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Research Paper

Quantum Thermodynamic Properties of a Moving Particle Next to a Surface in the Presence of Electromagnetic Field

Marjan Jafari
Department of Physics, Faculty of Science, Imam Khomeini International University, 34148–96818, Qazvin, Iran;
email: Jafary.marjan@gmail.com, m.jafari@sci.ikiu.ac.ir

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Abstract. In this paper, we investigate the effect of the motion of a magnetodielectric particle on the quantum thermodynamic properties of a system. Specifically, we study the behavior of a polarizable and magnetizable dielectric particle moving above a semi-infinite bulk dielectric in the presence of an electromagnetic field. Using a microscopic approach, we propose a covariant Lagrangian for the combined system and obtain a dynamical covariant response of the moving particle. We calculate the emitted power from the work extracted by the electric and magnetic dipoles of the moving particle. We explicitly determine the behavior of the main thermodynamic functions of the system, including thermal correlation functions, free energy, mean energy, entropy, and heat capacity. It is found that the thermodynamic properties of the moving particle in the electromagnetic field depend on the properties of the semi-infinite bulk dielectric. Moreover, we demonstrate that the formulation of quantum thermodynamics for an electromagnetic system in uniform relative motion differs from its formulation in the rest-frame.

Keywords: Thermal corralation function, Quantum Electrodynamics, Moving particle, Mean energy

1 Introduction

Recent progress in nanotechnology and nanoscale devices, such as scanning thermal microscopes, has raised questions about quantum thermodynamics and near-field effects [1–3]. Investigating the quantum thermodynamic properties of static nanoscale effects has attracted a lot of interest. Additionally, studying the effect of motion on quantum thermodynamic properties can be interesting in many branches of science, such as detectors and thermometers. Quantum thermodynamic properties of two objects, such as a microscope's tip near a substrate, are among the challenging problems [4,5].

Macroscopic electromagnetism is a fundamental part of theoretical physics, providing a detailed description of light-matter interactions. Studying the quantum thermodynamic properties of this kind of system is an attractive field that can appear in many branches of physics, such as the quantum thermometer, ion trapping, and optical trapping [6,7]. The electrodynamics of moving media is a fundamental subject, and a covariant theory of electrodynamics in moving media was presented by Minkowski using Einstein's special relativity theory [8,9]. The electrodynamics of moving media has been applied in various physical fields, such as the optics of moving media [10,11], radiation of fast charged particles in media [12], and astrophysics [13].

The quantization of the electromagnetic field in moving media has been investigated by Jauch and Watson [14]. After that, Horsley developed a canonical theory that includes dispersive and dissipation effects [15].

Studying the macroscopic electromagnetic system focusing on its thermodynamic properties is of great interest in quantum optics, solid-state physics, and material science [16–18]. Many applications of quantum electrodynamics in dielectric media require a quantum thermodynamics description for the electromagnetic field interacting with the matter field [19]. The lack of a straightforward useful method for measuring thermodynamic quantities in moving systems in the presence of electromagnetic fields encourages us to investigate this subject.

In this paper, based on a microscopic approach, we propose a Lagrangian for the combined system of a moving dielectric particle next to the semi-infinite bulk dielectric in the presence of electromagnetic vacuum fluctuation. The Lagrangian describing the whole system is the Lagrangian of the electromagnetic vacuum field plus terms modeling the moving particle and semi-finite dielectric and their interaction with the electromagnetic field [20,21]. The moving particle and the semi-infinite bulk interact with the electromagnetic vacuum field indirectly. The particle and semi-infinite dielectric are described by covariant fields. Inspired by the Caldeira-Leggett model, we modeled fields as a continuum of the Klein-Gordon field [6]. The coupling tensors couple the electromagnetic field to the magneto-dielectric particle and bulk dielectric. The coupling tensors play a vital role in the canonical quantization scheme, and the susceptibility tensor of the particle is expressed in terms of the coupling tensor. The moving particle produces electric and magnetic moments that can interact with the electric and magnetic components of the electromagnetic field. The Hamiltonian is diagonalized by the Fano model [22], a simple model describing the interaction of the electromagnetic field with a medium. A benefit of diagonalization of the Hamiltonian is that it allows for a straightforward calculation of thermodynamic quantities for the moving particle. We assume thermal equilibrium for all couplings, which makes diagonalization valid. Next, we obtain thermal correlation functions and the main thermodynamic functions, including free energy, mean energy, entropy, and heat capacity.

The aim of this work is to investigate the quantum thermodynamic properties of a moving dielectric particle above a plane interface in the framework of covariant canonical field quantization approach. The particle could be a photon detector placed outside of the material. The layout of the paper is as follows: In Section 2, a covariant Lagrangian for the total system is proposed, and using Euler-Lagrange equations, the equation of motion for moving nanoparticle above a semi-infinite dielectric is obtained in the presence of electromagnetic field. The total system is canonically quantized in section 3, and the Hamiltonian is diagonalized using the Fano diagonalization technique. In Section 4, the thermal correlation functions of dynamical variables, the thermal expectation value of the moving nanoparticle, mean force thermal energy, and free energy are obtained. Finally, the conclusion is given in section 5.

