potential is also plotted and it is seen that in mono-sized dust grains case, solitary waves are propagated. Whereas, for different-sized dust grains with power-law size distribution, the solitary waves transform into cnoidal ones.

Keywords: Dust acoustic waves, Quantum dusty plasma, Sagdeev potential, Dust size distribution, Much number, Power-law distribution

1 Introduction

Adding dust particles to a plasma creates new wave modes that have attracted the special attention of researchers. Specially, more practical interest have been focused on investigation of nonlinear waves in a complex plasma, which contains multi species of particles. Compared to classical plasmas, the quantum plasmas are recognized by high-plasma particle number densities and low temperatures. In this case, dimensions of the system and the de Broglie wavelength $\lambda_B = h/\sqrt{2m\pi K_B T}$, (where m is the mass of charge particle, T is the system temperature, and h is the Planck constant) are comparable. Therefore, dusty plasma acts like a Fermi gas. The dusty plasma can be included with electrons, ions, and very heavy charged dust particles. Such plasma has attracted attentions in recent years. A lot of dense astrophysical plasmas are categorized as this type of plasmas [1]. For example, in the laboratory plasmas [2,3], the interior of Jupiter, Saturn, Neptune and Saturn's rings [4–7] and

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brown or white dwarf stars [8]. Rao et al [8] theoretically investigated DAWs in a plasma included of electrons, ions and dust grains. These waves are confirmed experimentally in a lot of laboratories [9,10]. The range of dust particles in laboratory plasmas are limited, while dust patricles have different sizes in space. Although a lot of researchers investigated quantum effects on dusty plasmas, all of them assumed the dust grains as mono-sized particles [11–14]. The DSD is associated with dusty plasma's conditions and the environment. In the laboratory, the DSD can be expressed as Gaussian distribution [15], while in space plasmas, dust grains generally are polydisperse and expressed as a power-law distribution [16]. Furthermore, the arbitrary dust size distribution function has the form of a polynomial expansion [17]. According to the researchers, DSD influences the waves' properties in dusty plasmas [19–24]. El-Labany et al [18] studied the influences of the power-law DSD and the polynomial DSD on the linear and nonlinear quantum dust ion acoustic waves (DIAWs). The results showed that in case of power-law DSD, the wave's velocity is smaller and the amplitude is larger compared to polynomial DSD. Also, the influences of the power-law DSD and the polynomial DSD on a quantum dusty plasma are studied by Feng et al. [19]. With the help of Sagdeev's potential method, Ishak-Boushaki et al. [20] investigated the influence of arbitrary DSD on nonlinear DAWs. Banerjee et al. [21], also applied Sagdeev's potential method to study double layers and nonlinear ion acoustic waves (IAWs) in a dusty plasma with power-law DSD. Meuris [22] compared the dusty plasma frequency for the case of mono-sized dust with that of a dusty plasma having dust distribution when the total number density is constant and obtained some corrections due to different dust distributions. El-wakil et al. [23] studied the effects of the higher order nonlinearity on the DA solitary waves in a dusty plasma having power law dust size distribu-tions (DSDs). El-Shewy et al. [24] studied the effect of higher order dispersion corrections on the solitary wave properties for both the case of having monosized dust and DSD. The effect of DSD on large amplitude DAWs in a quantum dusty plasma has not been reported to the best of our knowledge.

In this paper, our aim is to investigate arbitrary amplitude DAWs in a quantum dusty plasma with arbitrary DSD. We present our article as follows. The governing equations are introduced in Section 2. Then, in Section 3, the effect of polynomial DSD on large amplitude solitons is investigated by using sagdeev potential method. The results are concluded in Section 4.

2 Basic Equations

Let us consider an unmagnetized plasma composed of inertialess ions and electrons and, N different species of mobile dust grains with dust particles density n_{dj}, mass m_{dj}, velocity u_{dj}, charge $Q_{dj} = eZ_{dj}$, where Z_{dj} is the number of charges residing on jth dust grain for $j = 1, 2, \dots, N$. Then, we assume that ions and electrons obey the pressure law

$$P_\alpha = \frac{m_\alpha V_{F\alpha}^2}{3n_{\alpha 0}} n_\alpha^3,$$

in a zero-temperature Fermi gas, where $n_{\alpha 0}$ is the equilibrium number density of the α th species and

$$V_{F\alpha} = \sqrt{\frac{2K_B T_{F\alpha}}{m_\alpha}}, \qquad \alpha = e, i,$$

