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Thermodynamic Phase Transition of Anti De Sitter Schwarzschild Scalar-Tensor-Vector-Black Holes

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Abstract. Instead of scalar-tensor gravity models which are applicable for description of cosmic inflation with unknown dark sector of matter/energy, at present tense there are presented different alternative scalar-tensor-vector gravities where meaningful dynamical vector fields can support cosmic inflation well without to use dark matter/energy concept. One of these gravity models was presented by Moffat which its modified Schwarzschild black hole solution is used to study thermodynamic phase transition in presence of the AdS space pressure in this article. To do so, we obtained an equation of state which asymptotically reaches to equation of state of ideal gas for large black holes but for small scale black holes we obtained a critical point at phase space where the black hole can be exhibited with a phase transition at processes of isothermal and isobaric. By looking at diagrams of the Gibbs free energy and the heat capacity at constant pressure which are plotted versus the temperature and the specific volume one can see an inflection point which means that the phase transition is second order type. In fact there is small to large phase transition for the black hole which is equivalent to the Van der Waals liquid-gas phase transition in ordinary thermodynamic systems. The phase transition happens below the critical point in phase space when the gravitational charge of the black hole is equal to its mass.

Keywords: Modified Gravity, Thermodynamics of black holes, Schwarzschild, de Sitter, phase transition

1 Introduction

General relativity (GR) is an elaborated theory of gravity which successfully has the most correspondence to the experiments [1–4] but nonetheless, there are some unresolved issues that can not been explained by GR. Therefore, various kinds of theories have been developed as generalization of GR in order to solve such problems [5–7]. By calculating velocity of galaxies in the clusters Zwicky concluded that the gravitational mass is more than the luminous matter [8]. Consequently, dark matter was defined, but it has not been detected yet and GR has failed to explain cosmological data without postulated non-baryonic dark matter. In fact, we require modified gravity theories where the gravity tensor is coupled non-minimally by dynamical vector fields and so dark sector of matter/energy problem could be resolved by understandable dynamical vector fields. These models are called as scalar-tensor-vector gravity (STVG) models [9–12]. These theories explain the phenomena without requirement to introduce dark matter [13] and successfully explain the rotation curves of galaxies [10], the dynamics of galactic clusters [14], the growth of structure [15] and the cosmic microwave background (CMB) acoustic power spectrum [16]. Also, Cai and Miao have analyzed the quasinormal modes of the generalized Ayon-Beato Garcia (ABG) black holes [17–20] in STVG theory [21]. One of us is studied some applications of the STVG model given by [11,12] in the references [22–25]. Discovery of evaporation of black holes in presence of interacting quantum matter fields by Hawking in 1974 [26,27] is inferred that there is a closed relationship between the unknown quantum gravity theory and the black hole thermodynamics. After his novel paper, many physicists encouraged to expand studies about the black hole thermodynamics while the quantum gravity theory is still maintained as unknown. For instance, Davis began to study the phase transition of black holes [28] and it became noticable since the Hawking discovered the black holes radiation [29]. Bardeen investigated the laws of black holes thermodynamics [30]. Bekenstein introduced that the black hole entropy should be a quarter of the surface area of the horizon $S = \frac{A}{4}$ [31]. Hawking and Page obtained a first order phase transition for the AdS Schwarzschild black hole [32] which is known as Hawking-Page phase transition. The next motivation had been made by Chambline et al in [33,34] where they discovered a small-large phase transition in AdS-RN black hole which is the same as Van der Waals liquid-gas phase transition. These studies have encouraged others to study thermodynamics of different kinds of AdS black holes in several gravitational theories: Kerr-AdS black hole [35] is studied in GR theory, AdS-Schwarzschild, AdS-RN and Kerr-AdS black holes thermodynamics are studied in modified f(R) gravity [36], Born-Infeld gravity [37], Gauss-Bonnet gravity [38–40], dilaton gravity [**?**,41,42], Lovelock gravity [43–45] or higher dimensions [46–48], respectively. In this regard, the effect of a positive cosmological constant was not forgotten and the evaporation of the quantum black holes in the presence of a cosmological constant prevented the final destruction of the quantum black holes. In practice, the cosmic parameter was a restraining force (see for instance [49–51]). In this sense, modified laws of black holes thermodynamics were generated where fluid hydrodynamics behavior of the black holes are described by VdP work, the same as ordinary thermodynamic systems in which V is the black hole thermodynamic volume and P is pressure of the surrounded environments. In fact, P is equal to inverse of radius square of AdS space (de Sitter space with negative cosmological constant). In a geometrical perspective, an AdS (dS) vacuum space is an open hyperbola (closed spherical) with negative (positive) Gaussian spatial curvature space time. In other words, negative (positive) values cosmological constant is related to negative (positive) values of vacuum energy density in the AdS (dS) space. There are many published papers which one can follow in the literature (see for instance [52–64] and references therein). When one studies thermodynamic behavior of the black holes without (with) to using the cosmological constant, she is appling in fact an ordinary (extended) thermodynamic phase space. Studying thermodynamics of black holes is one of the most remarkable and important subjects to investigate in all gravitational theories and physicists hope to obtain some acceptable proposals about the unknown essential quantum gravity theory via studies of black holes thermodynamics. Moffat presented a particular scalar-tensor-vector gravity (STVG) model [9] and he solved its gravitational metric equations to obtain a gravitationally charged spherically symmetric static black hole metric solution. His obtained solution is similar to the Reissner-Nordstrom electrically charged black hole metric solution [65] where we want to study effects of the black hole charge on its possible thermodynamic phase transition in presence of the AdS pressure. We will call this black hole as AdS Schwarzschild STVG black hole in what follows. The structure of the work is as follows.