2 Lagrangian

The system under consideration is a moving dielectric particle interacting with an electromagnetic field in the presence of a semi-infinite bulk dielectric. The moving dielectric particle and the semi-infinite bulk dielectric are defined by real vector fields $X_{\mu,\omega}(\mathbf{x},t)$ and $Y_{\mu,\omega}(\mathbf{x},t)$, respectively, which are assumed to be a continuum of real Klein-Gordon fields. Throughout the article, we assume that the fields are defined in $(1+3)$-dimensional space-time. The covariant Lagrangian density for the system is given by the following expression

$$L_{EM} = \frac{1}{2}\partial_\mu A_\nu \partial^\mu A^\nu; \tag{1}$$

$$L_s = \frac{1}{2}\int_0^\infty d\omega[(\gamma v_\alpha \partial^\alpha X_{\mu,\omega}(\mathbf{x},t))(\gamma v^\beta \partial_\beta X^\mu_\omega(\mathbf{x},t)) - \omega^2 X_{\mu,\omega}(\mathbf{x},t)X^\mu_\omega(\mathbf{x},t)], \qquad \alpha=\beta \tag{2}$$

$$L_{env} = \frac{1}{2}\int_0^\infty d\omega(\partial_\mu Y^\mu(\mathbf{x},t)\partial^\zeta Y_\zeta(\mathbf{x},t) - \omega_0^2 Y_\mu(\mathbf{x},t)Y^\mu(\mathbf{x},t)), \quad \mu=\zeta \tag{3}$$

$$L_{int} = \int_0^\infty d\omega X_{\mu,\omega}(\mathbf{x},t)f_{\mu,\nu}(\omega,t)\partial_\alpha A_\nu(\mathbf{x},t) + \int_0^\infty d\omega Y_{\mu,\omega}(\mathbf{x},t)g_{\mu,\nu}(\omega)\partial_\alpha A_\nu(\mathbf{x},t) \tag{4}$$

The first term, L_{EM}, is the covariant form of the Lagrangian density of the electromagnetic field. The second term is the covariant Lagrangian density of the moving particle, which is modified due to the Lorentz transformation of the time coordinate. The third term is the Lagrangian density of the bulk dielectric, and the last term is the interaction between the moving dielectric particle and the electromagnetic field, as well as the interaction between the bulk dielectric and the electromagnetic field. The velocity component of the moving particle is denoted by v_α, and $\gamma = \frac{1}{\sqrt{1-\frac{\mathbf{v}^2}{c^2}}}$. The second-order coupling tensor between the particle's field and electromagnetic field, $f_{\mu\nu}$, is time-dependent due to the motion of the particle, whereas the coupling tensor $g_{\mu\nu}$ between the electromagnetic field and static dielectric Y_ω is time-independent.

Dielectrics are considered anisotropic and homogeneous, and we define $P_A(\mathbf{x},t) = \int d\omega X^\mu_\omega(\mathbf{x},t)f_{\mu\nu}(\omega,t)$ and $P_B(\mathbf{x},t) = \int d\omega Y^\mu_\omega(\mathbf{x},t)g_{\mu\nu}(\omega)$ as the polarization components of the moving dielectric particle and semi-infinite bulk dielectric, respectively. The classical equation of motion from Euler-Lagrange equations for A_μ, X_ω, and Y_ω is obtained as follows.

$$-\partial_\mu\partial^\mu A^\nu = \partial_\mu[P_A(\mathbf{x},t) + P_B(\mathbf{x},t)], \tag{5}$$

$$[\gamma^2 v^\alpha v_\beta \partial^\alpha \partial_\beta + \omega^2]X_{\omega,\mu}(\mathbf{x},t) = f_{\mu,\nu}(\omega)\partial_\alpha A^\nu(\mathbf{x},t), \tag{6}$$

$$(\partial_\nu\partial^\nu + \omega_0^2)Y_{\omega,\mu}(\mathbf{x},t) = g_{\mu,\nu}(\omega)\partial_\alpha A^\nu(\mathbf{x},t). \tag{7}$$

For simplicity, we work in the reciprocal space, and the equation of motion is written in terms of spatial Fourier transforms. So, the equations of motion for the electromagnetic field, moving dielectric particle, and bulk dielectric field are derived as follows

$$(\partial_t^2 + \mathbf{k}^2)A_\mu(\mathbf{k},t) = (\partial_t - \mathbf{k}*)(P_A(\mathbf{k},t) + P_B(\mathbf{k},t)), \tag{8}$$

$$[\partial_t^2 + \Omega^2(k,\omega)]X_{\omega\mu}(\mathbf{k},t) = \frac{f_{\mu,\nu}(\omega)}{\gamma^2}(\partial_t A_\nu(\mathbf{k},t) + \mathbf{k}\times A_\nu(\mathbf{k},t)), \tag{9}$$

where $\Omega(k,\omega) = \sqrt{v^2k^2 + \frac{\omega^2}{\gamma^2}}$, and

$$[\partial_t^2 + \Omega'^2(k,\omega)]Y_{\omega\mu}(\mathbf{k},t) = g_{\mu,\nu}(\omega)\partial_t A^\nu(\mathbf{k},t), \tag{10}$$

where $\Omega'^2(\mathbf{k},\omega) = \mathbf{k}^2 + \omega_0^2$. The general solution of the moving dielectric particle field is

$$X_{\mu,\omega}(\mathbf{k},t) = X^N_{\mu,\omega}(\mathbf{k},t) + \frac{f_{\mu,\nu}(\omega)}{\gamma^2\Omega(k,\omega)}\int_0^t dt'(\partial_t A^\nu(\mathbf{k},t') + (\mathbf{k}\times A)^\nu)\sin\Omega(t-t')e^{-0^+(t-t')} \quad (11)$$

The fluctuating field X_N is the homogeneous solution of the equation of motion for the particle field,

$$X^N_{\mu,\omega}(\mathbf{k},t) = \dot{X}_{\mu,\omega}(\mathbf{k},0)\frac{\sin(\Omega t)}{\Omega(k,\omega)} + X_{\mu,\omega}(\mathbf{k},0)\cos(\Omega t). \quad (12)$$

The formal solution of the equation (10) is

$$Y_{\mu,\omega}(\mathbf{k},t) = Y^N_{\mu,\omega}(\mathbf{k},t) + g_{\mu,\nu}(\omega)\int_0^t dt'\partial_t A^\nu(\mathbf{k},t')\sin\Omega'(t-t')e^{-0^+(t-t')}. \quad (13)$$

The fluctuating field Y_N is the homogeneous solution of the equation of motion for the bulk dielectric field,

$$Y^N_{\mu,\omega}(\mathbf{k},t) = \dot{Y}_{\mu,\omega}(\mathbf{k},0)\frac{\sin(\Omega' t)}{\Omega'(k,\omega)} + Y_{\mu,\omega}(\mathbf{k},0)\cos(\Omega' t). \quad (14)$$

