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Scaling relations in dynamical evolution of star clusters

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Abstract.

We have carried out a series of small scale collisional N-body calculations of single-mass star clusters to investigate the dependence of the lifetime of star clusters on their initial parameters. Our models move through an external galaxy potential with a logarithmic density profile and they are limited by a cut-off radius. In order to find scaling relations between the lifetime of star clusters and their initial conditions including the initial mass, size and galactocentric distance, we vary the initial conditions and measure the final half mass radius and dissolution time of each cluster. We show that the lifetime of star clusters scales with the initial half-mass radius, galactocentric distance, and initial mass as $T_{diss} \sim R_h^{0.15}$, $T_{diss} \sim R_G^{0.94}$, and $T_{diss} \propto M_i^{0.45}$, respectively. Our results are in remarkable agreement with the previous works by Baumgardt & Makino (2003) and Haghi et al. (2014) who have found some scaling relations for the lifetime of multi-mass star clusters with a large number of stars including the stellar evolution. Moreover, we find that all clusters with the same mass and different initial half-mass radius, converge to an equilibrium value of half-mass radius, after core collapse that scales with galactocentric distance as $R_h \sim R_G^{0.8}$. We show that the exponent in this scaling relation is slightly larger for the massive star clusters.

Keywords: galaxies: star clusters, method: numerical

1 Introduction

A star cluster is a group of stars that shares a common origin and is gravitationally bound for some length of time. Star clusters are particularly useful to astronomers as they provide a way to study and model star formation and stellar evolution. They also provide an important laboratory to explore many aspects of evolution of stellar systems as driven by the effects of two-body encounters. In this regard, with advances in software and hardware and observational facilities during the last decade, plenty of theoretical and numerical investigations have been done by several authors to evaluate the dynamics of star clusters [18, 5, 9, 13, 14, 6, 19, 20]. In addition, many photometric and spectroscopic measurements of young star clusters as well as old ones have been carried out.

About 160 objects are classified as globular clusters (GCs) in the Milky Way (MW) [8], each containing a few thousand to millions of stars within a radius of a few pc. Their distances from the Galactic Centre range from 0.5 to 125 kpc. More than 50% are found within 10 kpc, but their distribution extends to the very outskirts of our Galaxy.

Dynamical evolution plays a key role in shaping the current properties of star clusters and star cluster systems. Simulations of GC systems suggest that the GC population we observe today is only the survival of a once much richer system [16]. Whether a star cluster survives in the tidal field of its host galaxy depends crucially on its size, mass, and their spatial distribution. For instance star clusters with larger radii, and lower mass orbiting in a smaller galactocentric distance are more susceptible to tidally induced mass loss, whereas compact and massive systems can survive even near the Galactic center.

With densities as high as $10^6 pc^{-3}$, GCs are among the few places in the Universe where stars interact via two-body encounters. While telescopes can provide us with a snapshot of what these dense clusters look like at present, we must rely on detailed numerical simulations to learn about their evolution. These simulations are quite challenging, however, since dense star clusters are described by a complicated set of physical processes occurring on many different length and time scales, including stellar and binary evolution, weak gravitational scattering encounters, strong resonant binary interactions, and tidal stripping by the host galaxy.

The long-term evolution of GCs is determined mainly by mass-loss due to stellar evolution, stellar dynamics and the effects from the tidal field. It is well known that the internal properties of GCs can undergo significant changes at birth but also during the course of the cluster's dynamical evolution (e.g. Heggie & Hut 2003). It is therefore essential to specify to what extent the present-day properties of GCs, such as their physical sizes and masses are imprinted by early evolution and formation processes and to what extent they are the outcome of long-term dynamical evolution.

A pioneering study of this research field was carried out by Vesperini & Heggie (1997) who investigated the effects of dynamical evolution on the mass function of globular clusters through direct N-body simulations. The total number of particles that Vesperini & Heggie (1997) were able to simulate was limited to $N \simeq 4000$ due to restrictions imposed by the hardware and included a basic treatment of stellar evolution. Despite the computational limitations to which Vesperini & Heggie (1997) were subject, they found a very important trend of increasing mass-loss with decreasing galactocentric distance. Baumgardt & Makino (2003) studied the stellar mass function of star clusters using $NBODY4$ but with more realistic particle numbers, going up to $N = 130000$. They found that owing to mass segregation low mass stars are prone to be depleted from the star cluster. They also showed that the crossing time (T_{cr}) is of importance and found for equal mass clusters that $T_{dis} \propto T_{relax}^{3/4} T_{cr}^{1/4}$. They as well found that this scaling also holds for models of clusters with a mass spectrum, stellar evolution and for different types of orbits in a logarithmic potential.

