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Central AdS Generalized Ayón-Beato-García Black Holes and Joule-Thomson Expansion

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Abstract. In this paper, the Joule-Thomson (JT) adiabatic expansion is investigated for generalized Ayon-Beato-Garcia (ABG) regular black hole. It has a magnetic charge which makes central region of the black hole metric to be AdS spacetime and so become non singular. Thus we not need to use an additional cosmological parameter coming from ADS/CFT correspondence for production of pressure coordinate in the black hole equation of state. Form this point of view our work is a new approach versus to conventional methods which are addressed in the references of this paper. However we will see important behavior of parameters of this modified ABG black hole on possibility of being of JT adiabatic expansion. This black hole is characterized by five parameters including mass m, magnetic charge q, and three other parameters related to form of used nonlinear electromagnetic interaction fields. Inversion points are in fact a particular T-P curve at constant enthalpy and it separates cooling and heating phase of the modified AdS ABG black hole.

Keywords: Nonsingular black holes, Magnetic charge, phase transition, black hole thermodynamic, Joule Thomson

1 Introduction

Due to the similar behavior of black holes to the ordinary thermodynamic systems [1], it is possible to attribute properties of the ordinary thermodynamic systems to the black holes. Clausius-Clapeyron relation, Joule-Thomson expansion, the heat engines subject, Maxwell‘s equal-area law, and the chemical behavior are some examples which have been investigated for AdS charged black holes in the literature [2–6]. Also, the mentioned subjects have been studied for AdS-Kerr black holes in the same way [7–11]. Moreover, these topics were studied for other black holes in the alternative theories of gravity [12–18]. In all of them, the cosmological constant plays as thermodynamic pressure in the black hole system which is called as extended phase space. Fortunately, central region of the ABG black hole behaves asymptotically as AdS space in which its magnetic charge produces a variable cosmological parameter or variable pressure. This can be considered as alternative AdS pressure instead of auxiliary cosmological constant which come from AdS/CFT correspondence and it is used in usual way [19]. In the present work, we specifically investigate the JT expansion for a generalized ABG black hole which is introduced by Cai and Miao [20]. Firstly, the ABG black holes was proposed by Ayón-Beato and García [21] as a black hole without a central singularity. This advantage is due to the presence of nonlinear interaction of electromagnetic fields. The generalized ABG black hole is defined in terms of five parameters including mass m, magnetic charge q, and three other constant parameters which is related to kind of nonlinear interaction of the electromagnetic fields. In other words, these three parameters can be control regularity of the central region of the ABG black hole. Thermodynamics and phase transition of the mentioned black hole is studied in our recently work [19] and we investigate effects of the parameters of this kind of regular black hole on the JT adiabatic expansion now.

As we said above, in fact, the study of the JT phenomenon in black holes is modeled from the behavior of real gases, which is studied in ordinary thermodynamics. As the small-to-large scales phase transition in black holes corresponds to the liquid-to-gas phase transition in the Van der Waals fluid, because the behavior of temperature and pressure changes is the same for both. Of course, for this phase transition to happen, conditions must be established which in black holes, the most important quantity is the pressure of the surrounding environment on the surface of the black hole's horizon and it is provided by the negative cosmological constant. It is because of AdS/CFT correspondence . In the case of JT adiabatic expansion, we follow similar approach too. Generally, if a gas become cooler (warmer) when we raise the affecting pressure, then its temperature decreases (increases) at constant entropy. This behavior for gases is called JT adiabatic expansion and is happens some times. Simplest tools to check if it happened for a particular gas is the 'inversion temperature-pressure curve'. In fact, it determines some points on the T-P phase space at constant enthalpy such that the gas under pressure can may be participates in the JT phenomena. This pattern is also followed in black holes. As an example, we like to investigate this effect for modified AdS ABG black hole here. Since a black hole's mass plays the role of enthalpy in the extended phase space, the JT expansion could be examined for black holes as well. In this way, the black hole's temperature changes as a result of its pressure variation while its mass remains constant. The temperature variation due to the pressure variation is exhibited as isenthalpic curves and it is introduced by a JT coefficient $\mu = \left(\frac{\partial T}{\partial P}\right)_m$. This is in fact slope of T-P diagrams at constant enthalpy. Hence, one can infer that the sign of the JT coefficient determines the behavior of the black hole as a heater or cooler. In this view, when $\mu > 0 (\mu < 0)$ then the black hole is heater (cooler) because of increase (decrease) of its temperature versus the raise of AdS pressure. So, it can be concluded that it is located in the heating (cooling) phase. Inversion temperature and inversion pressure are another factors are studying in this way. The inversion point (T_i, P_i) is obtained from $\mu = 0$ which is the maximum point of the isenthalpic curves. In fact, the inversion curve as a separation line between two heating and cooling phases passes through the inversion points of the different isenthalpic curves. In other words, the intersection of the inversion curves and isenthalpic curves are positions where the black hole participates in the cooling-heating phase transition. One can follows [22–38] where the JT expansion are examined for various types of black holes.