In the frequency domain, (11) and (13) are

$$\begin{aligned} X_{\mu,\omega}(\mathbf{k},\omega') =& \pi(X_{\mu,\omega}(\mathbf{k},0) + i\frac{\dot{X}_{\mu,\omega}(\mathbf{k},0)}{\Omega})\delta(\Omega-\omega') \\ &+ \pi(X_{\mu,\omega}(\mathbf{k},0) - i\frac{\dot{X}_{\mu,\omega}(\mathbf{k},0)}{\Omega})\delta(\Omega+\omega') \\ &+ \mathbf{P}\frac{f_{\mu,\nu}(\omega)}{\gamma^2}\frac{\omega' A_\nu(\mathbf{k},\omega') + (\mathbf{k}\times A(\mathbf{k},\omega'))_\nu}{\Omega^2-\omega'^2} \\ &+ \frac{f_{\mu,\nu}(\omega)}{2\gamma^2\Omega}[\delta(\omega'-\Omega)+\delta(\omega'+\Omega)]\mathbf{P}\int d\zeta\frac{\zeta A_\nu(\mathbf{k},\zeta) + (k\times A(\mathbf{k},\zeta))_\nu}{\zeta-\omega'} \end{aligned} \quad (15)$$

and

$$\begin{aligned} Y_{\mu,\omega}(\mathbf{k},\omega') =& \pi(Y_{\mu,\omega}(\mathbf{k},0) + i\frac{\dot{Y}_{\mu,\omega}(\mathbf{k},0)}{\Omega'})\delta(\Omega'-\omega') \\ &+ \pi(Y_{\mu,\omega}(\mathbf{k},0) - i\frac{\dot{Y}_{\mu,\omega}(\mathbf{k},0)}{\Omega'})\delta(\Omega'+\omega') + \mathbf{P}g_{\mu,\nu}(\omega)\frac{\omega' A_\nu(\mathbf{k},\omega')}{\Omega'^2-\omega'^2} \\ &+ \frac{g_{\mu,\nu}(\omega)}{2\Omega'}[\delta(\omega'-\Omega')+\delta(\omega'+\Omega')]\mathbf{P}\int d\zeta\frac{\zeta A_\nu(\mathbf{k},\zeta)}{\zeta-\omega'}. \end{aligned} \quad (16)$$

The Fourier transform(s) of the Euler-Lagrange equations (8), (9) and (10) are

$$(-\omega'^2 + \mathbf{k}^2)A_\nu(\mathbf{k},\omega') = \omega' P_B(\mathbf{k},\omega') + (\omega' + \mathbf{k}\times)P_A(\mathbf{k},\omega'), \quad (17)$$

$$(-\omega'^2 + \Omega'^2(\omega,k))Y_{\mu,\omega}(\mathbf{k},\omega') = g_{\mu,\nu}(\omega)\omega' A_\nu(\mathbf{k},\omega'), \quad (18)$$

$$(-\omega'^2 + \Omega^2(\omega,k))X_{\mu,\omega}(\mathbf{k},\omega') = \frac{f_{\mu,\nu}(\omega)}{\gamma}(\omega' A_\nu(\mathbf{k},\omega') + (\mathbf{k}\times A(\mathbf{k},\omega'))_\nu). \quad (19)$$

In this section, the classical equations of motion that follow from (1) are obtained. To obtain the quantum dynamical description, we need to quantize the system canonically.

3 Quantum dynamics

A Hamiltonian must be derived from the local Lagrangian to quantize the system(1). The associated canonical momentum density for the electromagnetic field, moving dielectric particle, and bulk dielectric are as follows

$$\mathbf{p}(\mathbf{x},t) = \frac{\partial L}{\partial \dot{A}} = \dot{A}(\mathbf{x},t) + P_A(\mathbf{x},t) + P_B(\mathbf{x},t) \tag{20}$$

$$\mathbf{\Pi} = \frac{\partial L}{\partial \dot{\mathbf{X}}} = \gamma^2 \dot{\mathbf{X}}, \tag{21}$$

$$\mathbf{Q} = \frac{\partial L}{\partial \dot{\mathbf{Y}}} = \dot{\mathbf{Y}}. \tag{22}$$

The following equal-time communication relations are required on the fields and their conjugate momenta to quantize the system canonically

$$\left[\hat{A}_\mu(x,t), \hat{p}_\nu(x',t)\right] = i\hbar\,\delta(x-x')\delta_{\mu\nu}, \tag{23}$$

$$\left[\hat{X}_{\mu,\omega}(x,t), \hat{\Pi}_{\nu,\omega'}(x',t)\right] = i\hbar\gamma^2\delta(\omega-\omega')\delta_{\mu\nu}\,\delta(x-x'), \tag{24}$$

$$\left[\hat{Y}_{\mu,\omega}(x,t), \hat{Q}_{\nu,\omega'}(x',t)\right] = i\hbar\,\delta(\omega-\omega')\delta_{\mu\nu}\,\delta(x-x'). \tag{25}$$

From these relations, we can obtain the Hamiltonian of the system as follows

$$\begin{aligned}
\hat{H} &= \frac{1}{2}\int d\omega\Big(\frac{\hat{\mathbf{\Pi}}^2_\omega(\mathbf{x},t)}{\gamma^2} + \omega^2\hat{\mathbf{X}}^2_\omega(\mathbf{x},t) + \gamma^2 v^2(\partial_x \hat{\mathbf{X}}_\omega(\mathbf{x},t))^2\Big) \\
&+ \frac{1}{2}\int d\omega\big(\hat{\mathbf{Q}}^2_\omega(\mathbf{x},t) + \omega_0^2\hat{\mathbf{Y}}^2_\omega(\mathbf{x},t) + (\partial_x\hat{\mathbf{Y}}_\omega(\mathbf{x},t))^2\big) \\
&+ \int d\omega \hat{X}_{\mu\omega}(\mathbf{x},t) f_{\mu\nu}(\omega)\partial^i\hat{A}^\nu(\mathbf{x},t) + \frac{1}{2}\partial_i\hat{A}_\nu\partial^i\hat{A}^\nu \\
&+ \frac{1}{2}(\hat{\mathbf{p}}(\mathbf{x},t) - P_A(\mathbf{x},t) - P_B(\mathbf{x},t))^2.
\end{aligned} \tag{26}$$