In section 2, we introduce in summary the AdS Schwarzschild STVG black hole metric. In section 3, we calculate its thermodynamic variables such as entropy, temperature, heat capacity, Gibbs free energy etc., and try to give a modified Smarr relation. By plotting diagrams of the thermodynamic variables, we analyze possible phase transition of the black hole and situations where the black hole exhibits two coexisting subsystems. Section 3 is devoted to the concluding remarks and outlook of the work.

2 AdS Schwarzschild STVG black hole

Let us start with scalar-tensor-vector gravity (STVG) model which was presented at the first time by Moffat [9] as follows.

$$S = S_{Grav} + S_\phi + S_S + S_M, \tag{1}$$

where gravity part of the above action S_{Grav} is defined by the Einstein-Hilbert action with additional cosmological constant Λ

$$S_{Grav} = \frac{1}{16\pi}\int d^4x\sqrt{-g}[\frac{1}{G}(R+2\Lambda)], \tag{2}$$

the modified massive vector part of the action S_ϕ is given by

$$S_\phi = -\int d^4x\sqrt{-g}\Big[\omega\left(\frac{1}{4}B^{\mu\nu}B_{\mu\nu} + V(\phi)\right)\Big], \tag{3}$$

and the scalar part of the action S_S is given by

$$\begin{aligned} S_S = &\int d^4x\sqrt{-g}\left[\frac{1}{G^3}\left(\frac{1}{2}g^{\mu\nu}\nabla_\mu G\nabla_\nu G - V(G)\right)\right] \\ &+\int d^4x\sqrt{-g}\frac{1}{G}\left(\frac{1}{2}g^{\mu\nu}\nabla_\mu\omega\nabla_\nu\omega - V(\omega)\right) \\ &+\int d^4x\sqrt{-g}\left[\frac{1}{\mu^2 G}\left(\frac{1}{2}g^{\mu\nu}\nabla_\mu\mu\nabla_\nu\mu - V(\mu)\right)\right], \end{aligned} \tag{4}$$

respectively and S_M denotes model dependent ordinary matter source. g is absolute value of the metric determinant and $R = g_{\mu\nu}R^{\mu\nu}$ is the Ricci scalar. ϕ_μ refers to a massive vector field with mass parameter μ and self interaction potential $V(\phi) = -\frac{1}{2}\mu^2\phi^\mu\phi_\mu$ and $B_{\mu\nu} = \partial_\mu\phi_\nu - \partial_\nu\phi_\mu$ is anti symmetric linear tensor field and ω is a dimensionless scalar field with self interaction potential $V(\omega)$. $V(G)$ is self interaction potential of non material scalar field $G(x)$ (namely variable Newtonian gravity coupling parameter) and $V(\mu)$ denotes self interaction potential according to the scalar field $\mu(x)$. ∇_μ refers to the covariant derivative for a metric tensor field $g_{\mu\nu}$. Such alternative gravity models are called as creative models against GR, because without the last term S_M these models can create gravity by self interaction of the fields, while in GR with S_{Grav}, the external matter source S_M is necessary to produce the gravity. In fact, effects of the vector field mass ϕ_μ dose not vanish just at kiloparsec scales from gravitational sources, so it can be neglected near the black holes solutions of the model. At the slow varying regime of the Newton's gravity coupling parameter, one can consider $G = G_N(1+\alpha)$ where G_N is the well known Newton's gravity coupling constant at the Newton and General relativity approach of the model in which the dimensionless parameter α comes from alternative contribution of the above action at the slow varying regime of the scalar field $G(x)$. In other words, α is usually called as the gravitational charge which for $\alpha = 0$ the STVG gravity reduces to GR, so we can regard deviation of the STVG theory with respect to GR given by α parameter. For vacuum sector of the action (1), the Moffat himself solved metric field equations and obtained spherically symmetric static metric vacuum solutions without Λ. He obtained a singular asymptotically flat metric same as the Reissner-Nordstrom one with particular gravitational charge $q = \pm\sqrt{G_N G}M$ [65] as

$$\begin{aligned} ds^2 = &-\left(1 - 2GM/r + \alpha G_N GM^2/r^2\right)dt^2 \\ &+\left(1 - 2GM/r + \alpha G_N GM^2/r^2\right)^{-1}dr^2 + r^2 d(d\theta^2 + \sin^2\theta d\varphi^2), \end{aligned} \tag{5}$$