At last, in general, only when a black hole participates in the Joule-Thomson phase transition does the inversion curve intersect with the temperature-pressure isenthalpic curves, and this possibility depends on the parameters and characteristics of the black hole system. The layout of this work is as follows.

In section 2, we give a short review about the generalized ABG black hole defined by [20] and in section 3, we generate some suitable equations related to the JT coefficient. In this section, we plot inversion curves and isenthalpic diagrams for different values of the black hole parameters. Last section denotes to concluding remark.

2 Generalized ABG black hole and its thermodynamic

Let us start with line element of the generalized ABG magnetic spherically symmetric static black hole which in the Schwarzschild frame is defined by [20]

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2(d\theta^2 + sin\theta^2 d\varphi^2), \tag{1}$$

in which the metric potential has the following form.

$$f(r) = 1 - \frac{2mr^{\frac{\alpha\gamma}{2}-1}}{(q^\gamma + r^\gamma)^{\alpha/2}} + \frac{q^2 r^{\frac{\beta\gamma}{2}-2}}{(q^\gamma + r^\gamma)^{\beta/2}}, \tag{2}$$

where m and q refer to the mass and magnetic charge of the generalized ABG black hole. The three parameters of α, β, and γ are related to each other through $\alpha\gamma \geq 6$, $\beta\gamma \geq 8$ and $\gamma > 0$ [20]. In fact, regularity of this black hole depends to these inequalities between the parameters. If we choose different values for these three parameters by regarding the above inequalities, we can obtain different kinds of regular modified ABG black holes. In this paper, we use ansatz $\alpha\gamma = 6$ and $\alpha\beta = 8$ for which the metric potential (2) takes on simpler form given in the paper [19]. In that work we studied thermodynamic phase transition of the modified AdS ABG black hole. Also, we showed that the above metric can be viewed as a Schwarzschild-de Sitter form of the black hole metric if we set a variable mass function and the variable cosmological parameter as follows.

$$M(r) = \frac{mr^{\frac{\alpha\beta}{2}}}{(r^\gamma + q^\gamma)^{\frac{\alpha}{2}}}, \quad \Lambda(r) = -\frac{3q^2 r^{\frac{\beta\gamma}{2}-4}}{(r^\gamma + q^\gamma)^{\frac{\beta}{2}}}. \tag{3}$$

Regarding the AdS space pressure definition $P = \frac{-\Lambda}{8\pi}$ for this variable cosmological parameter and by substituting it into the event horizon equation $f(r_+) = 0$, we can obtain pressure equation versus the black hole mass m and the other parameters such that

$$P(m, r_+) = \frac{6mr_+^{\frac{\alpha\gamma}{2}-1} - 3(q^\gamma + r_+^\gamma)^{\alpha/2}}{8\pi r_+^2 (q^\gamma + r_+^\gamma)^{\alpha/2}}. \tag{4}$$