To have a better understanding of the quantum dynamics of the system, especially moving dielectric particle, the diagonal form of Hamiltonian is useful. The Hamiltonian operator is quadratic in the fields; therefore it can be diagonalized into a continuum of normal modes by the Fano diagonalization technique as follows

$$\hat{H} = \int_0^\infty d\mathbf{k}\int_0^\infty \hbar\omega\,(\hat{C}^\dagger(\mathbf{k},\omega,t)\hat{C}(\mathbf{k},\omega,t) + h.c). \tag{27}$$

The $\hat{C}$ operators appearing in (27) satisfy bosonic commutation relations and are combinations of the electromagnetic field, particle, and bulk dielectric field operators.

$$[\hat{C}(\mathbf{k},\omega,t), \hat{C}^\dagger(\mathbf{k}',\omega',t)] = \delta(\omega-\omega')\,\delta(\mathbf{k}-\mathbf{k}'). \tag{28}$$

The diagonalizing operators can be expressed as a linear combination of the canonical operators

$$\begin{aligned}
\hat{C}_\mu(\mathbf{k},\omega,t) =& \frac{-i}{\hbar}\big(f^*_{p,\mu\nu}(\mathbf{k},\omega)\hat{A}_\nu(\mathbf{k},t) - f^*_{A,\mu\nu}(\mathbf{k},\omega)\hat{p}_\nu(\mathbf{k},t) \\
&+ \int_0^\infty d\omega'\, f^*_{\Pi,\mu\nu}(\mathbf{k},\omega,\omega')\hat{X}_{\omega',\nu}(\mathbf{k},t) - \int_0^\infty d\omega'\, f^*_{X,\mu\nu}(\mathbf{k},\omega,\omega')\hat{\Pi}_{\omega',\nu}(\mathbf{k},t)\big) \\
&+ \int_0^\infty d\omega'\, f^*_{Q,\mu\nu}(\mathbf{k},\omega,\omega')\hat{Y}_{\omega',\nu}(\mathbf{k},t) - \int_0^\infty d\omega'\, f^*_{Y,\mu\nu}(\mathbf{k},\omega,\omega')\hat{Q}_{\omega',\nu}(\mathbf{k},t)\big),
\end{aligned} \tag{29}$$

where $f_p, f_A, f_\pi, f_X, f_Q, f_Y$ are bi-tensorial coefficients, and they must be obtained to establish diagonalization. The following relations among bi-tensorial coefficients are achieved using $[\hat{H}, \hat{C}] = \hbar\omega\hat{C}$ and fundamental commutation relations

$$(\gamma^2\omega^2 - \gamma^2\Omega^2(\mathbf{k},\omega'))f_{X,\mu\nu}(\omega,\omega',\mathbf{k}) = -f_{\mu\alpha}(\omega')(\mathbf{k} - \frac{\mathbf{k}^2}{\omega})f_{A,\alpha\nu}(\mathbf{k},\omega) + \int d\omega' f^2_{\mu\alpha}(\omega')(1 + \frac{\mathbf{k}}{\omega'})f_{X,\alpha\nu}(\omega,\omega',\mathbf{k}) + \int d\omega' g_{\mu\alpha}(\omega')f_{\alpha\beta}(\omega')f_{Y,\beta\nu}(\omega,\omega',\mathbf{k}); \quad (30)$$

$$(\omega^2 - \Omega'^2(\mathbf{k},\omega'))f_{Y,\mu\nu}(\omega,\omega',\mathbf{k}) = g_{\mu\alpha}(\omega')\frac{\mathbf{k}^2}{\omega}f_{A,\alpha\nu}(\mathbf{k},\omega) + \int d\omega' f_{\mu\alpha}\omega' g_{\mu\nu}(\omega')(1 + \frac{\mathbf{k}}{\omega'})f_{X,\alpha\nu}(\omega,\omega',\mathbf{k}) + \int d\omega' g^2_{\mu\alpha}(\omega')f_{Y,\beta\nu}(\omega,\omega',\mathbf{k}); \quad (31)$$

$$(\omega^2 - \mathbf{k}^2)f_{A,\mu\nu}(\mathbf{k},\omega) = -\int_0^\infty d\omega'\, f_{\mu\alpha}(\omega')(\mathbf{k} - \omega')f_{X,\alpha\nu}(\omega,\omega',\mathbf{k}) + \int_0^\infty d\omega'\, g_{\mu\alpha}(\omega')\omega' f_{Y,\alpha\nu}(\omega,\omega',\mathbf{k}); \quad (32)$$

$$(\gamma^2\omega^2 - \eta^2)f_{X,\mu\nu}(\omega,\omega',\mathbf{k}) = -A_{\mu\alpha}(\mathbf{k},\omega)f_{A,\alpha\nu}(\mathbf{k},\omega) \quad (33)$$

$$(\omega^2 - \eta'^2)f_{A,\mu\nu}(\mathbf{k},\omega) = -\int_0^\omega B_{\mu\alpha}(\mathbf{k},\omega)f_{X,\mu\nu}(\omega,\omega',\mathbf{k}) \quad (34)$$

$$(\omega^2 - \Lambda'^2)f_{Y,\mu\nu}(\omega,\omega',\mathbf{k}) = g_{\mu\alpha}\frac{\mathbf{k}\times\mathbf{k}}{\omega}f_{A,\alpha\nu}(\mathbf{k},\omega) + \int d\omega f_{\mu\alpha}g_{\alpha\alpha'}f_{X,\alpha'\nu}(\omega,\omega',\mathbf{k}) \quad (35)$$