for a gauge field $\phi_t(r) \neq 0$ with $\phi_r(r) = \phi_\theta(r) = \phi_\varphi(r) = 0$. Haydarov et al [66] studied effects of the gravitational charge α on stabilization of orbits of test particles moving on the space time (5) recently. Moffat himself obtained also another nonsingular asymptotically flat black hole metric (not shown) by choosing other form for the vector gauge field ϕ_μ in the ref. [65] where central region of the obtained metric solution reduces to vacuum de Sitter space which makes it as nonsingular. By looking at his metric solutions, one can infer that the produced effective cosmological constant versus α and the black hole mass M is depended to kind of the used vector gauge field ϕ_μ. But, we should point that these asymptotically flat solutions are different with non asymptotically flat metric solutions which are obtained usually by regarding a nonzero cosmological constant $\Lambda \neq 0$. These solutions approach asymptotically to the vacuum de Sitter ($\Lambda > 0$) or the vacuum anti de Sitter ($\Lambda < 0$) spaces. In this sense, we want to study thermodynamic behavior of the metric solution (5) with $\Lambda \neq 0$ in this work. Thermodynamic behavior of the modified Schwarzschild black hole metric (5) is studied by Mureika et al [67] where the black hole heat capacity exhibits change of the sign at the critical mass in presence of the Hawking radiation. This means the black hole can has a phase transition same as the Reissner-Nordsrom black hole itself (i.e. with electric charge) by regarding the Hawking temperature and the backreaction correction of the interacting quantum fields which is studied by one of us previously [52]. In general, the first law of (the asymptotically flat) black hole thermodynamics is usually written as

$$dM = TdS + \Omega dJ + \Phi dQ, \tag{6}$$

where $T = \frac{\kappa}{4\pi}$ is the Hawking temperature of the black hole (κ is the surface gravity), $S = \frac{A}{4}$ is the Bekenstein-Hawking entropy of the black hole (A is the event horizon surface area), Ω is the angular velocity, J is the angular momentum, Φ is the electrostatic potential difference between infinity and the horizon, Q is the electric charge and M is the black hole mass. Usually M is considered to be internal energy of the black hole U in the ordinary thermodynamic sense, but it was suggested in [68] that it is more correctly interpreted as the enthalpy $M = H = PV + U$ of the black holes in presence of the cosmological constant. In this sense, the first law of the black hole thermodynamics (6) should be extended with variation of the cosmological term VdP as follows [69].

$$dM = TdS + VdP + \Omega dJ + \Phi dQ, \tag{7}$$

where $P = \frac{-\Lambda}{8\pi}$ is pressure of the AdS vacuum space and it is related to radius of the AdS space as $\ell_{AdS} = \sqrt{\frac{-3}{\Lambda}}$ for $\Lambda < 0$. One can infer that the AdS black hole thermodynamic volume can be calculated from the above extended first law of the thermodynamics of the AdS black holes as $V = \frac{\partial M}{\partial P}\big|_{S,J,Q}$. In this sense, V is interpreted as conjugate thermodynamic variable for the pressure P of the AdS space. In other words, V is a finite, effective volume for the region outside the AdS black hole horizon, which can also be interpreted as minus the volume excluded from a spatial slice by the black hole horizon. In fact, the black hole solutions with a non-vanishing cosmological constant Λ have received considerable recent attention because of two reasons as follows: This is due both to the role they play in the phenomenology of the AdS/CFT correspondence [70–72] which associates the cosmological constant with the rank of the gauge group originally and also, of course, to the observational data suggesting that the universe may have a small positive value of Λ [73]. In four dimensions, the extension of the Kerr-Newman family of solutions to non-zero Λ term was found many years ago by Carter [74]. In fact, many authors are investigating to bring our understanding of certain properties of AdS black holes more closely in parallel with well known results in the asymptotically flat case. Dolan showed in ref. [69] that the cosmological parameter raises efficiency from Penrose process in the AdS black hole with respect the case where $\Lambda = 0$. Because of importance of Λ term which we introduced here and also as an extension of our previous work [52], we like to study thermodynamic behavior of the AdS gravitationally charged Schwarzschild STVG black hole which is given by the subsequent line element,

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2, \tag{8}$$

where the metric potential $f(r)$ is

$$f(r) = 1 - \frac{2(1+\alpha)M}{r} + \frac{\alpha(1+\alpha)M^2}{r^2} + \frac{8\pi}{3}Pr^2, \tag{9}$$

for which we have substituted $P = \frac{-\Lambda}{8\pi}$, $G = G_N(1+\alpha)$ and $G_N = 1$. To study thermodynamic behavior of this AdS Schwarzschild STVG black hole, let us to use an equipotential surface $f(r, M, P, \alpha) = constant$ to obtain first law of thermodynamics for this black hole by varying this equipotential surface with respect to the variables r, M, α, P and by setting $df(r, M, P, \alpha) = 0$ as follows

$$dM = TdS + VdP + \Phi_\alpha d\alpha, \tag{10}$$

where the event horizon hypersurface $r = r_h$ is determined by setting $f(r_h, M, P, \alpha) = 0$ as follows.

$$1 - \frac{2(1+\alpha)M}{r_h} + \frac{\alpha(1+\alpha)M^2}{r_h^2} + \frac{8\pi r_h^2 P}{3} = 0. \tag{11}$$