This form of the state equation is suitable to study isenthalpic phenomena because the black hole mass m plays the enthalpy role, so it remains constant during the JT process. Moreover, the Hawking temperature is obtained through the black hole surface gravity on the horizon, $T_H = \frac{\kappa}{2\pi}|_{r=r_+}$ [19] such that

$$\begin{aligned} T(m, r_+) =& \frac{mr_+^{\frac{\alpha\gamma}{2}-2}}{4\pi(q^\gamma + r_+^\gamma)^{\alpha/2}} \left[\frac{(2-\alpha\gamma)(q^\gamma + r_+^\gamma) + \alpha\gamma r_+^\gamma}{(q^\gamma + r_+^\gamma)}\right] \\ &+ \frac{q^2 r_+^{\frac{\beta\gamma}{2}-3}}{8\pi(q^\gamma + r_+^\gamma)^{\beta/2}} \left[\frac{(\beta\gamma - 4)(q^\gamma + r_+^\gamma) - \beta\gamma r_+^\gamma}{(q^\gamma + r_+^\gamma)}\right] \end{aligned} \tag{5}$$

Considering the dimensionless parameter of $x = \frac{r_+}{q}$, relations of (4) and (5) are rewritten as

$$P(m, x) = \frac{6mx^{\frac{\alpha\gamma}{2}-1} - 3q(1 + x^\gamma)^{\alpha/2}}{8\pi q^3 x^2 (1 + x^\gamma)^{\alpha/2}}, \tag{6}$$

and

$$\begin{aligned} T(m, x) =& \frac{mx^{\frac{\alpha\gamma}{2}-2}}{4\pi q^2 (1 + x^\gamma)^{\alpha/2}} \left[\frac{(2-\alpha\gamma)(1 + x^\gamma) + \alpha\gamma x^\gamma}{(1 + x^\gamma)}\right] \\ &+ \frac{x^{\frac{\beta\gamma}{2}-3}}{8\pi q(1 + x^\gamma)^{\beta/2}} \left[\frac{(\beta\gamma - 4)(1 + x^\gamma) - \beta\gamma x^\gamma}{(1 + x^\gamma)}\right]. \end{aligned} \tag{7}$$

We are now in position to calculate JT coefficient and investigate cooling-heating phase transition of this kind of black hole.

3 Joule-Thomson Expansion

JT expansion in ordinary thermodynamics is known as an irreversible process, where a gas moves from a region with high pressure to a region with low pressure through a porous plug. Investigating the temperature changes during this expansion is the main aim of this study. Since the system's enthalpy remains constant during this process, a set of (T,P) values forms the isenthalpic curves, where JT coefficient defined as the isenthalpic curve slope and is given as follows

$$\mu_{JT} = \left(\frac{\partial T}{\partial P}\right)_H . \tag{8}$$

During the JT expansion, the pressure always reduces. So, the sign of the JT coefficient determines the heating (the temperature increase) or cooling (the temperature decrease) behavior of the system. For this purpose, the JT coefficient for a black hole under consideration is obtained as follow

$$\mu(m, x) = \left(\frac{\partial T}{\partial P}\right)_m = \left(\frac{\partial T}{\partial x}\right)_m \left(\frac{\partial x}{\partial P}\right)_m . \tag{9}$$

As we said in the previous section to choose a particular form of the modified regular AdS ABG black hole metric given in the ref [19] we substitute $\alpha = \frac{3\gamma}{4}$ and $\beta = \frac{8}{\gamma}$ into the relations of (6) and (7), then we calculate (9) by keeping m as constant enthalpy such that

$$\mu(m, x) = \left\{ mqx^{\frac{3\gamma^2}{8}}(1 + x^\gamma)^{\frac{4}{\gamma}} \left[-128x^{2\gamma} + 8(3\gamma^3 + 9\gamma^2 - 32)x^\gamma + (-9\gamma^4 + 72\gamma^2 - 128)\right] \right.$$
$$\left. +64q^2x^3(1 + x^\gamma)^{\frac{3\gamma}{8}} \left[+3x^{2\gamma} - 2(\gamma + 2)x^\gamma + 1\right] \right\} \Big/$$
$$\left\{ 12(1 + x^\gamma)^{\frac{4}{\gamma}+1} \left[3mx^{\frac{3\gamma^2}{8}-1} \left(\gamma^2 - 8x^\gamma - 8\right) + 8q(1 + x^\gamma)^{\frac{3\gamma}{8}+1} \right] \right\}. \tag{10}$$