where

$$A_{\mu\alpha}(\mathbf{k},\omega) = f_{\mu\nu}(\omega)(\mathbf{k} - \frac{\mathbf{k}\times\mathbf{k}}{\omega}) + \int d\omega g^2_{\mu\alpha}f_{\alpha\nu}\frac{\mathbf{k}\times\mathbf{k}}{\omega(\omega^2 - \Lambda'^2)}; \quad (36)$$

$$B_{\mu\alpha}(\mathbf{k},\omega) = f_{\mu\nu}(\omega)(\omega - \mathbf{k}) + \int d\omega g^2_{\mu\alpha}f_{\alpha\nu}(1 - \frac{\mathbf{k}}{\omega})\frac{\omega}{\omega^2 - \Lambda'^2}; \quad (37)$$

$$\eta^2 = \Lambda^2 + \int d\omega \int d\omega' g^2_{\mu\alpha}f^2_{\alpha\mu}(1 - \frac{\mathbf{k}}{\omega})\frac{1}{\omega^2 - \lambda'^2}; \quad (38)$$

$$\eta'^2 = \mathbf{k}^2 + \int d\omega g^2_{\mu\alpha}\frac{\mathbf{k}\times\mathbf{k}}{\omega}\frac{1}{\omega^2 - \lambda'^2}; \quad (39)$$

$$\Lambda^2 = \gamma^2\Omega^2 + \int d\omega' f_{\mu\nu}f_{\nu\mu}(1 - \frac{\mathbf{k}}{\omega'}); \quad (40)$$

$$\Lambda'^2 = \Omega'^2 + \int d\omega' g_{\mu\nu}g_{\nu\mu}; \quad (41)$$

After some long straightforward calculation, we find the required bi-tensorial coefficients which completes the diagonalization procedure. Now, the electromagnetic field operators in the frequency-domain can be written in terms of the diagonalizing operators as follows

$$\hat{A}_\mu(\mathbf{k},\omega) = \pi\sqrt{\frac{2\hbar}{\eta(\mathbf{k},\omega)}}B_{\alpha\alpha'}(\mathbf{k},\omega)G_{\alpha'\mu}(\mathbf{k},\omega)\hat{C}_\alpha(\mathbf{k},\omega), \quad (42)$$

that green's function G of the system is as follows

$$G_{\mu\nu}(\mathbf{k},\omega) = -(\omega^2 - \eta'^2 + P\int_0^\infty d\omega'\frac{A_{\mu\alpha}(\mathbf{k},\omega')B_{\alpha\nu}(\mathbf{k},\omega')/\gamma^2}{\eta^2(\mathbf{k},\omega)/\gamma^2 - \omega^2} + \frac{i\pi A_{\mu\alpha}(\mathbf{k},\omega)B_{\alpha\nu}(\mathbf{k},\omega')}{2\eta(\mathbf{k},\omega')})^{-1} \quad (43)$$

and memory function of the system is

$$\kappa_{\mu\nu} = P\int_0^\infty d\omega' \frac{A_{\mu\alpha}(\mathbf{k},\omega')B_{\alpha\nu}(\mathbf{k},\omega')/\gamma^2}{\eta^2(\mathbf{k},\omega)/\gamma^2-\omega^2} + \frac{i\pi A_{\mu\alpha}(\mathbf{k},\omega)B_{\alpha\nu}(\mathbf{k},\omega')}{2\eta(\mathbf{k},\omega')}. \tag{44}$$

The moving dielectric particle field in terms of the diagonalization operators is

$$\begin{aligned}\hat{X}_{\omega,\mu}(\mathbf{k},\omega') =& 2\pi\sqrt{\frac{\hbar}{2\eta(\mathbf{k},\omega)}}\delta(\omega-\frac{\eta(\mathbf{k},\omega)}{\gamma})\hat{C}_\mu(\mathbf{k},\omega)\\ &+\frac{A_{\nu\mu}}{2\gamma^2\eta(\mathbf{k},\omega)}(\frac{1}{\omega+\frac{\eta(\mathbf{k},\omega)}{\gamma}}+\frac{1}{\frac{\eta(\mathbf{k},\omega)}{\gamma}-\omega-i0^+})\hat{A}_\nu(\mathbf{k},\omega),\end{aligned} \tag{45}$$

and the bulk dielectric field according to diagonalize operators is

$$\begin{aligned}\hat{Y}_{\omega,\mu}(\mathbf{k},\omega') =& 2\pi\sqrt{\frac{\hbar}{2\Lambda'(\mathbf{k},\omega)}}\delta(\omega'-\Lambda')\hat{C}_\mu(\mathbf{k},\omega)\\ &\frac{D_{\mu\nu}(\mathbf{k},\omega)}{2\Lambda'(\mathbf{k},\omega)}(\frac{1}{\omega'-\Lambda'(\mathbf{k},\omega)}+\frac{1}{\Lambda'-\omega'-i0^+})\hat{A}_\alpha(\mathbf{k},\omega'),\end{aligned} \tag{46}$$

where we defined

$$\begin{aligned}D_{\mu\nu}(\mathbf{k},\omega) =& g_{\mu\nu}\frac{\mathbf{k}\times\mathbf{k}}{\omega}+\frac{A_{\mu\nu}(\mathbf{k},\omega)}{2\gamma^2\eta(\mathbf{k},\omega)}(\frac{1}{\omega+\frac{\eta}{\gamma}}+\frac{1}{\frac{\eta}{\gamma}-\omega-i0^+})\\ &+\int d\omega f_{\mu\alpha}(\omega)g_{\alpha\nu}(\omega)(1-\frac{\mathbf{k}}{\omega})\end{aligned} \tag{47}$$

Having explicit form of the fields in terms of creation and annihilation operators, quantum dynamical and quantum thermodynamical properties of the system can be calculated. In the next section, we will obtain some interesting thermal properties of the system.