In the equation (10), the Hawking temperature is given versus the surface gravity of the event horizon as

$$T = \frac{1}{4\pi}\frac{df}{dr}\bigg|_{r_h, M, P, \alpha} = \frac{4r_h P}{3} + \frac{(1+\alpha)M}{2\pi r_h^2}\left(1 - \frac{\alpha M}{r_h}\right), \tag{12}$$

and

$$dS = \frac{2\pi r_h dr_h}{(1+\alpha)\left(1 - \frac{\alpha M}{r_h}\right)}, \tag{13}$$

is the entropy difference, and thermodynamic volume

$$V = \frac{\frac{4\pi r_h^3}{3}}{(1+\alpha)\left(1 - \frac{\alpha M}{r_h}\right)}, \tag{14}$$

is conjugate variable of the AdS space pressure P and

$$\Phi_\alpha = -\frac{M}{2(1+\alpha)}\left(\frac{2r_h - (1+2\alpha)M}{r_h - \alpha M}\right) \tag{15}$$

is the conjugate variable for dimensionless gravitational charge α. One can see that the entropy deference (13) reads the Bekenstein-Hawking entropy $S = \frac{A}{4} = \pi r_h^2$ for $\alpha = 0$ and the corresponding thermodynamic volume (14) reduces to the geometric volume of the black hole $V = \frac{4\pi r_h^3}{3}$. While for $\alpha \neq 0$ and $M = H = constant$ the integration of the entropy difference (13) leads to the following equation containing a logarithmic term

$$S = \int_0^{r_h} \frac{2\pi r_h dr_h}{(1+\alpha)\left(1 - \frac{\alpha M}{r_h}\right)} = \frac{2\pi}{(1+\alpha)}\left\{r_h^2 + \alpha M r_h + \alpha^2 M^2 \ln\left|1 - \frac{r_h}{\alpha M}\right|\right\}. \tag{16}$$

In fact, this logarithmic term originates from quantum aspect which can be followed via [75] and references therein. To obtain a relationship between the thermodynamic variables of this black hole to be independent of the geometric parameter, we can do as follows: By substituting $(1-\alpha M/r_h)$ from (14) into the equation (12) we obtain

$$r_h = \frac{3TV}{2(M+2PV)}. \tag{17}$$

Next, we substitute the latter equation into the conjugate potential (15) to obtain the following identity between thermodynamic variables of the black hole.

$$\Phi_\alpha = \frac{-M}{4(1+\alpha)}\left[\frac{(1+2\alpha)M(M+2PV)-3TV}{2\alpha M(M+2PV)-3TV}\right]. \tag{18}$$

This differs with the well known Smarr relation [76] obtained from dimensional approach as $M=2TS+\alpha\Phi_\alpha$[1] for the metric equation (9). However, there is do no worry because authors of the ref. [77] showed also that there are some black hole solutions which do not obey completely the Smarr relation of the black holes. In this sense, we can claim that equation (18) is in fact modified Smarr relation for the AdS Schwarzschild STVG black hole under consideration. The equation (18) is a hypersurface $F(\Phi_\alpha,\alpha,P,V,T,M)=0$ defined in a 6-dimensional phase space. It is useful to obtain inflection point of this hypersurface by calculating

$$\left.\frac{\partial\Phi_\alpha}{\partial\alpha}\right|_{P,V,T}=0,\quad \left.\frac{\partial^2\Phi_\alpha}{\partial\alpha^2}\right|_{P,V,T}=0, \tag{19}$$

which reads 3 different critical points in 6 dimensional phase space $\{\Phi_\alpha,\alpha,P,V,T,M\}$ as follows.

$$I:\qquad \alpha\to\infty,\qquad \Phi_\alpha=0,\qquad M(2PV+M)=0, \tag{20}$$

$$II:\qquad \alpha=-1,\qquad \Phi_\alpha\to-\infty,\qquad 2M^2+4MPV+3TV=0, \tag{21}$$

and

$$III:\qquad \alpha=-\frac{3}{4},\qquad \Phi_\alpha=4M,\qquad M^2+2MPV+3TV=0. \tag{22}$$

In the following, we will see that none of the above values for $\alpha=-1,-\frac{3}{4}$ and/or $\alpha\to\infty$ do not describe physical situations and so possibility of phase transition is done just for $\alpha=1$.

We proceed now to obtain possible critical points of this black hole system as follows. By regarding the black hole evaporation in presence of the Hawking radiation and beak-reaction corrections of interacting quantum fields, the mass of the black hole is lost [49], [50], [52] and so thermodynamics study of black holes should be done with variable enthalpy M. This leads us to obtain an equation of state for the AdS Schwarzschild STVG black hole (9) which is independent of the mass M. To do so, we first obtain the possible critical points which occur through the following conditions for the inflection points

$$\left.\frac{\partial T}{\partial r_h}\right|_{P,M,\alpha}=0,\quad \left.\frac{\partial^2 T}{\partial r_h^2}\right|_{P,M,\alpha}=0, \tag{23}$$

which by substituting the Hawking temperature (12) reads

$$r_c=2\alpha M,\quad P_c=\frac{3(1+\alpha)}{128\pi\alpha^3M^2},\quad T_c=\frac{(1+\alpha)}{8\pi\alpha^2M}. \tag{24}$$

[1] To obtain $M=2TS+\alpha\Phi_\alpha$ we set $Q=\alpha M$ and $\Phi_Q=\frac{\Phi_\alpha}{M}$ and substitute them into the Smarr relation of AdS Reissner-Nordstrom black hole which was given by $M=2TS+\Phi_QQ$ in ref. [77].