To have inversion curves we should obtain roots of the equation $\mu(m, x) = 0$ which reads to the following parametric identity.

$$m_i(x) = \left\{ 64qx(1 + x^\gamma)^{\frac{3\gamma}{8}} \left[3x^{2\gamma} - 2(\gamma + 2)x^\gamma + 1\right] \right\} \Big/$$
$$\left\{ x^{\frac{3\gamma^2}{8}-2}(1 + x^\gamma)^{4/\gamma} \left[128x^{2\gamma} - 8(3\gamma^3 + 9\gamma^2 - 32)x^\gamma + 9\gamma^4 - 72\gamma^2 + 128\right] \right\}. \tag{11}$$

In this step, the inversion pressure P_i and inversion temperature T_i are accessible by substituting m_i into (6) and (7) as follows

$$P_i(x) = \Big\{ \left[1152x^{2+2\gamma} - (768\gamma + 1536)x^{2+\gamma} + 384x^2\right](1 + x^\gamma)^{-4/\gamma}$$
$$-384x^{2\gamma} + (72\gamma^3 + 216\gamma^2 - 768)x^\gamma - 27\gamma^4 + 216\gamma^2 - 384 \Big\} \Big/$$

$$\Big\{8\pi q^2 x^2 \left[128x^{2\gamma} - (24\gamma^3 + 72\gamma^2 - 256)x^\gamma + 9\gamma^4 - 72\gamma^2 + 128\right]\Big\} \tag{12}$$

and

$$T_i(x) = \Big\{x(1+x^\gamma)^{\frac{-(4+\gamma)}{\gamma}} \left[64x^{3\gamma} + (24\gamma^3 - 128\gamma - 192)x^{2\gamma} \right.$$
$$\left. -(9\gamma^4 - 24\gamma^3 - 96\gamma^2 + 128\gamma + 64)x^\gamma + 9\gamma^4 - 96\gamma^2 + 192\right]\Big\}\Big/$$
$$\Big\{2\pi q \left[128x^{2\gamma} - (24\gamma^3 + 72\gamma^2 - 256)x^\gamma + 9\gamma^4 - 72\gamma^2 + 128\right]\Big\}. \tag{13}$$

Now, it is possible to plot the inversion curve in the T-P diagram for the black hole under consideration by choosing different values for γ and q (see black colored dash lines in Figures 1,2,3,4,5,6,7,8). In fact, the inversion curve is the separation boundary between the heating and cooling areas of the black hole which intersects the isenthalpic curves at the maximum points given by $\mu = 0$ or the inversion points (T_i, P_i). By looking at these diagrams, one can infer that the top of the isenthalpic curves versus to the inversion curve ($\mu > 0$) represents the heating phase where the temperature increases by raising the pressure while the below part of these diagrams with respect to the inversion curve $\mu < 0$ shows that the black hole participates in the cooling phase where the temperature decreases by raising the pressure. In this paper, we plotted T-P diagrams at constant enthalpy (the black hole mass) for various values of the black hole mass parameter (m) by considering the positive values for γ which comes from regularity condition of the black hole metric given in the [20] and we mentioned in the previous section, but both positive and negative signs for the magnetic charge q.
The isenthalpic T-P curves plotted in Figures 1, 4 and 7 show that for $\gamma = 3$ there is just one inversion curve for $q < 0$ while two inversion curves for $q > 0$. In other choices for fixed $\gamma = 10, 20$ there are obtained two inversion curves for both positive and negative magnetic charges. Looking at the diagrams one can infer that for a fixed γ the JT expansion appears at lower pressures by raising the absolute value of the magnetic charge $|q|$. All figures show that there is not appear the JT expansion or cooling/heating phase transition for the lightest black hole (with smallest mass). In these diagrams, intersection of inversion curves (the black dash lines) and isenthalpic curves (colored solid lines) namely the maximum point of the isenthalpic curves, show some positions where the JT expansion are happened. In other words, for some (smallest black holes) which there is no any intersection point between the isenthalpic curves and the inversion curve one can result that the central AdS modified ABG black hole does not participate in the JT adiabatic expansion. With this regard we obtain that smallest black holes can not expand adiabatically and remain in thermal equilibrium with its surroundings. We should point that when mass of an evaporating quantum black hole remain as constant and so the entropy remains constant too. In this case, the black hole maybe participates in the JT adiabatic expansion or may not be. Such a black hole exchanges heat with its surroundings if it participates in the Joule-Thomson expansion process, otherwise it will be in thermal equilibrium.