4 Quantum Thermal Properties

When objects are set in motion, they may extract real photons from the quantum electromagnetic field and experience a non-contact force. Firstly, we calculate the emitted power by extracting work from the electric and magnetic dipoles of the moving particle. The power of the system is determined by the rate of work done by the electromagnetic field on a dielectric at a differential volume $d\mathbf{r}$ in the relativistic regime.

$$\langle P\rangle = \int_V d\mathbf{r}\,\langle\partial_\mu(v_\mu P_{A\nu}(\mathbf{k},\omega)+P_{B\nu}(\mathbf{k},\omega))(v_\alpha\partial_\alpha A^\nu(\mathbf{k},\omega))\rangle\,, \tag{48}$$

where $\partial_\mu(v_\mu P_{A\nu}(\mathbf{k},\omega)+P_{B\nu}(\mathbf{k},\omega))$ is relativistic current density in the matter. By substituting Eqs.(42,45,46) into Eq.(48), we have

$$\begin{aligned}\langle P\rangle =& 2\pi^2\hbar\int_V d\mathbf{r}\int d\omega\int dk^\mu[(\frac{\mathbf{k}^2v^2 f_{\alpha\alpha''}(\omega)\delta(\omega-\frac{\eta}{\gamma})}{\eta(\mathbf{k},\omega)}+\frac{\mathbf{k}^2 v g_{\alpha\alpha''}(\omega)\delta(\omega-\Lambda')}{\sqrt{\eta(\mathbf{k},\omega)\Lambda'(\mathbf{k},\omega)}})\\ &\times B_{\alpha\alpha'}(\mathbf{k},\omega)G_{\alpha'\alpha''}(\mathbf{k},\omega)N(\omega)\\ &+(f_{\alpha\alpha'}(\omega)\mathbf{k}^2v^2\beta_{\alpha'\alpha''}(\omega)+g_{\alpha\alpha'}(\omega)\mathbf{k}^2v\beta'_{\alpha'\alpha''}(\omega))N(\omega)\Im G_{\alpha''\alpha}(\mathbf{k},\omega))],\end{aligned} \tag{49}$$

where

$$\Im G_{\mu\nu} = \frac{\pi^2\hbar}{\eta(\mathbf{k},\omega)} B^2_{\mu\alpha}(\omega) G^*_{\alpha\beta}(\mathbf{k},\omega) G_{\beta\nu}(\mathbf{k},\omega),$$

and we defind

$$\beta_{\mu\nu}(\omega) = \frac{A_{\nu\mu}(\mathbf{k},\omega)}{2\gamma^2\eta(\mathbf{k},\omega)}\left(\frac{1}{\omega + \frac{\eta(\mathbf{k},\omega)}{\gamma}} + \frac{1}{\frac{\eta(\mathbf{k},\omega)}{\gamma} - \omega - i0^+}\right) \tag{50}$$

$$\beta'_{\mu\nu}(\omega) = \frac{D_{\mu\nu}(\mathbf{k},\omega)}{2\Lambda'(\mathbf{k},\omega)}\left(\frac{1}{\omega' - \Lambda'(\mathbf{k},\omega)} + \frac{1}{\Lambda' - \omega' - i0^+}\right) \tag{51}$$

In the non-relativistic limit, where terms containing velocity are ignored, the same results as those obtained in previous examples [20,23] are obtained. This relation allows us to interpret thermal and fractional radiation, heat transfer, and the effect of movement on the particle's radiation power.

In the second step, we calculate the thermal correlation functions of the subsystems. In thermal equilibrium, the expectation value of each eigenmode of the system is

$$\left\langle \hat{C}^\dagger_\mu(k,\omega)\hat{C}_\nu(k',\omega')\right\rangle = N(\omega)\delta(k-k')\delta(\omega-\omega')\delta_{\mu\nu},$$

and

$$\left\langle \hat{C}_\mu(k,\omega)\hat{C}_\nu(k',\omega')\right\rangle = 0,$$

where $N(\omega) = \exp(\hbar\omega/K_BT) - 1$. Using equation (42), we can calculate the frequency-domain correlation function for the electromagnetic field.

$$\begin{aligned}\left\langle \hat{A}^\dagger_\mu(\mathbf{k},\omega)\hat{A}_\nu(\mathbf{k}',\omega')\right\rangle =& \frac{\pi^2\hbar}{\eta(\mathbf{k},\omega)} B^2_{\mu\alpha}(\omega) G^*_{\alpha\alpha'}(\mathbf{k},\omega) G_{\alpha'\nu}(\mathbf{k},\omega) N(\omega)\,\delta(\omega-\omega')\delta(\mathbf{k}-\mathbf{k}')\delta_{\mu\nu}\\ =& \frac{N(\omega)}{N(\omega)+1}\left\langle \hat{A}_\mu(\mathbf{k},\omega)\hat{A}^\dagger_\nu(\mathbf{k}',\omega')\right\rangle,\end{aligned} \tag{52}$$

and the thermal expectation value of the particle in the frequency domain is

$$\begin{aligned}\left\langle \hat{X}^\dagger_\mu(\mathbf{k},\omega)\hat{X}_\nu(\mathbf{k}',\omega')\right\rangle =& \frac{2\pi\hbar}{\eta(\mathbf{k},\omega)}(1 + \beta_{\mu\nu}(\omega)\beta^*_{\mu\nu}(\omega) + \beta_{\mu\alpha}(\omega)B_{\alpha\alpha'}(\mathbf{k},\omega)G_{\alpha'\nu}(\mathbf{k},\omega)\\ &+ \beta^*_{\mu\alpha}(\omega)B^*_{\alpha\alpha'}(\mathbf{k},\omega)G^*_{\alpha'\nu}(\mathbf{k},\omega)) \times N(\omega)\,\delta(\omega-\omega')\delta(\mathbf{k}-\mathbf{k}')\delta_{\mu\nu}.\end{aligned} \tag{53}$$