In the present paper we aim at shedding light on the effect of different initial conditions on the dynamical evolution, focusing on the effect of initial size, mass and galactocentric distance of star clusters on the dissolution time by means of direct N-body simulations. We obtain several scaling relations for the dissolution time of stellar systems in terms of their initial parameters such as initial mass and radius.

The plane of the paper is as follow: The set-up of our N-body models is described in Sec. 2. In Sec. 3, we present our results for the evolution of star clusters and scaling relations for dissolution time. In Sec. 4, we will also derive some simple scaling relations for the final radius of star clusters and in Sec. 5, we finally draw our conclusions.

2 N-body Models

We performed a series of N-body simulations using the high-level, up-to-date collisional N-body code NBODY6 [2, 1]. NBODY6 follow the orbits of cluster members using a 4th-order Hermite integration scheme and invokes regularization schemes to deal with the internal evolution of small-N subsystems. In addition to the tidal effects of the Galaxy, NBODY6 includes the stellar and binary evolution using the dynamical integrated SSE/BSE routines developed by Hurley et al. (2000, 2002). However, in this study we assume all clusters contain single-mass stars; and hence the stellar evolution is switch-off here to focus on pure dynamical evolution. The clusters are set up as Plummer (1911) profile in virial equilibrium defined as

$$\rho(r) = \frac{3Ma^2}{4\pi}(a^2 + r^2)^{-5/2}, \tag{1}$$

where M is the total cluster mass, and a is a scale radius. The half-mass radius, R_h, of this profile is related to a by $R_h \simeq 1.305a$. The Plummer-model is the simplest plausible and self-consistent model for a star cluster.

The initial half-mass radii and the number of stars in all computed models are set in the range $R_h = 1 - 5$ pc and $N = 1000 - 6000$, respectively. We evolve all models until they dissolve (i.e., 10% of initial stars are retained). The clusters move on a circular orbit through logarithmic potential of the Milky Way at different galactocentric radius from $R_G = 8.5$ until 32 kpc with circular velocity $V_G = 220$ kms^{-1}. The galaxy model is adopted as Allen Santillan potential model (1991).

3 Dissolution time of star clusters

Here, we investigate how the dissolution time of modeled star clusters depends on their initial conditions. We examine how varying the initial half-mass radius, initial mass and the galactocentric distance of a star cluster influences its dissolution time. The main aim is to find the scaling relations between these parameters and the dissolution time and final radius of star clusters. This will allow us to make inferences towards the sensitivity of the results of this paper on choosing this crucial initial parameter. All will be discussed in the following.

3.1 Impact of initial cluster mass

To determine the effect of initial cluster mass on the dissolution time, we change the total cluster mass from $1000M_\odot$ to $5000M_\odot$ evolving on a circular orbit with galactocentric distance of 8.5 kpc. The dissolution time of clusters increases with increasing the initial cluster mass. Fig. 1 depicts the cluster lifetime as a function of initial cluster mass for different initial half-mass radius (i.e., R_h =1 and 5 pc). A very clear scaling law between the dissolution time and the initial mass of star cluster is obtained by linear fitting to the simulation results (data points in Fig.2) as follow

$$R_h = 1\ pc \ \Rightarrow \log(T_{diss}) = 0.41(\pm 0.04)\log(M_i) + 2.11(\pm 0.15), \tag{2}$$

$$R_h = 5\ pc \ \Rightarrow \log(T_{diss}) = 0.49(\pm 0.03)\log(M_i) + 1.96(\pm 0.08). \tag{3}$$

The obtained scaling laws can be written as $T_{diss} \propto M_i^{\alpha}$, where $\alpha = 0.41$ for cluster with $R_h = 1$ pc. The cluster with larger radius ($R_h = 5$ pc) shows a slightly steeper scaling law with $\alpha = 0.49$. According to the uncertainties in the slope of the fitted lines, it can be seen that both values are approximately comparable to what Baumgardt & Makino found (2003), where the dissolution time was scaled with the two-body relaxation time (T_{relax})[1] as $T_{diss} \sim T_{relax}^{x}$, where $x \simeq 0.8$. This implies a scaling relation as $T_{diss} \sim M_i^{0.4}$. Note that

[1] Two-body relaxation arises from close encounters between cluster stars and leads to a slow diffusion of stars over the tidal boundary. It acts on a timescale (Spitzer 1987) as $T_{relax} = 0.138\frac{N^{1/2}R_h^{3/2}}{<m>^{1/2}G^{1/2}\ln\Lambda}$, where, $< m >$ is the mean stellar mass and $\ln\Lambda$ is the Coulomb logarithm.