4 Concluding remark

In this work, we use central AdS generalized ABG magnetically charged black hole with multiple parameters and studied cooling/heating phase transition for it. This is done by studying the isenthalpic T-P curves intersecting with inversion curves. Inversion curves are obtained when the JT coefficient takes on zero value. Our mathematical calculation predicted that for constant magnetic charges the JT expansion appears for massive generalized ABG black hole by raising the γ parameter of the black hole and low mass black holes do not participate in the JT expansion phenomena. Because there is not a intersection point between the isenthalpic curves of the lightest generalized ABG black hole and corresponding inversion curve. In other words, smallest scale of this kind of black holes remain equilibrium thermodynamically with its surroundings. Also, for a fixed γ parameter the JT expansion appears for massive generalized ABG black holes at lower pressures by raising the magnetic charge parameter. As an future work about the generalized ABG black hole we like to study effect of holographic entanglement entropy and its thermalization affects.

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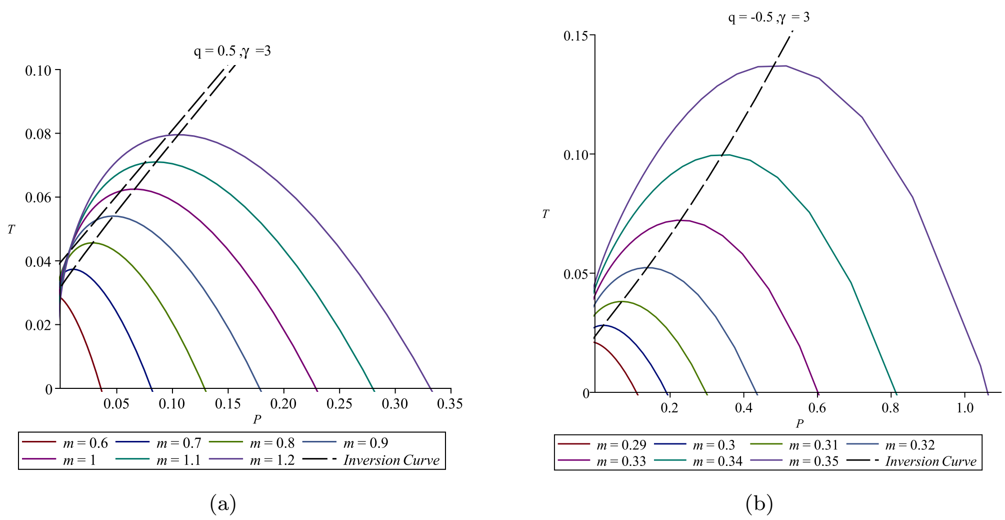
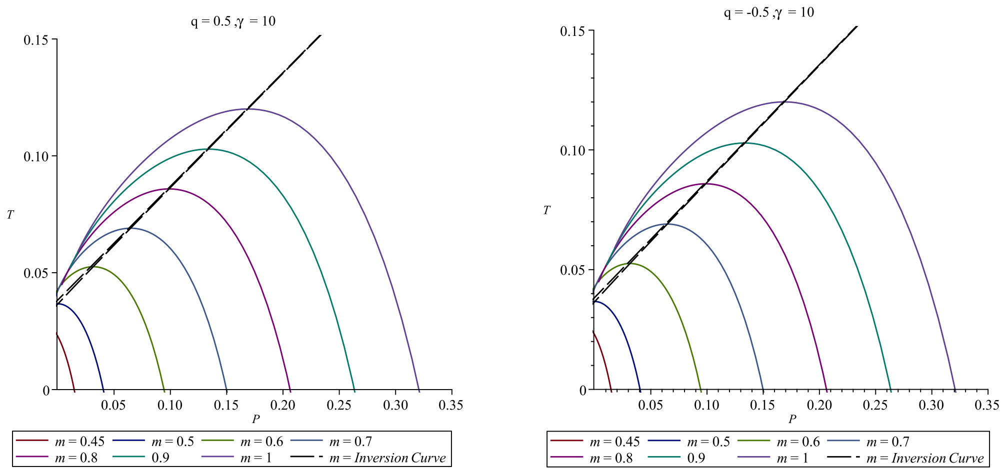
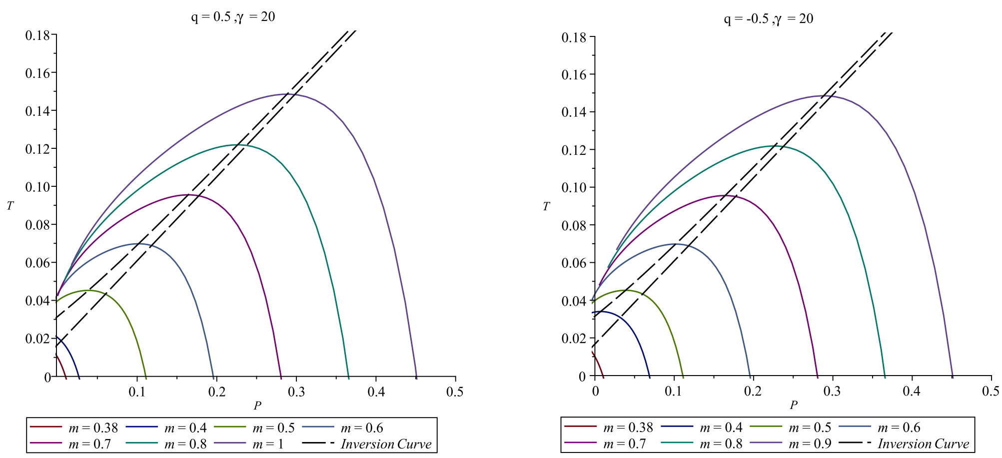


Figure 1: Intersection of the inversion curve (the black line) with isenthalpic curves for $q = \pm 0.5$ and $\gamma = 3$



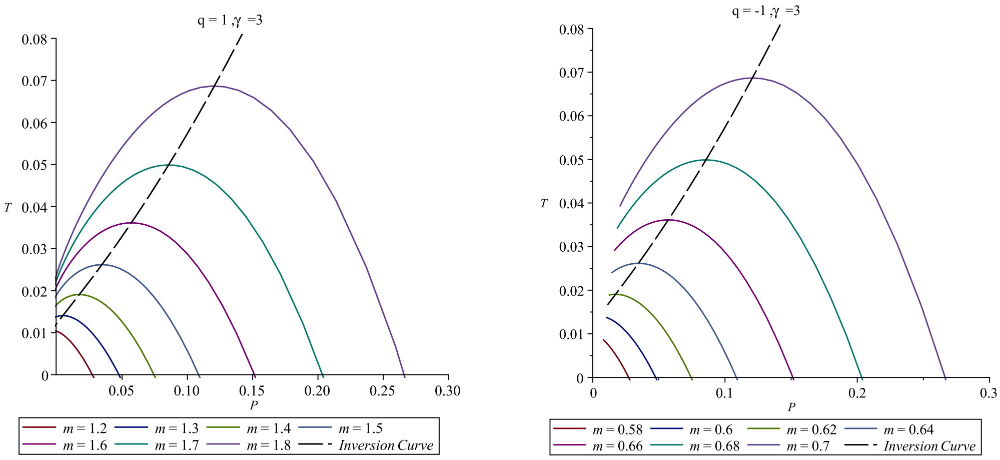
(a) (b)