By substituting (24) into the thermodynamic volume (14), we will have for critical thermodynamic volume

$$V_c = \frac{64\pi\alpha^3 M^3}{3(1+\alpha)}. \tag{25}$$

The critical horizon radius r_c and the pressure P_c given by (24) must obey the equation of horizon (11) for which we obtain the following condition.

$$\alpha = 1. \tag{26}$$

This corresponds to extremal condition on the Reissner-Nordstrom charged black hole which is called as Lukewarm black hole [51] where the electric charge of the black hole is equal to the mass and so the black hole temperature vanishes [52]. To make a dimensionless equation of state we defined

$$t = \frac{T}{T_c}, \quad p = \frac{P}{P_c}, \quad v = \frac{1}{2}\left(\frac{r_h}{r_c}\right), \tag{27}$$

and substitute into the temperature equation (12) so that

$$t = pv + \frac{1}{4v^2} - \frac{1}{16v^3}. \tag{28}$$

The above equation of state reduces to the well known ideal gas equation of state for large black holes $v \to \infty$ where we must call v as specific volume of this black hole. In this sense, the obtained critical point will be

$$(t_c, p_c, v_c) = \left(1, 1, \frac{1}{2}\right). \tag{29}$$

This critical point is applicable for every AdS Schwarzschild STVG black hole with arbitrary mass. Now, since we have found a mass-independent equation of state, we investigate its possible phase transition for different processes of isotherm and isobaric, namely for processes at constant temperature and constant pressure respectively. Substituting (24) and (27) into the entropy (16) and the potential (15) one can obtain dimensionless forms respectively for these quantities as follows.

$$s = 4v^2 + v + \frac{1}{4}\ln|1-4v|, \tag{30}$$

and

$$\tilde{\phi} = \frac{8v-3}{4v-1}, \tag{31}$$

where we defined $s = \frac{S}{4\pi M^2}$ and $\tilde{\phi} = \frac{\Phi_\alpha}{\Phi_c}$ in which critical potential is $\Phi_c = \Phi_\alpha(r_c) = -\frac{M}{4}$. Substituting the above dimensionless thermodynamic variables into the Gibbs free energy $G = M - TS$ one can obtain its dimensionless form as follows.

$$g = -\frac{1}{2} + 3ts, \tag{32}$$

in which we have defined $g = \frac{G}{G_c}$ and $G_c = -\frac{M}{2}$ is critical value of the Gibbs free energy.

3 Thermodynamic phase transition

To study possible phase transition in $p-v$ plane which is applicable for isothermal processes, we can rewrite the equation of state (28) versus the pressure as follows

$$p = \frac{t}{v} - \frac{1}{4v^3} + \frac{1}{16v^4}. \tag{33}$$

We plot diagram of the above equation of state for constant temperatures below and upper of the critical temperature $t_c = 1$ and also corresponding Gibbs free energy in Figure 1. The pressure diagram in the Figure 1 shows the black hole is made from two subsystems for $t < t_c$ but it has a single state for $t > t_c$. Furthermore, in the Gibbs free energy diagram we can infer that the system is maintain stable for negative values of the Gibbs free energy. Physical interpretation of this diagrams is a phase transition from small to large black holes because the Gibbs energy diverges to positive infinite value in limits $v \to 0$. The system will be unstable where the Gibbs free energy has positive values. Also, one can see that the pressure diagram in Figure 1 is the same as the one which is happened for a Van der Waals fluid in ordinary thermodynamics (see [61]). In Figure 2, we plot temperature versus the entropy, the Gibbs free energy versus the specific volume and the temperature for isobaric processes. Diagram of the temperature shows that for a given temperature one can obtain two different values for the entropy at $p < p_c$ which means that the black hole system under consideration is included with two subsystems (two different phases). Variation of the Gibbs free energy versus the specific volume at constant pressure in Figure 2 shows that for $p < p_c$, the system exhibits with the small to large black holes phase transition. Looking at this diagram one can infer that at constant pressure for large black holes $v > v_c(= 0.5)$ the Gibbs free energy takes negative values which means the system becomes stable. In Figure 3, we plot heat capacity of the system at constant pressure. Changing of the sign of the heat capacity at constant pressure for $(t, v) \leq (t_c, v_c)$ means that a phase transition happens for the system. When heat capacity has positive (negative) values the system is called as diathermal or heat absorber (exothermic or heat repellent) respectively. Variation of the heat capacity diagram versus the temperature and the specific volume and its divergency shows that the phase transition is the same as the one which is called as second kind phase transition in ordinary thermodynamic processes. It may be useful if we study behavior of the compressibility coefficient κ and coefficient of the volume expansion β near the critical point. In ordinary thermodynamics these are called as follows.

$$\kappa = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_t, \quad \beta = \frac{1}{v}\left(\frac{\partial v}{\partial t}\right)_p, \tag{34}$$

which corresponding diagrams are plotted in Figure 4. These diagrams show that β and κ diverge to infinity at the critical point (29). Looking at the compressibility coefficient diagram in Figure 4 one can see a divergency at small scale black holes which means that the black hole exhibits a phase transition. Diagram of the volumetric expansion coefficient versus the temperature shows that this coefficient decreases by raising the temperature at constant pressure. By looking at the diagram of the volumetric expansion coefficient vs the volume one can see that by rasing the specific volume and reducing it to below the critical volume, the volumetric expansion coefficient decreases to zero. In short, one can obtain the same interpretation for the system from β diagram in Figure 4.