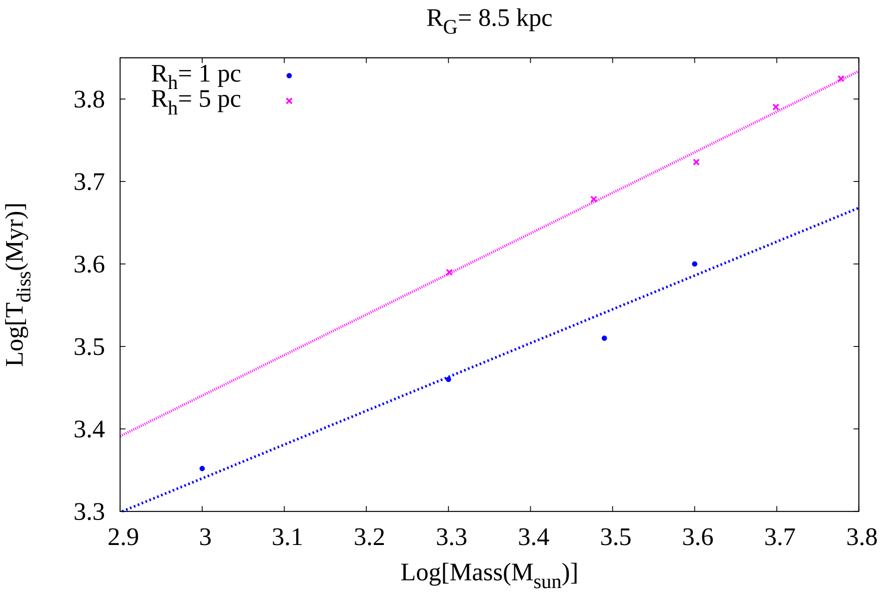

Figure 1: Dissolution time versus the initial cluster mass for different initial cluster half-mass radius. Points are the simulation results. Solid lines show the best linear fit to the simulation results that show a scaling relation in the form of $T_{diss} \propto M_{init}^{\alpha}$. We obtained $\alpha \simeq 0.45$ in agreement with [5] who showed $\alpha \simeq 0.4$ for multi-mass star clusters with large number of stars (i.e. $N = 10^5$).

the results of Baumgardt & Makino (2003) were based on a multi-mass star clusters with more that 10^5 stars including the stellar evolution, while here we calculate small-number, single-mass clusters without stellar evolution.

3.2 Impact of galactocentric distance

Fig. 2 depicts the dependence of the cluster dissolution time on the galactocentric distance for a cluster with $M_i = 3000\ M_{\odot}$. We let the galactocentric distance vary between 8.5 and 30 kpc. The dissolution time of star clusters increases linearly with galactocentric distance. The dissolution time scales with galactocentric distance as

$$T_{diss} \sim R_G^{0.94} \tag{4}$$

This is in agreement with Haghi et al. (2014) who found the dissolution time of a cluster with 10^5 stars and initial radius of 6.2 pc scales with the galactocentric distance as $T_{dis} \propto R_G^{\alpha}$, where $\alpha = 1.12 \pm 0.1$. The obtained scaling relation is also very close to that derived by [5] for large number multi-mass models. They found that the dependency of dissolution time of star clusters on galactocentric distance as $T_{diss} \sim R_G$ which marginally agree within the 1σ error bars with our results. It should be noted that the exponent α, in general, depends on the degree of mass segregation. For highly segregated clusters, the exponent is $\alpha = 1.31 \pm 0.08$, while unsegregated models have a weaker dependence on R_G, with a slope of $\alpha = 1.12 \pm 0.13$ [6]. This implies that for non-segregated models, the dissolution time is significantly larger than for the segregated systems, that could be due to the larger amount of binding energy which is carried away during stellar evolution from the preferentially centrally located massive stars.

3.3 The impact of initial half-mass radius

In order to study the influence of initial half-mass radius on the lifetime of star clusters in detail, we performed three sets of models evolving on three different circular orbits with radius of R_G =8.5, 15, and 30 kpc. For each set, all clusters have identical initial mass of $M = 3000M_{\odot}$, but the initial half-mass radius varies from 0.5 to 5 pc to extract a scaling relation between the dissolution time and the initial half-mass radius.