Figure 2: Intersection of the inversion curve (the black line) with isenthalpic curves for $q = \pm 0.5$ and $\gamma = 10$



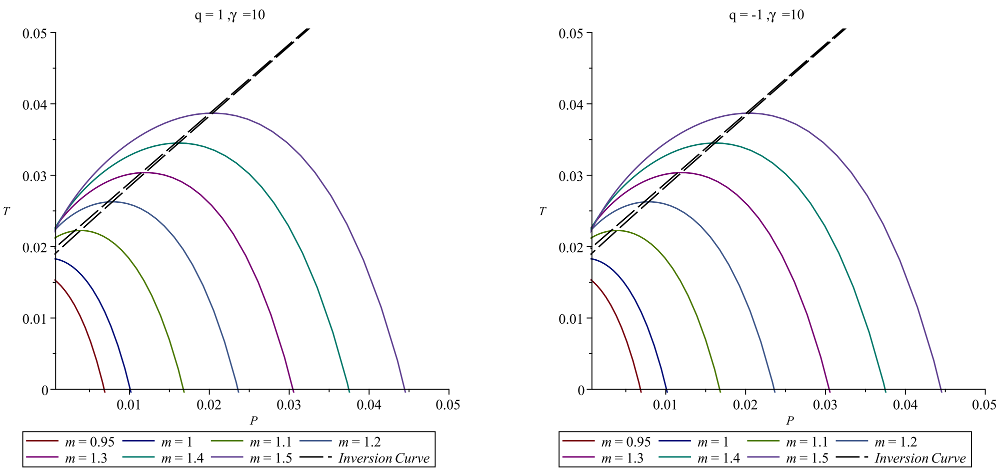
(a) (b)

Figure 3: Intersection of the inversion curve (the black line) with isenthalpic curves for $q = \pm 0.5$ and $\gamma = 20$



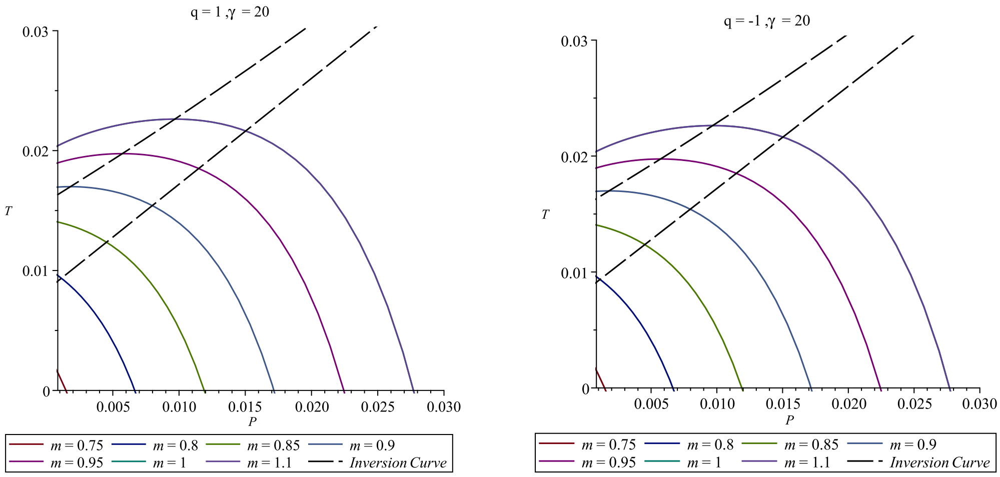
(a) (b)