is Fermi speed. We describe the neutrality condition as

$$n_{i0} = n_{e0} + \sum_{j=1}^{N} Z_{dj} n_{dj0}.$$

We now introduce the normalized variables, [25,26],

$$n_{dj} \to N_{tot}\tilde{n}_{dj}, \qquad u_{dj} \to V_0\tilde{u}_{dj}, \qquad t \to T_0\tilde{t}, \qquad Z_{dj} \to Z_d\tilde{Z}_{dj},$$
$$m_{dj} \to m_d\tilde{m}_{dj}, \qquad \varphi \to \varphi_0\tilde{\varphi}, \qquad x \to L_0\tilde{x},$$

where

$$V_0 = \frac{L_0}{T_0}, \qquad T_0 = \omega_{pd}^{-1} = \Big(\frac{m_d}{4\pi e^2\overline{Z}_d^2 N_{tot}}\Big)^{1/2},$$
$$\varphi_0 = \frac{2K_BT_{Fi}}{e}, \qquad L_0 = \lambda_d = \Big(\frac{2K_BT_{Fi}}{4\pi e^2\overline{Z}_d N_{tot}}\Big)^{1/2}.$$

Furthermore

$$\sigma = \frac{T_{Fi}}{T_{Fe}}, \qquad \mu_i = \frac{n_{i0}}{\overline{Z}_d N_{tot}}, \qquad \mu_e = \frac{n_{e0}}{\overline{Z}_d N_{tot}}, \qquad H = \sqrt{\hbar^2\omega_{pe}^2/(2K_BT_{Fe})}.$$

The dynamic of DAWs is governed by the following set of dimensionless equations in model plasma

$$\frac{\partial u_{dj}}{\partial t} + u_{dj}\frac{\partial u_{dj}}{\partial x} = \frac{Z_{dj}}{m_{dj}}\frac{\partial \varphi}{\partial x}, \tag{1}$$

$$-\frac{\partial \varphi}{\partial x} - n_i\frac{\partial n_i}{\partial x} + \frac{H_i^2}{2}\frac{\partial}{\partial x}\Big(\frac{\frac{\partial^2}{\partial x^2}\sqrt{n_i}}{\sqrt{n_i}}\Big) = 0, \tag{2}$$

$$-\frac{\partial \varphi}{\partial x} - \sigma n_e\frac{\partial n_e}{\partial x} + \frac{H^2}{2}\frac{\partial}{\partial x}\Big(\frac{\frac{\partial^2}{\partial x^2}\sqrt{n_e}}{\sqrt{n_e}}\Big) = 0, \tag{3}$$

$$\frac{\partial n_{dj}}{\partial t} + \frac{\partial}{\partial x}(n_{dj}u_{dj}) = 0, \tag{4}$$

$$\frac{\partial^2 \varphi}{\partial x^2} = \mu_e n_e - \mu_i n_i + \sum_{j=1}^{N} Z_{dj} n_{dj}. \tag{5}$$

3 Arbitrary Amplitude DAWs

In this section, we study the effect of DSD on arbitrary amplitude DAWs in model plasma. We will consider a quantum dusty plasma including N different species of negatively charged dust grains with different sizes, inertialess quantum electrons and ions. The range of dust particles are limited as $[r_{d1}, r_{d2}]$, where r_{d1} and r_{d2} are the minimum and maximum radius of dust grains, respectively. The arbitrary DSD is described by a polynomially expressed distribution function as follows

$$h(r_d)dr_d = (a_0 + a_1 r_d + a_2 r_d^2 + \cdots)dr_d, \tag{6}$$

which satisfies the total number density

$$N_{tot} = \int_{d_{d1}}^{r_{d2}} h(r_d)dr_d, \tag{7}$$

where $a_0, a_1, a_2, \ldots$ are constants and r is the radius of dust particle.
We consider $h(r_d) = 0$ for the radii outside of the limited range. To study the properties

of large amplitude DAWs, Sagdeev approach is employed to find a suitable pseudopotential which appropriately describes the propagation of these waves. To this purpose, all the dependent variables in equations (1)–(5) are made to be depended on time and space coordinate as $\xi = x - Mt$. From equations (1)–(5) the following expression for dust, ion, and electron densities are obtained

$$n_{dj} = \frac{n_{dj0}}{\sqrt{1 + \dfrac{2Z_{dj}\varphi}{M^2 m_{dj}}}}, \tag{8}$$

$$n_i = \Big[1 - 2\varphi + H_i^2(1-2\varphi)^{-1/4}\frac{\partial^2}{\partial\xi^2}(1-2\varphi)^{1/4}\Big]^{1/2}, \tag{9}$$

$$n_i = \Big[1 + \frac{2\varphi}{\sigma} + \frac{H^2}{\sigma}(1+\frac{2\varphi}{\sigma})^{-1/4}\frac{\partial^2}{\partial\xi^2}(1+\frac{2\varphi}{\sigma})^{1/4}\Big]^{1/2}. \tag{10}$$