4 Conclusion

In this paper, we have studied physical effects of a dynamical vector field on thermodynamic phase transition of modified AdS Schwarzschild black hole. In short, dynamical vector field creates a dimensionless α parameter which behaves as charge parameter for the black hole and so the metric is the same as the Reissner-Nordstrom one. We obtained generalized Smarr relation for the black hole. Mathematical calculations show small to large black hole phase transition which is happened at below the critical point in phase space when the gravitational charge α of the black hole is equal to the mass M. Motivation for using the

AdS space pressure in studying the black hole thermodynamics shows that this black hole behaves as ideal gas in its large scale but at smale scales transits to an imperfect Van der Waals fluid phase. In this sense, without using the AdS pressure, the phase transition does not happen for the black hole. The results of this paper are from studies on the behavior of the Gibbs free energy, heat capacity, volumetric expansion coefficient and compressibility coefficient on the PVT phases space. In future, we are going to see if other thermodynamic phenomena are possible via the effects such as the holographic entanglement entropy on the thermalization are possible for this kind of black hole and whether it has the Joule-Thomson expansion or not.

References

[1] Will, C. M. 2014, Living reviews in relativity, 17, 1.

[2] Berti, E., *et al.* 2015, Classical and Quantum Gravity, 32, 243001.

[3] Abbott, B. P., *et al.* 2016, Physical review letters, 116, 061102.

[4] Hees, A., *et al.* 2017, Physical Review Letters, 118, 211101.

[5] Heisenberg, L. 2019, Physics Reports, 796, 1.

[6] Clifton, T., Ferreira, P. G., Padilla, A., & Skordis, C. 2012, Physics reports, 513, 1.

[7] Weinberg, S. 1972, Gravitation And Cosmology: Principles And Applications Of The General Theory Of Relativity, Wiley, New York.

[8] Trimble, V. 1987, Annual review of astronomy and astrophysics, 25, 425.

[9] Moffat, J. W. 2006, Journal of Cosmology and Astroparticle Physics, 2006, 004.

[10] Moffat, J., & Rahvar, S. 2013, Monthly Notices of the Royal Astronomical Society, 436, 1439.

[11] Ghaffarnejad, H. 2008, General Relativity and Gravitation, 40, 2229.

[12] Ghaffarnejad, H. 2009, General Relativity and Gravitation, 41, 2941.

[13] Brownstein, J., & Moffat, J. 2006, Monthly Notices of the Royal Astronomical Society, 367, 527.

[14] Moffat, J., & Rahvar, S. 2014, Monthly Notices of the Royal Astronomical Society, 441, 3724.

[15] Moffat, J. 2014, arXiv preprint arXiv:1410.2464.

[16] Moffat, J. 2014, arXiv preprint arXiv:1409.0853.

[17] Ayon-Beato, E., & Garcia, A. 1998, Physical review letters, 80, 5056.

[18] Ayon-Beato, E., & Garcıa, A. 1999, Physics Letters B, 464, 25.

[19] Ayon-Beato, E., & Garcia, A. 1999, General Relativity and Gravitation, 31, 629.

[20] Ayón-Beato, E., & Garcia, A. 2005, General Relativity and Gravitation, 37, 635.

[21] Cai, X.-C., & Miao, Y.-G. 2020, arXiv preprint arXiv:2008.04576.

[22] Ghaffarnejad, H. 2009, Classical and Quantum Gravity, 27, 015008.

[23] Ghaffarnejad, H. 2015, Journal of Physics: Conference Series, IOP Publishing.

[24] Ghaffarnejad, H., & Dehghani, R. 2019, The European Physical Journal C, 79, 1.

[25] Ghaffarnejad, H. 2019, Journal of Physics: Conference Series, IOP Publishing.

[26] Hawking, S. W. 1974, Nature, 248, 30.

[27] Hawking, S. W. 1975, Commun. Math. Phys. 43, 199.

[28] Davies, P. C. 1977, Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences, 353, 499.

[29] Hawking, S. W. 1971, Physical Review Letters, 26, 1344.

[30] Bardeen, J. M., Carter, B., & Hawking, S. W. 1973, Communications in mathematical physics, 31, 161.

[31] Bekenstein, J. D. 1973, Physical Review D, 7, 2333.

[32] Hawking, S. W., & Page, D. N. 1983, Communications in Mathematical Physics, 87, 577.

[33] Chamblin, A., Emparan, R., Johnson, C. V., & Myers, R. C. 1999, Physical Review D, 60, 064018.

[34] Chamblin, A., Emparan, R., Johnson, C. V., & Myers, R. C. 1999, Physical Review D, 60, 104026.

[35] Altamirano, N., Kubizňák, D., Mann, R. B., & Sherkatghanad, Z. 2014, Galaxies, 2, 89.

[36] Chen, S.-B., Liu, X.-F., & Liu, C.-Q. 2013, Chinese Physics Letters, 30, 060401.

[37] Zou, D.-C., Zhang, S.-J., & Wang, B. 2014, Physical Review D, 89, 044002.