The temporal and symmetric correlation function of the electromagnetic field is obtained as follows using equation (52)

$$\begin{aligned}&\frac{1}{2}\left\langle \hat{A}^\dagger_\mu(\mathbf{x},t)\hat{A}_\nu(\mathbf{x}',t') + \hat{A}_\mu(\mathbf{x},t)\hat{A}^\dagger_\nu(\mathbf{x}',t')\right\rangle =\\ &\frac{\hbar}{4\pi^3}\int_0^\infty d\omega\int_0^\infty d\mathbf{k}\coth\left(\frac{\hbar\omega}{2K_BT}\right)\cos[\omega(t-t') - \mathbf{k}(\mathbf{r}-\mathbf{r}')]\,\Im G_{\mu\nu}(\mathbf{k},\omega).\end{aligned} \tag{54}$$

The thermal expectation value of the moving dielectric particle in thermal equilibrium is

$$\begin{aligned}&\frac{1}{2}\int_0^\infty d\omega\left\langle \gamma^2 v^2_\alpha \partial^\alpha X_{\mu,\omega}(\mathbf{x},t)\partial^\alpha X_{\nu,\omega}(\mathbf{x},t) - \omega^2 X_{\mu,\omega}(\mathbf{x},t)X_{\nu,\omega}(\mathbf{x},t)\right\rangle =\\ &\frac{\hbar}{2\pi}\Im\int_0^\infty d\omega\int_0^\infty d\mathbf{k}\,\mathbf{k}^2\frac{A_{\alpha'\alpha''}(\omega)}{B_{\alpha'\alpha''}(\omega)}\coth\left(\frac{\hbar\omega}{2K_BT}\right)\frac{d[\kappa_{\mu\alpha}(\mathbf{k},\omega)\Omega(\mathbf{k},\omega)]}{d\omega}G_{\alpha\nu}(\mathbf{k},\omega)\end{aligned} \tag{55}$$

Hamiltonian (26) expectation value in thermal equilibrium contains some sentence that we need to calculate. The expectation value of the Hamiltonian of the free moving dielectric particle is

$$\frac{1}{2}\int_0^\infty d\omega \left\langle \partial^\alpha Y_{\mu,\omega}(\mathbf{x},t)\partial^\alpha Y_{\nu,\omega}(\mathbf{x},t) - \omega_0^2 Y_{\mu,\omega}(\mathbf{x},t)Y_{\nu,\omega}(\mathbf{x},t)\right\rangle =$$
$$\frac{\hbar}{2\pi}\Im\int_0^\infty d\omega\int_0^\infty d\mathbf{k}\,\mathbf{k}^2\frac{D_{\alpha'\alpha''}(\omega)}{B_{\alpha'\alpha''}(\omega)}\sqrt{\frac{\eta(\mathbf{k},\omega)}{\eta'(\mathbf{k},\omega)}}\coth(\frac{\hbar\omega}{2K_BT})\frac{d[\kappa'_{\mu\alpha}(\mathbf{k},\omega)\Omega'(\mathbf{k},\omega)]}{d\omega}G_{\alpha\nu}(\mathbf{k},\omega), \tag{56}$$

and the expectation value of Hamiltonian of the bulk dielectric is as follows

$$\kappa'_{\mu\nu} = P\int_0^\infty d\omega'\frac{D_{\mu\alpha}(\mathbf{k},\omega')B_{\alpha\nu}(\mathbf{k},\omega')}{\Lambda'^2(\mathbf{k},\omega)-\omega^2} + \frac{i\pi D_{\mu\alpha}(\mathbf{k},\omega)B_{\alpha\nu}(\mathbf{k},\omega')}{2\Lambda'(\mathbf{k},\omega')} \tag{57}$$

and the thermal expectation value of the Hamiltonian interaction part is

$$\int_0^\infty d\omega f_{\mu\nu}\left\langle X_{\mu,\omega}(x,t)\partial^\alpha A_\nu(x,t)\right\rangle =$$
$$\frac{\hbar}{2\pi}\Im\int_0^\infty d\omega\int_0^\infty d\mathbf{k}\,\mathbf{k}f_{\alpha\alpha'}(\omega)\frac{A_{\alpha'\alpha''}(\omega)}{B_{\alpha'\alpha''}(\omega)}\coth(\frac{\hbar\omega}{2K_BT})\kappa_{\mu\alpha'}(\mathbf{k},\omega)G_{\alpha\nu}(\mathbf{k},\omega), \tag{58}$$

and

$$\int_0^\infty d\omega g_{\mu\nu}\left\langle Y_{\mu,\omega}(x,t)\partial^\alpha A_\nu(x,t)\right\rangle =$$
$$\frac{\hbar}{2\pi}\Im\int_0^\infty d\omega\int_0^\infty d\mathbf{k}\,\mathbf{k}g_{\alpha\alpha'}(\omega)\frac{D_{\alpha'\alpha''}(\omega)}{B_{\alpha'\alpha''}(\omega)}\sqrt{\frac{\eta}{\Lambda'}}\coth(\frac{\hbar\omega}{2K_BT})\kappa'_{\mu\alpha'}(\mathbf{k},\omega)G_{\alpha\nu}(\mathbf{k},\omega). \tag{59}$$

So having these quantities, the thermal equilibrium expectation value of the internal energy and free energy of the system are accessible. The internal energy of the system is

$$\begin{aligned}U =&\frac{\hbar}{2\pi}\Im\int_0^\infty d\omega\int_0^\infty d\mathbf{k}\,\coth(\frac{\hbar\omega}{2K_BT})G_{\mu\nu}(\mathbf{k},\omega)\\
&(\mathbf{k}^2I-\omega^2I+f_{\mu\alpha}(\omega)\frac{A_{\alpha'''\alpha''}(\omega)}{B_{\alpha'''\alpha''}(\omega)}(\frac{d[\kappa_{\alpha\nu}(\mathbf{k},\omega)\Omega(\mathbf{k},\omega)]\mathbf{k}^2}{d\omega}+\omega\kappa_{\mu\nu}(\mathbf{k},\omega))\\
&+g_{\mu\alpha}(\omega)\frac{D_{\alpha'''\alpha''}(\omega)}{B_{\alpha'''\alpha''}(\omega)}\sqrt{\frac{\eta(\mathbf{k},\omega)}{\Lambda'(\mathbf{k},\omega)}}(\frac{d[\kappa'_{\alpha\nu}(\mathbf{k},\omega)\Omega'(\mathbf{k},\omega)]\mathbf{k}^2}{d\omega}+\omega\kappa'_{\mu\nu}(\mathbf{k},\omega)))\end{aligned}$$