Upon substituting equations (8)–(10) in equation (5) and after integrating once with suitable boundary conditions, namely , $\varphi \to 0$ and $\partial\varphi/\partial x \to 0$ as $\xi \to \pm\infty$, The quasi particle's energy equation is worked out as

$$\frac{1}{2}(\frac{d\varphi}{d\xi})^2 + V(\varphi, M) = 0. \tag{11}$$

In which the Sagdeev potential is

$$\begin{aligned} V(\varphi) = \Big[-1 + \frac{\mu_e H^2}{4\sigma^2}(1+\frac{2\varphi}{\sigma})^{-3/2} + \frac{\mu_i H_i^2}{4}(1-2\varphi)^{-3/2}\Big]^{-1} \\ \times \big[\frac{\mu_e\sigma}{3}(1+\frac{2\varphi}{\sigma})^{3/2} - \frac{\mu_e\sigma}{3} + \frac{\mu_i}{3}(1-2\varphi)^{3/2} - \frac{\mu_i}{3} + Z_{d0}MK(\varphi, M)\big]. \end{aligned} \tag{12}$$

While,

$$K(\varphi, M) = \sum_j m_{dj}n_{dj}\left(\sqrt{M^2 + \frac{2Z_{dj}}{m_{dj}}\varphi} - M\right). \tag{13}$$

For the continuous case, equations (6) and (7) lead to

$$n_{tot} = \sum_{j=1}^{N} n_{dj0} = \int_1^c h(r)dr = \int_1^c (a_0 + a_1 r + a_2 r^2 + \cdots)dr, \tag{14}$$

where $r = r_d/r_{d1}$, and $c = r_{d2}/r_{d1}$ shows the extent of the distribution and defined as the maximum dust size divided by the minimum dust size. Therefore, equation (13) can take the following form

$$K(\varphi, M) = \int_1^c r^3\left(\sqrt{M^2 + \frac{2\varphi}{r^2}} - M\right)h(r)dr. \tag{15}$$

If we consider $h(r) = a_0$, then all different-sized dust grains possess the same number density. A more complicated case would be $h(r) = a_0 + a_1 r = a_0(1 + ar)$, where $a = a_1/a_0$. In case of $a > 0$, larger dust grains possess the biggest number densities in comparison to smaller ones. In contrast, if $a < 0$, then larger dust grains have smaller number densities. As we mentioned before, power-law distribution which satisfies $a < 0$, are used to describe space plasmas. So, we can rewrite the relation (24) as follows

$$K(\varphi, M) = Ma_0\Big\{(\frac{ac^2}{5} - \frac{14a\varphi}{15M^2} + \frac{c}{4})(c^2 + \frac{2\varphi}{M^2})^{3/2} - (\frac{a}{5} - \frac{14a\varphi}{15M^2} + \frac{1}{4})(1 + \frac{2\varphi}{M^2})^{3/2} \quad (16)$$
$$\frac{1}{2}\frac{\varphi^2 \ln\left(1 + \sqrt{1 + \frac{2\varphi}{M^2}}\right)}{4M^2} - \frac{1}{2}\frac{\varphi^2 \ln\left(c^2 + \sqrt{c^2 + \frac{2\varphi}{M^2}}\right)}{4M^2} + \frac{\varphi\sqrt{1 + \frac{2\varphi}{M^2}}}{4M^2}$$
$$- \frac{c\varphi\sqrt{c^2 + \frac{2\varphi}{M^2}}}{4M^2} - \frac{a(c^5 - 1)}{5} - \frac{(c^4 - 1)}{4}\Big\},$$

where $a_0 = \dfrac{n_{tot}}{(c-1)(1 + \dfrac{1}{2a(c+1)})}$. The possibility of solitary wave propagation relies on the following conditions.