[38] Cai, R.-G., Cao, L.-M., Li, L., & Yang, R.-Q. 2013, J. High Energy Physics, 2013, 1.

[39] Wei, S.-W., & Liu, Y.-X. 2014, Physical Review D, 90, 044057.

[40] Zou, D.-C., Liu, Y., & Wang, B. 2014, Physical Review D, 90, 044063.

[41] Dehghani, M., Kamrani, S., & Sheykhi, A. 2014, Physical Review D, 90, 104020.

[42] Stetsko, M. 2020, Physical Review D, 101, 124017.

[43] Dolan, B. P., Kostouki, A., Kubizňák, D., & Mann, R. B. 2014, Classical and Quantum Gravity, 31, 242001.

[44] Frassino, A. M., Kubizňák, D., Mann, R. B., & Simovic, F. 2014, J. High Energy Physics, 2014, 1.

[45] Xu, H., Xu, W., & Zhao, L. 2014, The European Physical Journal C, 74, 1.

[46] Gunasekaran, S., Kubizňák, D., & Mann, R. B. 2012, J. High Energy Physics, 2012, 1.

[47] Belhaj, A., Chabab, M., El Moumni, H., & Sedra, M. 2012, Chinese Physics Letters, 29, 100401.

[48] Dolan, B. P. 2014, Classical and Quantum Gravity, 31, 165011.

[49] Ghafarnejad, H. 2006, Physical Review D, 74, 104012.

[50] Ghaffarnejad, H. 2007, Physical Review D, 75, 084009.

[51] Ghaffarnejad, H., Neyad, H., & Mojahedi, M. 2013, Astrophysics and Space Science, 346, 497.

[52] Ghaffarnejad, H. 2016, Astrophysics and Space Science, 361, 1.

[53] Ghaffarnejad, H., & Farsam, M. 2019, Advances in High Energy Physics, 2019.

[54] Ghaffarnejad, H., Yaraie, E., & Farsam, M. 2018, International J. Theoretical Physics, 57, 1671.

[55] Farsam, M., Ghaffarnejad, H., & Yaraie, E. 2019, Nuclear Physics B, 938, 523.

[56] Ghaffarnejad, H., Farsam, M., & Yaraie, E. 2020, Advances in High Energy Physics, 2020.

[57] Ghaffarnejad, H., Yaraie, E., & Farsam, M. 2019, General Relativity and Gravitation, 51, 1.

[58] Ghaffarnejad, H., & Yaraie, E. 2018, Physics Letters B, 785, 105.

[59] Ghaffarnejad, H., & Farsam, M. 2019, The European Physical Journal Plus, 134, 110.

[60] Yaraie, E., Ghaffarnejad, H., & Farsam, M. 2018, The European Physical Journal C, 78, 1.

[61] Ghaffarnejad, H., Yaraie, E., & Farsam, M. 2020, The European Physical Journal Plus, 135, 1.

[62] Ghaffarnejad, H., Yaraie, E., Farsam, M., & Bamba, K. 2020, Nuclear Physics B, 952, 114941.

[63] Farsam, M., Yaraie, E., Ghaffarnejad, H., & Ghasami, E. 2020, arXiv preprint arXiv:2010.05697.

[64] Ghaffarnejad, H., Yaraie, E., & Farsam, M. 2020, arXiv preprint arXiv:2010.07108.

[65] Moffat, J. 2015, The European Physical Journal C, 75, 1.

[66] Haydarov, K., *et al.* 2020, The European Physical Journal C, 80, 1.

[67] Mureika, J. R., Moffat, J. W., & Faizal, M. 2016, Physics Letters B, 757, 528.

[68] Kastor, D., Ray, S., & Traschen, J. 2009, Classical and Quantum Gravity, 26, 195011.

[69] Dolan, B. P. 2011, Classical and Quantum Gravity, 28, 235017.

[70] Maldacena, J. 1999, International J. theoretical physics, 38, 1113.

[71] Gubser, S. S., Klebanov, I. R., & Polyakov, A. M. 1998, Physics Letters B, 428, 105.

[72] Witten, E. 1998, arXiv preprint hep-th/9802150.

[73] collaboration, W., *et al.* 2007, Astrophys. J. Suppl, 170, 377.

[74] Carter, B. 1968, Communications in Mathematical Physics, 10, 280.

[75] Page, D. N. 2005, New J. Physics, 7, 203.

[76] Smarr, L. 1973, Physical Review Letters, 30, 71.

[77] Hu, S.-Q., Kuang, X.-M., & Ong, Y. C. 2019, General Relativity and Gravitation, 51, 1.

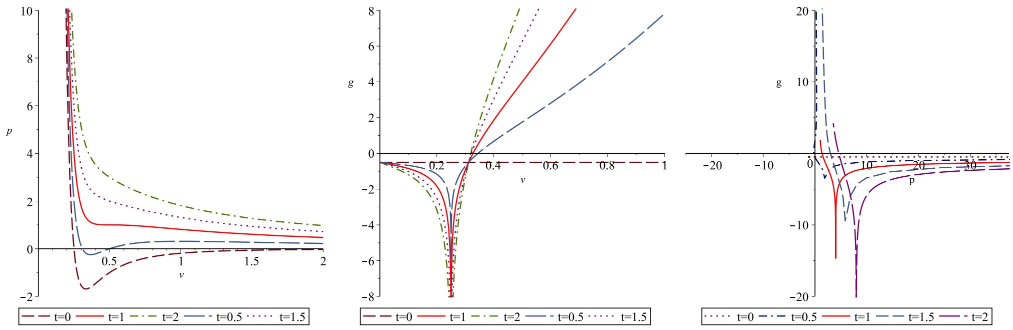

Figure 1: Diagrams of the pressure p and the Gibbs free energy g are plotted versus the specific volume for isotherm processes $t = constnat$.