Fig. 3 shows the dissolution time of each cluster versus its initial half-mass radius in logarithmic scales. As can be seen, by increasing the initial half-mass radius, the dissolution time slowly increases. Different lines in Fig. 3 show satisfactory fit for each set of models. Independent of the galactocentric distance, the dissolution time of star clusters scales with half-mass radius as follow

$$T_{diss} \sim R_h^{0.15} \tag{5}$$

Such a little dependency of dissolution time to the adopted initial half mass radius for a given cluster mass is marginally in agreement with the independency of the lifetime of a star cluster from its initial size, if the half-mass radius in a cluster assumed to be scaled with its tidal radius (See Eq. 7 of [5]). The slight difference could be due to the fact that our models are single-mass clusters while those models calculated by [5] are multi-mass clusters. Comparision between single-mass star and multi-mass clusters is our upcoming work.

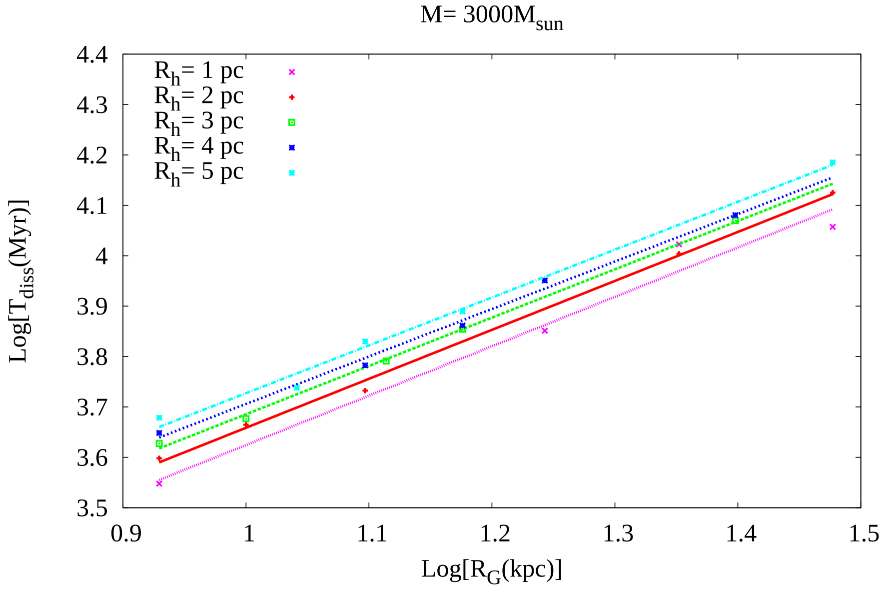

Figure 2: The dissolution time versus the galactocentric distance for star clusters with $M = 3000M_\odot$ and five different initial half-mass radius. Clusters orbiting in the inner part of galaxy dissolve very fast in strong tidal fields. Power-law fit of the form $T_{diss} \propto R_G^\alpha$ is indicated as a line with a slope of $\alpha = 0.94$.

4 The evolution of half-mass radius of star clusters

Before addressing the scaling relations for half-mass radius, we investigated the evolution of half-mass radius for clusters with different initial half-mass radius, but identical initial mass $1000M_\odot$ located at $R_G = 8.5kpc$. The star clusters with the same mass and different initial half-mass radius, converge to an equilibrium value of half-mass radius, after core collapse (Fig. 4).

The half-mass radius of clusters with same mass and different initial half-mass radius settles to the same equilibrium value after core collapse for large fraction of their life time. The virial equilibrium implies that, reducing the half-mass radius raises the mean velocity dispersion of member stars. Therefore, a cluster with the half mass radius, which is twice the other, exhibits a half mean velocity dispersion of that. But since the clusters have the same tidal radius, they redistribute their masses through two-body relaxation such that after core collapse the initially smaller or larger clusters do not differ in their properties, from clusters with same initial mass. This holds especially for the half-mass radius. The value of final equilibrium radius *slightly* depends on the clusters mass such that for a larger initial mass, the equilibrium value is larger. In the next section, we asses the dependency of the value of plateau on the galactocentric distance of orbiting star cluster.

4.1 $R_h - R_G$ scaling relation

In order to show how the final radius of star clusters are sensitive to the initial mass and galactocentric distance, the evolution of half-mass radius of a set of star clusters with dif-