1) $V(\varphi)\mid_{\varphi=0} = \dfrac{dV(\varphi)}{d\varphi}\mid_{\varphi=0} = 0, \qquad \dfrac{d^2V(\varphi)}{d\varphi^2}\mid_{\varphi=0} < 0$
2) $V(\varphi_m) = 0$, where φ_m is either the minimum or maximum value of φ and $V(\varphi)$ is negative for $\varphi_m < \varphi < 0$ (rarefactive soliton) or $\varphi_m > \varphi > 0$ (compressive soliton), where there is no other roots

on

the range of $[0, \varphi_m]$.

1) If $V(\varphi_m) \geq 0$ at nonzero φ_m, then a quasi-particle with zero total energy will be reflected at $\varphi = \varphi_m$.

The existence conditions of the solitary wave propagation relay on the Mach number M and physical parameters of the system determine the permitted values of M. As we could see in Figure 1 (a), from the first condition which is based on the sign of the second derivative of $V(\varphi)$ at $\varphi = 0$, a lower limit of $M(M_{cl})$ can be obtained. The fact that dust number density should remain real leads us to the following inequality

$$\varphi \geq -\frac{M^2}{2}\frac{m_{dj}}{Z_{dj}}\Big(= -\frac{M^2}{2}(\frac{r_{dj}}{r_{d0}})\Big). \tag{17}$$

It is to be noted that the mass m_{dj} and the dust grain charge Z_{dj} are normalized by the mass and the charge of the most probable grain's radius having the most probable radius, viz., $m_{d0} = m_d(r_{d0})$ and $Z_{d0} = Z_d(r_{d0})$. In model plasma, it is assumed that the smaller dust grains are more abundant. Therefore one can write $\varphi_m \geq -\dfrac{M^2}{2}$ as the maximum value of φ. Then, we substitute φ_m into the sagdeev potential and apply the third condition. Therefore, the upper limit of $M(M_{ch})$ is determined (Figure 1 (b)) in case of $a = 0$, $a = -0.1$ and $a = -0.2$. One may notice that, when different-sized dust grains do not have the same number density, the allowed Mach number's range is reduced. Considering $M_{cl} = 0.85$, in case of $a = -0.1$, $a = -0.2$, and $M_{cl} = 1.1$ for $a = 0$, Sagdeev potential $V(\varphi)$ is plotted with respect to φ in Figure (2). It is seen that when all different-sized dust grains possess the same number density ($a = 0$), solitary waves are propagated. However, considering different number densities for different-sized dust grains leads to propagating cnoidal waves.

Also, potential' depth increases with increase in a. Considering a small amplitude limit, we examine solitary wave structure for Mach number close to M_{cl}. For this purpose, near $\varphi = 0$, we express $V(\varphi)$ as

$$V(\varphi) = V_0^{'}\varphi + \frac{1}{2}V_0^{''}\varphi^2 + \frac{1}{6}V_0^{'''}\varphi^3. \tag{18}$$

If $V_0^{'} = 0$, upon substituting equation (18) in equation (11) and integrating, a solitary solution is obtained as

$$\varphi(\xi) = -3\frac{V_0^{''}}{V_0^{'''}}\frac{1}{\cosh^2(\frac{1}{2}\sqrt{-V_0^{''}}\xi)}. \tag{19}$$

Note that a Korteweg–de Vries (KdV) equation obtained with the help of reductive perturbation method has the equivalent solitary solution as (19). However, when $V_0^{'} \neq 0$, the cnoidal waves are created. Cnoidal waves can be propagated only for the allowed Mach number ($M_d < M < M_{ch}$). Considering φ_0, φ_1 and φ_2 as the roots of sagdeev potential for the allowed Mach number, $V(\varphi)$ becomes negative between $\varphi = 0$ and $\varphi = \varphi_2$. There is an oscillatory solution between $\varphi = 0$ and $\varphi = \varphi_2$; therefore, we can use the Taylor expansion of $V(\varphi)$ around $\varphi = 0$ to obtain equation (18). Equation (11) can be expressed otherwise as

$$\frac{\partial\varphi}{\partial\xi} = \pm(\frac{F(\varphi)}{3k}), \tag{20}$$

where

$$F(\varphi) = \varphi^3 - 3B\varphi^2 - 6A\varphi = (\varphi - \varphi_0)(\varphi - \varphi_1)(\varphi - \varphi_2), \qquad k = 1/V_0^{'''},$$

while

$$A = -V_0^{'}/V_0^{''}, \qquad B = -V_0^{''}/V_0^{'''}.$$

The roots of $F(\varphi)$ are given by

$$\varphi_1 = 0, \qquad \varphi_{0,2} = \frac{3}{2}B \pm \sqrt{(\frac{3}{2}B)^2 + 6A}.$$