and free energy of the system is

$$\begin{aligned}F =&\frac{K_BT}{\pi}\int_0^\infty d\omega\int_0^\infty d\mathbf{k}\,\ln(\sinh\frac{\hbar\omega}{2K_BT})\Im G_{\mu\nu}(\mathbf{k},\omega)\\
&(\mathbf{k}^2I-\omega^2I+f_{\mu\alpha}(\omega)\frac{A_{\alpha'''\alpha''}(\omega)}{B_{\alpha'''\alpha''}(\omega)}(\frac{d[\kappa_{\alpha\nu}(\mathbf{k},\omega)\Omega(\mathbf{k},\omega)]\mathbf{k}^2}{d\omega}+\omega\kappa_{\mu\nu}(\mathbf{k},\omega))\\
&+g_{\mu\alpha}(\omega)\frac{D_{\alpha'''\alpha''}(\omega)}{B_{\alpha'''\alpha''}(\omega)}\sqrt{\frac{\eta(\mathbf{k},\omega)}{\Lambda'(\mathbf{k},\omega)}}(\frac{d[\kappa'_{\alpha\nu}(\mathbf{k},\omega)\Omega'(\mathbf{k},\omega)]\mathbf{k}^2}{d\omega}+\omega\kappa'_{\mu\nu}(\mathbf{k},\omega)))\\
&+K_BT\ln 2.\end{aligned}$$

The quantum work of the electromagnetic field on the particle and heat power of the system can be achieved using the free energy of the system and the first law of thermodynamics [24]. Also, the entropy of the system and the specific heat at constant volume of the system can be obtained as follows

$$\begin{aligned} S =& \frac{K_B}{\pi} \int_0^\infty d\omega \int_0^\infty d\mathbf{k} \, [\ln(\sinh \frac{\hbar\omega}{2K_BT}) - \frac{\hbar\omega}{2K_BT} \coth(\frac{\hbar\omega}{2K_BT})] \Im G_{\mu\nu}(\mathbf{k},\omega) \\ & (\mathbf{k}^2 I - \omega^2 I + f_{\mu\alpha}(\omega) \frac{A_{\alpha'''\alpha''}(\omega)}{B_{\alpha'''\alpha''}(\omega)} (\frac{d[\kappa_{\alpha\nu}(\mathbf{k},\omega)\Omega(\mathbf{k},\omega)]\mathbf{k}^2}{d\omega} + \omega\kappa_{\mu\nu}(\mathbf{k},\omega)) \\ & + g_{\mu\alpha}(\omega) \frac{D_{\alpha'''\alpha''}(\omega)}{B_{\alpha'''\alpha''}(\omega)} \sqrt{\frac{\eta(\mathbf{k},\omega)}{\Lambda'(\mathbf{k},\omega)}} (\frac{d[\kappa'_{\alpha\nu}(\mathbf{k},\omega)\Omega'(\mathbf{k},\omega)]\mathbf{k}^2}{d\omega} + \omega\kappa'_{\mu\nu}(\mathbf{k},\omega))) \\ & + K_B \ln 2, \end{aligned}$$

and

$$\begin{aligned} C =& \frac{\hbar}{2\pi K_B T^2} \int_0^\infty d\omega \int_0^\infty d\mathbf{k} \, \omega \, csch^2(\frac{\hbar\omega}{2K_BT}) \Im G_{\mu\nu}(\mathbf{k},\omega) \\ & (\mathbf{k}^2 I - \omega^2 I + f_{\mu\alpha}(\omega) \frac{A_{\alpha'''\alpha''}(\omega)}{B_{\alpha'''\alpha''}(\omega)} (\frac{d[\kappa_{\alpha\nu}(\mathbf{k},\omega)\Omega(\mathbf{k},\omega)]\mathbf{k}^2}{d\omega} + \omega\kappa_{\mu\nu}(\mathbf{k},\omega)) \\ & + g_{\mu\alpha}(\omega) \frac{D_{\alpha'''\alpha''}(\omega)}{B_{\alpha'''\alpha''}(\omega)} \sqrt{\frac{\eta(\mathbf{k},\omega)}{\Lambda'(\mathbf{k},\omega)}} (\frac{d[\kappa'_{\alpha\nu}(\mathbf{k},\omega)\Omega'(\mathbf{k},\omega)]\mathbf{k}^2}{d\omega} + \omega\kappa'_{\mu\nu}(\mathbf{k},\omega))). \end{aligned}$$

The information transmitted by the system can be investigated by the entropy of the system under the influence of the moving dielectric particle.

5 Conclusion

In this paper, we study the behavior of a particle moving in an electromagnetic field in the presence of a dielectric body. We introduce a canonical relativistic quantization of the electromagnetic field in the presence of a moving dielectric particle next to the semi-infinite bulk dielectric with a surface in the interface. We investigate the quantum thermodynamic properties of the system and explicitly determine the behavior of the main thermodynamic functions: the free energy, the mean energy, the entropy, and heat capacity. We show that the formulation of quantum thermodynamics for an electromagnetic system in uniform relative motion differs from its formulation in the rest-frame.

Authors' Contributions

The author contributed to data analysis, drafting, and revising of the paper and agreed to be responsible for all aspects of this work.

Data Availability

No data available.

Conflicts of Interest

The author declares that there is no conflict of interest.

Ethical Considerations

The author has diligently addressed ethical concerns, such as informed consent, plagiarism, data fabrication, misconduct, falsification, double publication, redundancy, submission, and other related matters.

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