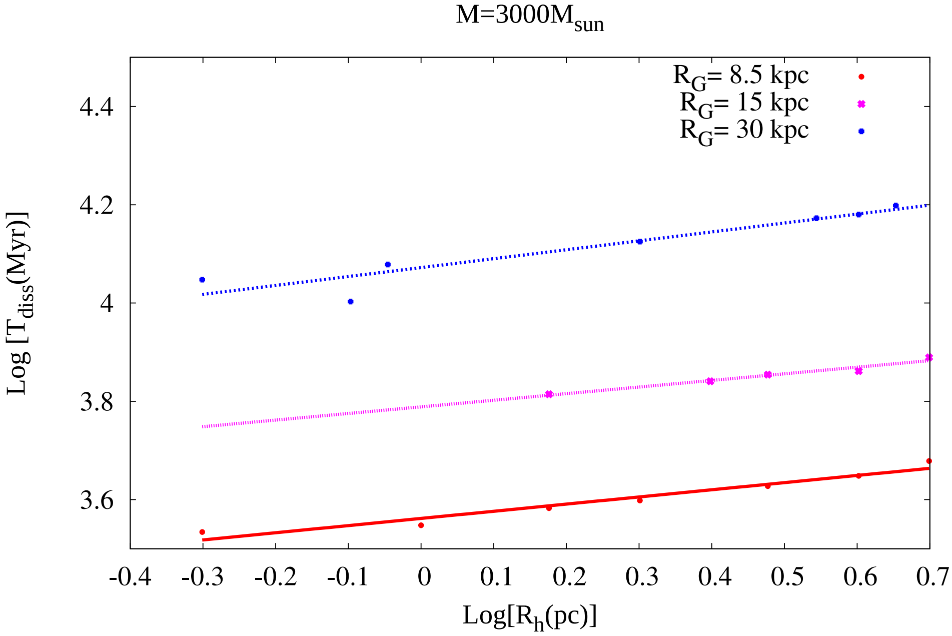

Figure 3: Lifetime of clusters with different initial half-mass radius moving on different circular orbits. All clusters have identical mass of $3000 M_{\odot}$. Lines show the linear fit to the data points.

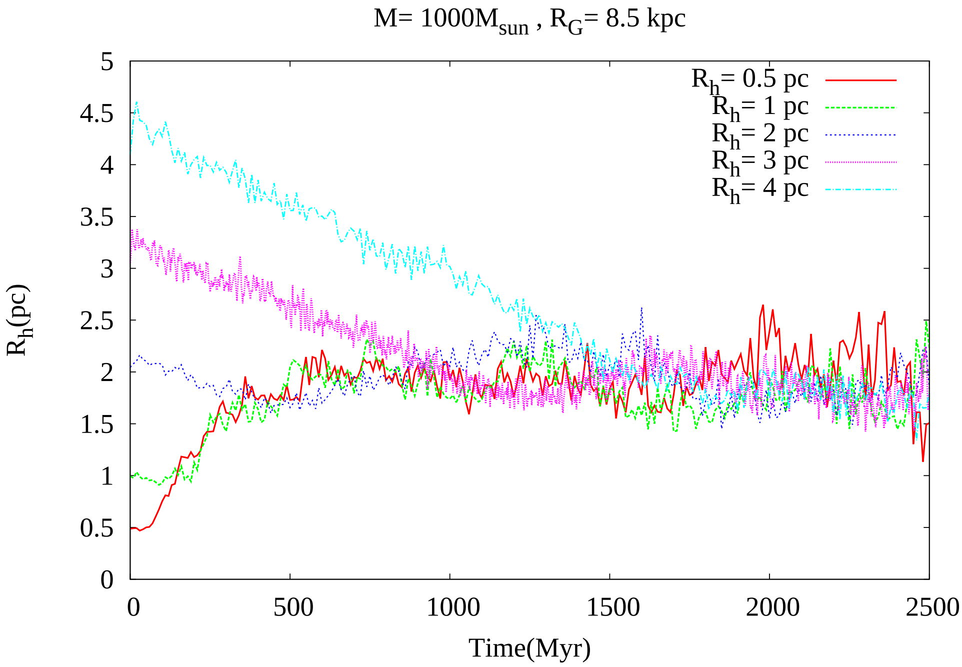

Figure 4: The evolution of half-mass radii of models with initial R_h of 0.5, 1, 2, 3, and 4 pc, but with the same initial mass ($1000M_{\odot}$) orbiting on a circular orbit with $R_G = 8.5$ kpc. The final size is independent of the initial half-mass radius and all models converge to an equilibrium value of 2 pc after core collapse, when the clusters have lost about 3050 per cent of their initial mass.