So, equation (20) has an oscillatory solution may be formulated as

$$\varphi(\xi) = \varphi_2 + (\varphi_1 - \varphi_2)cn^2\Big(\sqrt{\frac{(\varphi_2 - \varphi_0)\varphi_2}{12K}}\xi, S\Big). \tag{21}$$

Here, cn and S represents Jacobian elliptic function and modulus, respectively. The modulus is defined as $S^2 = \varphi_2/(\varphi_2 - \varphi_0)$. $S = 1$ is equivalent to $\varphi_0 = \varphi_1$ and shows a limit case for soliton solution.

The phase portraits of DA solitons and cnoidal waves have been drawn in Figure 3. Two different sets of orbits are seen in this figure, a periodic orbit and a homoclinic one. They correspond to periodic travelling and solitary wave solutions, respectively. It means the dashed curve represents soliton and the solid curve shows cnoidal waves. In phase portrait diagram, the dashed curve is also called a separatrix that corresponds to soliton solution when $S \to 1$. In other words, cnoidal wave structure must lie inside the separatrix.

The variations of potential φ versus ξ associated to Sagdeev potential are presented in Figure 4. it shows DA soliton and cnoidal wave structures. As it is mentioned, in mono-sized

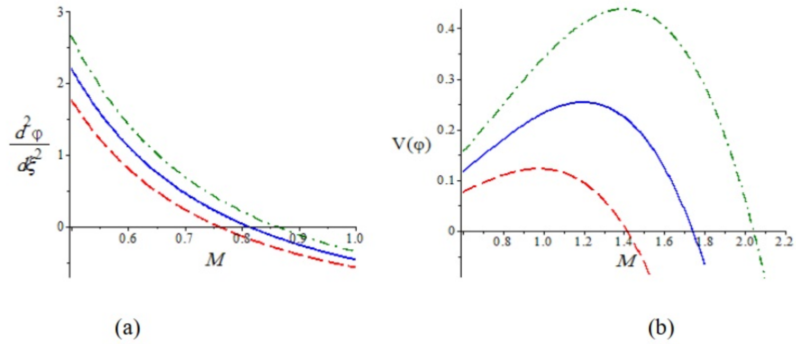

Figure 1: (a) the lower limit and (b) the upper limit of M for $a = -0.1$ (solid curve), $a = -0.2$ (dashed curve) and $a = 0$ (dash dotted curve) with fixed values $\mu_e = 0.33$, $\mu_i = 1.33$, $\sigma = 20$, $H = 0.6$, $H_i = 3 \times 10^{-3}$.

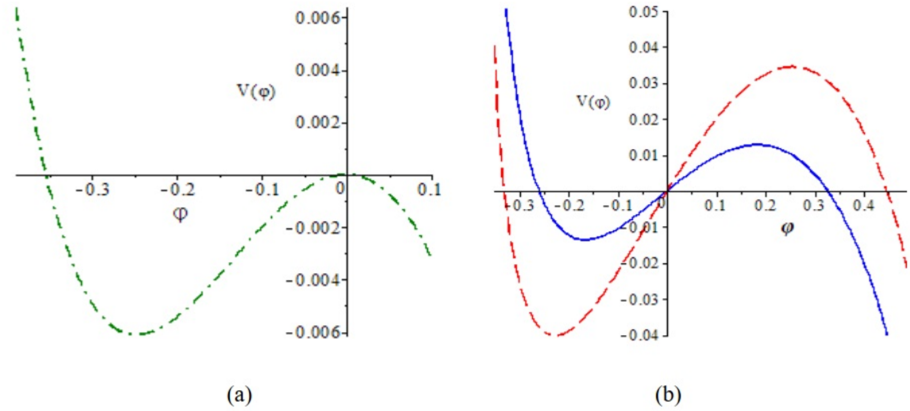

Figure 2: Variation of Sagdeev potential $V(\varphi)$ vs. φ for (a) $a = 0$, $M = 1$ (dash dotted curve) and (b) for $a = -0.1$ (solid curve) and $a = -0.2$ (dashed curve), $M = 0.85$ with the same fixed values as in Figure 1.