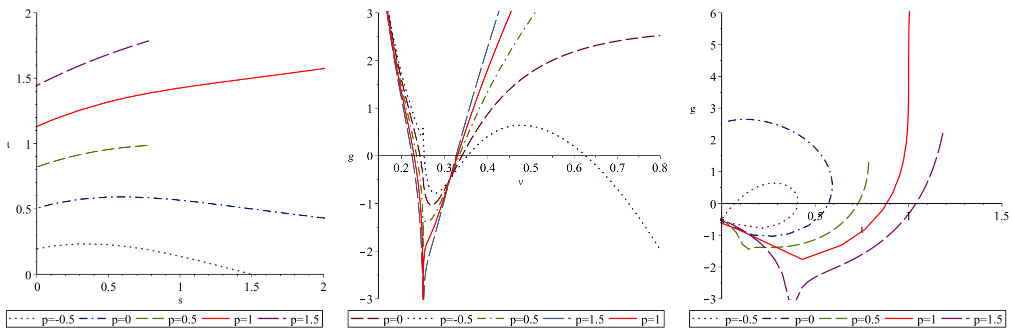

Figure 2: Diagrams for the temperature is plotted vs the entropy at constant pressure and the Gibbs energy is plotted vs temperature and specific volume for isobaric processes.

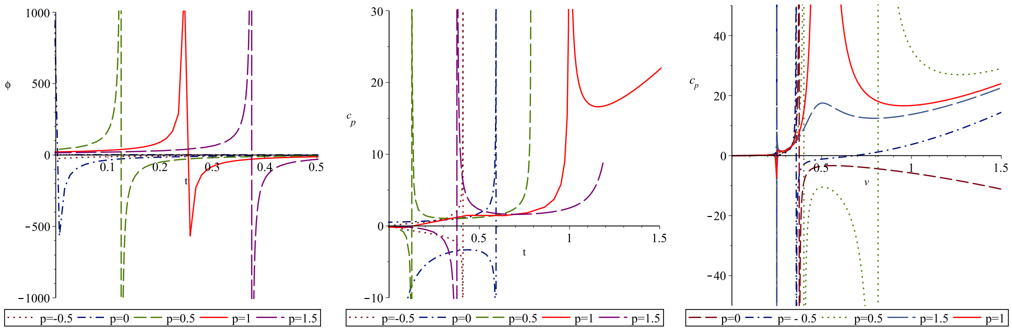

Figure 3: Diagrams for the potential is plotted vs the temperature at constant pressure and heat capacity is plotted vs the temperature and the specific volume at constant pressure.

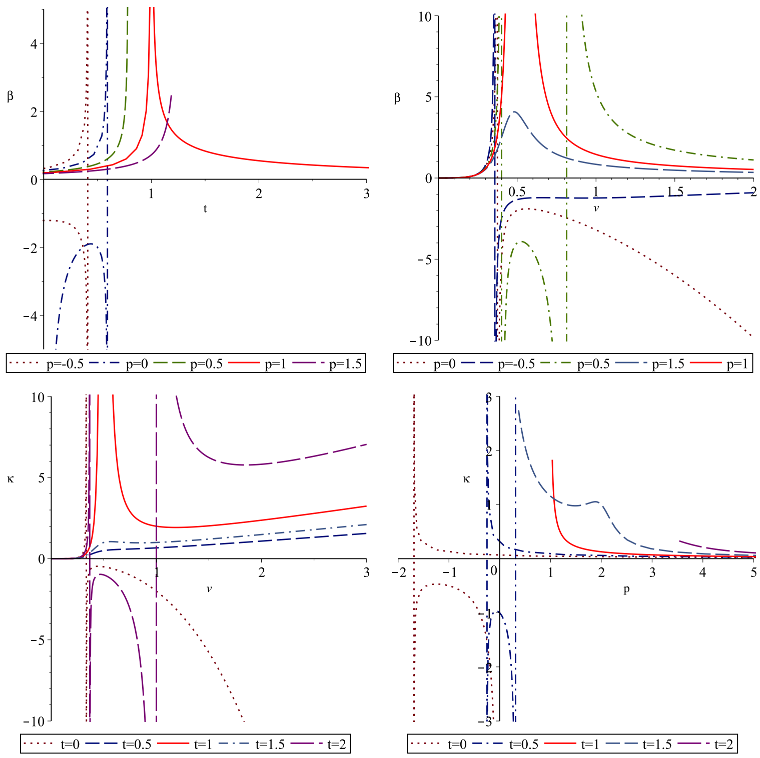

Figure 4: Diagrams for the coefficient of volume expansion β and compressibility coefficient κ are plotted vs the temperature at constant pressure and vs the specific volume at constant pressure respectively.