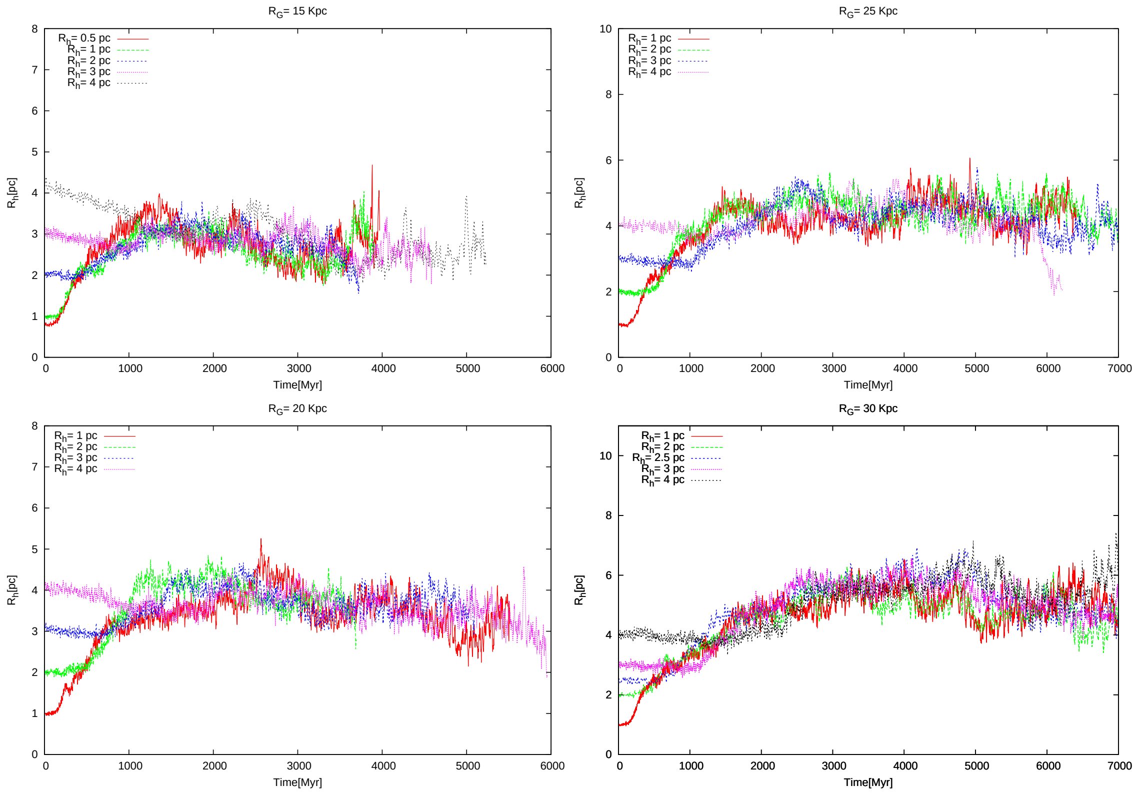

Figure 5: The evolution of half-mass radius for clusters with initial mass $1000M_\odot$ and R_G =15, 20, 25, 30 kpc. For larger galactocentic distance, the equilibrium value of half-mass radius is larger.

ferent initial masses and orbiting in different circular orbits are investigated in details in this section. For each set, we vary the initial R_h between 0.5 and 5 pc. The evolution of half-mass radius for clusters with initial mass of $1000M_\odot$ and $R_G = 15, 20, 25, 30Kpc$ is shown in Fig. 5. Star clusters with the same mass and different initial half-mass radius, converge to an equilibrium value of half-mass radius, after core collapse.

As can be seen, the maximum half-mass radius reaches a plateau independent of the initial half-mass radius, while the value of plateau strongly depends on galactocentric distances. By increasing the cluster mass and Galactic distance, the asymptotic value of half-mass radius increase. This scaling form of the $r_h - R_G$ relation is also plotted in Fig. 7. Moreover, although, the radius of the cluster increases slowly with increasing galactocentric distance, the onset of the plateau in the $r_h - R_G$ relation occurs at nearly the same time ($T \simeq 1$ Gyr) after the beginning of evolution. Therefore, the plateau of the $r_h - R_G$ relation tells us something about the galactocentric distance of star clusters.

To show the sensitivity of these results on the initial cluster mass, we calculate the evolution of a set of star clusters with the initial mass of $3000M_\odot$ at $R_G = 8.5$, 15 and 30 kpc. Comparing Fig. 5 and Fig. 6, one can see that the values of final radius slightly depends on the initial mass of star clusters. For massive clusters, the equilibrium final radius is larger.