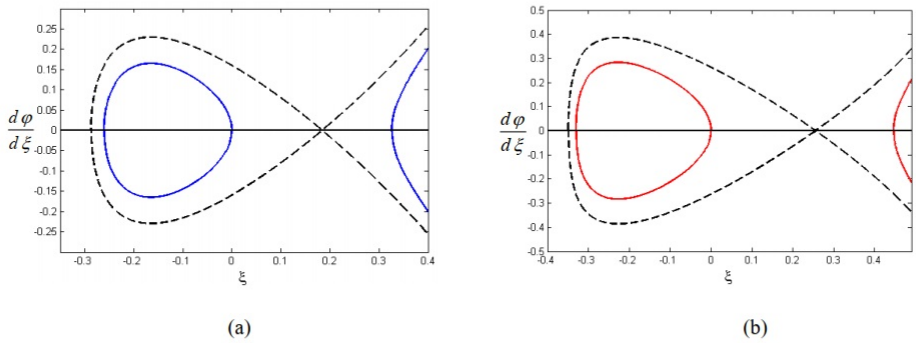

Figure 3: The phase portrait of DA soliton (dashed curve) and cnoidal wave (solid curve) for (a) $a = -0.1$ and (b) $a = -0.2$ with the same fixed values as in Figure 1.

dust grains case ($a = 0$) solitary waves are created. Moreover, if $\varphi_0 = \varphi_1$ then $S = 1$. In this case $V_0^{'} = 0$, therefore

$$\varphi(\xi) \to \sec h^2\Big(\frac{\mid \varphi_2 \mid}{12K}\xi\Big),$$

which corresponds to solitary solution when all different-sized dust grains have the same number.

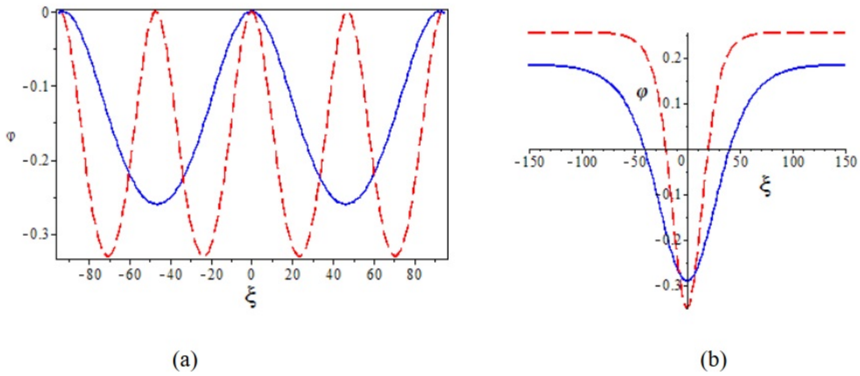

Figure 4: (a) the DA cnoidal wave structures, and (b) The DA solitons for $a = -0.1$ (solid curves) and for $a = -0.2$ (dashed curves) with the same fixed values as in Figure 1.

4 Conclusion

In this study, using the Sagdeev pseudopotential method for large amplitude waves, arbitrary amplitude DAWs are studied in a quantum dusty plasma, including the effect of dust size distribution (DSD). Considering the power-law distribution which satisfies $a < 0$, Sagdeev potential $V(\varphi)$ is obtained. Two cases are studied, a mono-sized dust grains case ($a = 0$) and a dust grains possessing power-law size distribution case ($a = -0.1, a = -0.2$). Applying the conditions in which a solitary solution can be existed, the upper and lower limits of the Mach number are plotted. The result shows that, the allowed Mach number's range is increased for mono-sized dust grains case. Thereafter, Sagdeev potential $V(\varphi)$ versus φ is plotted and it is seen that in mono-sized dust grains case, solitary waves are propagated. But, for different-sized dust grains having power-law size distribution, the solitary waves transform into cnoidal ones. The phase portrait is also plotted and comprised of Two different sets of orbits, a periodic orbit and a homoclinic one that correspond to periodic traveling wave solution and solitary wave solution, respectively. Finally, The variations of potential φ versus ξ associated to Sagdeev potential are plotted and represented DA soliton and cnoidal wave structures.

Authors' Contributions

All authors have the same contribution.

Data Availability

No data available.

Conflicts of Interest

The authors declare that there is no conflict of interest.

Ethical Considerations

The authors have diligently addressed ethical concerns, such as informed consent, plagiarism, data fabrication, misconduct, falsification, double publication, redundancy, submission, and other related matters.

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