Figure 4: Intersection of the inversion curve (the black line) with isenthalpic curves for $q = \pm 1$ and $\gamma = 3$



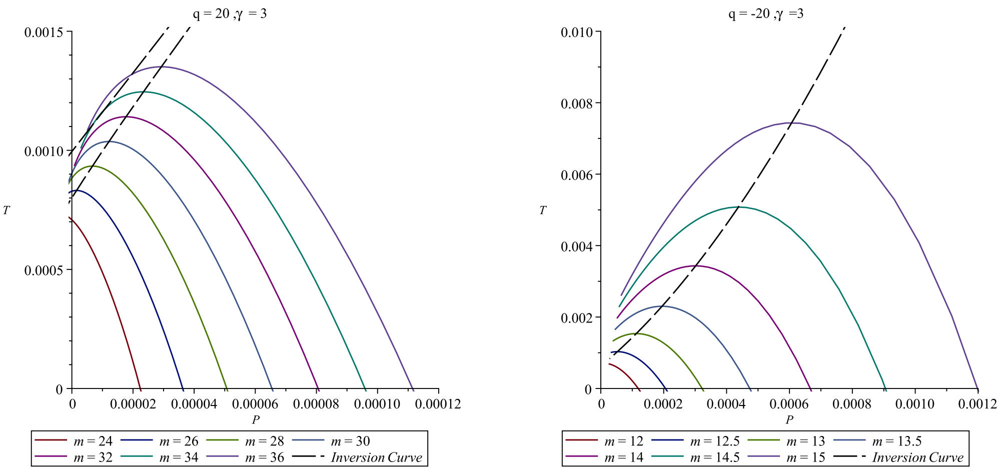
(a) (b)

Figure 5: Intersection of the inversion curve (the black line) with isenthalpic curves for $q = \pm 1$ and $\gamma = 10$



(a) (b)

Figure 6: Intersection of the inversion curve (the black line) with isenthalpic curves for $q = \pm 1$ and $\gamma = 20$



(a) (b)

Figure 7: Intersection of the inversion curve (the black line) with isenthalpic curves for $q = \pm 20$ and $\gamma = 3$

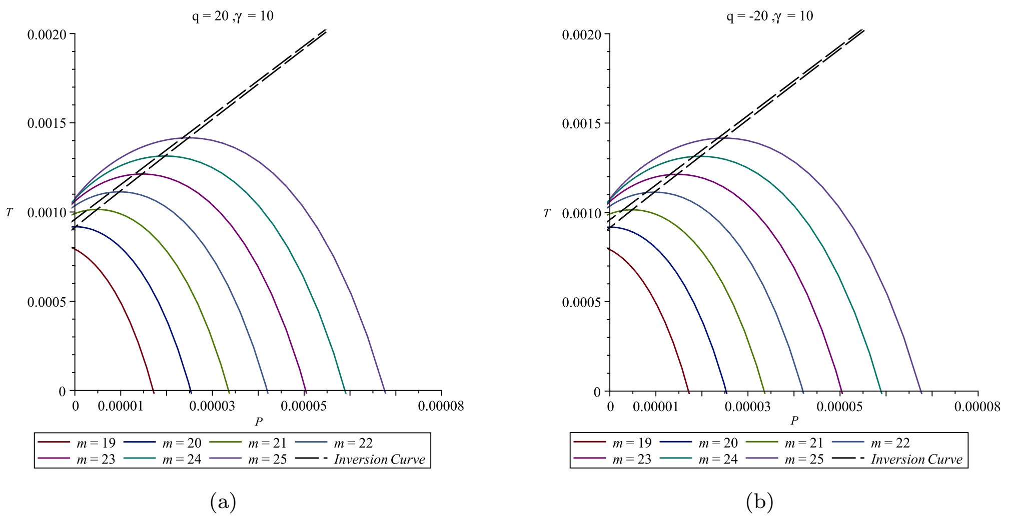


Figure 8: Intersection of the inversion curve (the black line) with isenthalpic curves for $q = \pm 20$ and $\gamma = 10$