Finally, in order to obtain a relation between the equilibrium half-mass radius (R_h) and

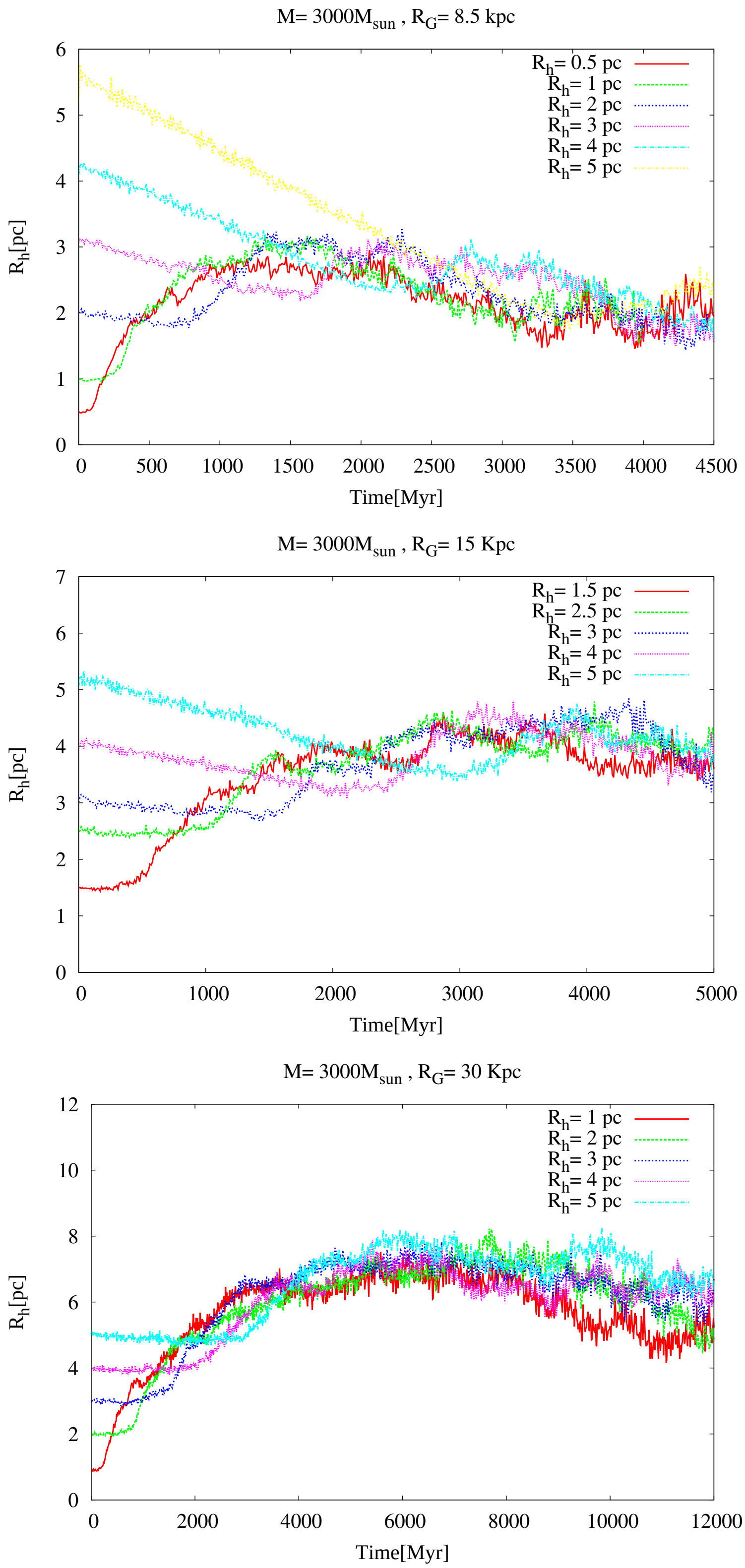

Figure 6: The same as Fig. 5, but for a cluster with $3000M_{\odot}$.

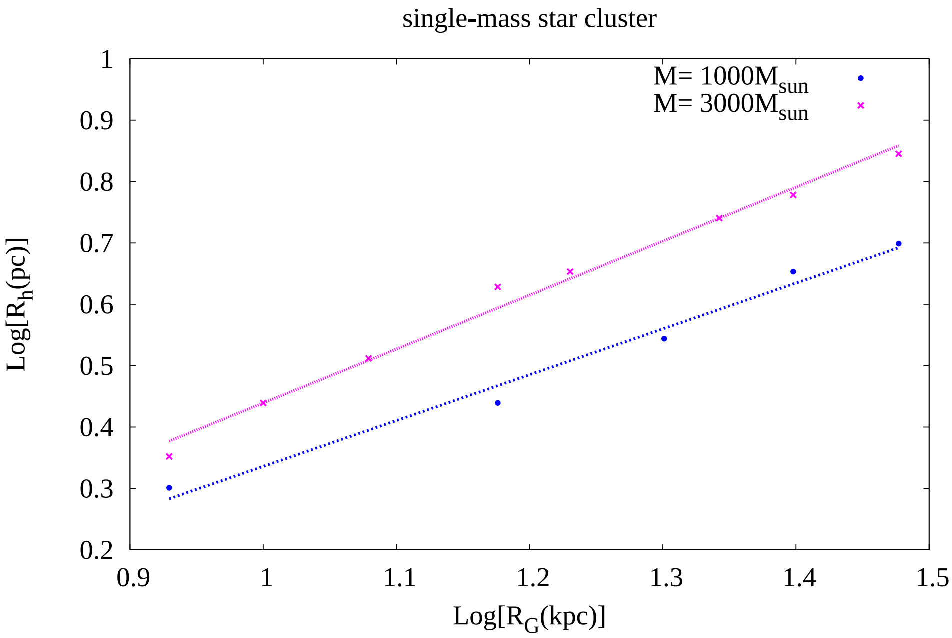

Figure 7: Galactocentric distance versus the equilibrium half-mass radius value for clusters with initial masses of $1000M_{\odot}$, $3000M_{\odot}$.

galactocentric distance (R_G) we plot the correlation between r_h and R_G for clusters with two different initial mass of $1000M_{\odot}$, $3000M_{\odot}$. The best fitted line to each set of simulation shows a clear scaling relation as follow

$$R_{h0} \sim R_G^{0.78}\ for\ M = 1000M_{\odot} \tag{6}$$

$$R_{h0} \sim R_G^{0.88}\ for\ M = 3000M_{\odot} \tag{7}$$

Such a correlation between R_h and R_G could be due to the expansion of initially compact GCs up to the respective Jacobi radius, which is roughly proportional to $R_G^{2/3}$ for a given GC mass. Our results is in remarkable agreement with this proposal.

5 Conclusions

We have performed a set of small scale N-body simulations of single-mass star clusters moving through an external galaxy with a logarithmic density profile with the collisional N-body code NBODY6. Our simulations included two-body relaxation and a fully consistent treatment of the external tidal field. We aimed at finding scaling relations between the lifetime of star clusters and their initial conditions including the initial mass, size and galactocentric distance. All simulated clusters dissolve mainly as a result of two-body relaxation and external tidal truncation. All computed clusters show a tight relation between dissolution time and cluster parameters. Our key results are:

1. The dissolution time scales with galactocentric distance as $T_{diss} \sim R_G^{0.94}$, in agreement with Haghi et al. (2014) who found the dissolution time of a cluster with 10^5 stars and initial

radius of 6.2 pc scales with the galactocentric distance as $T_{dis} \propto R_G^{1.2\pm0.1}$. This result is also compatible with the results of numerical computations of [5] for large number multi-mass models, who found the dependency of dissolution time of star clusters on galactocentric distance as $T_{diss} \sim R_G$.

2. The lifetimes of clusters moving on the same orbit scales proportional to the initial cluster mass as $T_{diss} \propto M_i^{0.45\pm0.04}$. Within the uncertainties in the slope of the fitted lines, this result is approximately comparable to what Baumgardt & Makino found (2003), where the dissolution time was scaled with the two-body relaxation time.

3. For all galactocentric distance, the lifetime of star clusters scales with half-mass radius as $T_{diss} \sim R_h^{0.15}$, that is marginally in agreement with the independency of the dissolution time of a star cluster from its initial size for clusters that the half-mass radius is scaled with their tidal radius.

4. In agreement with kuepper et al 2008, all clusters with the same mass and different initial half-mass radius, converge to an equilibrium value of half-mass radius, after core collapse. The final equilibrium radius depends on the clusters initial mass and orbital radius. For larger initial mass of star cluster and larger galoctocentric distance, the equilibrium value is larger. We found that the final half-mass radius of star cluster shows a clear scaling relation as $R_h \sim R_G^{0.78}$ and $R_h \sim R_G^{0.88}$ for cluster with the initial mass of $M = 1000$ and $3000 M_\odot$, respectively. That is, clusters with larger initial mass show slightly larger exponents than less massive clusters.

It should be noted that for the single-mass star clusters, the stellar evolution is switched-off and the pure dynamical effect plays role in the long-term evolution of star clusters. The reason that we used the low-number single-mass cluster in this work is indeed the low-computational costs. The run-time of each simulation in this work that is carried out on a normal CPU is about one hour (because of the low-number of particles) , whereas for a typical 10^5 solar-mass globular cluster, it takes 1-2 weeks on a desktop workstations with $Nvidia$ 690 Graphics Processing Units (GPU). Interestingly, we showed an acceptable consistency between our results and the more realistic simulations.

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