Iranian Journal of Astronomy and Astrophysics
Vol. 8, No. 2, Autumn 2021
©Available online at http://journals.du.ac.ir
DOI:10.22128/ijaa.2021.541.1118

Iranian Journal of Astronomy and Astrophysics

Effect of Fluctuation of Dust Grain Charge on Electrostatic Sheath Formation in Dusty Plasma with Tsallis Electron Distribution

Fatemeh Kashkooei Jahromi[1] · Iman Motie*[2] · Ali Bakhshayeshi[3] · Taghi Mirzaye[4] · Babak Haghighi [5]

[1] Department of Physics, Mashhad Branch, Islamic Azad University, Mashhad, Iran;
email: adineh8888@gmail.com
[2] Department of Physics, Mashhad Branch, Islamic Azad University, Mashhad, Iran;
*email: imanmoti@gmail.com
[3] Department of Physics, Mashhad Branch, Islamic Azad University, Mashhad, Iran;
email: bakhshayeshi.ali@gmail.com
[4] Department of Physics, Mashhad Branch, Islamic Azad University, Mashhad, Iran;
email: tmirzaye@gmail.com
[5] Department of Physics, Mashhad Branch, Islamic Azad University, Mashhad, Iran;
email: haghighi_236@mail.ru

Abstract. The effect of dust particle charge fluctuation on the electrical potential in the plasma sheath is determined by solvation of Poisson' equation with the help OLM theory and a Tsallis distribution for current carried by electrons, a cold ion fluid and negatively charged immobile dust grains. It is indicated that the nature of electrical potential in plasma sheath dependent to the Sagdeev potential and the properties of electrostatic sheath strongly affected by fluctuations of dust grain charge that modified the Bohm criterion condition. For the first time, by solving the Sagdeev potential, a function is obtained which shows that when this function becomes negative, the ions accelerate into the region of the sheath and form an electrostatic sheath. This nonlinear function is heavily dependent on nonextensive degree of electron, the ion density ratio to electron and kind of plasma gases. It is showed that the farther we go from the Maxwell equilibrium distribution function, the ions need a higher initial velocity to be able to separate from the main body of the plasma and move towards the plasma sheath and the wall. Finally, the results are indicated that considering the dust particle charge fluctuations and the electron nonextensivity degree plays an important role on electrostatic sheath formation and modifies Bohm criterion.

Keywords: Fluctuation of charge, Tsallis distribution, Nonextensivity, Bohm criterion, Electrostatic sheath, Dusty plasma.

1 Introduction

To date, about 9 decades have passed since the use of the term plasma by Langmuir and Tonks, and during this time, much theoretical and experimental research has been done on plasmas [1–4]. Plasmas are estimated to make up about 99% of the visible world, that dust particles being one of the most important components. Therefore, the study of dusty plasmas is very important, especially because of their complex behavior compared to ordinary plasmas [1,2]. Many phenomena and space systems such as comets, nebulae, zodiacal light, northeastern light, the noctilucent clouds at Earth's polar summer mesopause, planetary belts can be a good cosmological laboratory for studying the behavior and properties of dusty plasmas [4–10]. Cosmic, planetary and astrophysical plasmas consist of charged dust particles that have been proven by observation of spacecrafts and they are found in a wide range such as cometary tails, interstellar clouds, the Earth's mesosphere, ionosphere, Saturn's ring, Jupiter's gas ring, as well as in laboratory experiments [11–16]. Dusty plasmas can also be seen in many artificial systems and phenomena such as exhausts of satellites and space shuttles, fire, fission reactors, plasma processing and so on [4,5,8,9].

The velocity distribution function of particles in plasma is important in determining their properties. The Space plasmas are far from thermal equilibrium that is known as Lorentzian dusty plasma and, the suprathermal electron and ion are often found in theirs [11,13,15]. In plasmas that are naturally observed in planetary magnetosphere and in the solar wind, the particle velocity distribution is non-Maxwellian [11,15]. The statistical behavior of space plasma is associated with their nature as collisionless, weakly coupled plasmas and also, particles in a Debye sphere. Weakly coupled plasma is governed by the overall collective electrostatic forces of many particles. The intense interactions between individual particles are relatively rare, thus space plasmas exhibit strong collective behavior that characterizes correlated particles in a Debye sphere, without the phenomenon of localization between individual particles due to interaction or collision. This behavior cannot be understood by classical statistical description of thermal equilibrium [17,18]. The nonextensive distributions have been utilized in increasingly numerous studies across the space plasma process, from solar wind and planetary magnetospheres to heliosheath, and beyond to interstellar and intergalactic plasmas. Permeating dusty plasmas are far away from the equilibrium and can be created in the inter-penetrating solar/stellar wind that is dominated by electron-ion plasma in surrounding cometary plasma which consists of charged particles other than electrons and ions [19–22]. These distributions represent suprathermal deviations from Maxwell's equilibrium and is expected to occur in any lowdensity plasma in the universe, where the binary collision of charges is sufficiently rare [23,24]. The dust charging characteristics and the electrical potential of dust grains in Lorentzian plasma are different in terms of their properties in Maxwellian dusty plasma [11]. To understand the physics of this process, it is enough that we only know the raw details of all states, statistical mechanics, and distribution functions [25,26]. The Tsallis distribution is the result of a nonextensive entropy and a generalization of Boltzmann-Gibbs entropy that is presented by Tsallis in 1988 [19]. This distribution can be considered as more appropriate basis for a theoretical framework where small systems with long-range and complex interactions are present such as space plasmas [19,20,27]. Meanwhile, the q index is the boundary between Boltzmann-Gibbs and Tsallis. This means that BoltzmannGibbs mechanics will appear in $q = 1$, and we will have the statistical mechanics of the Tsallis for $q \neq 1$ [19]. Silva et al. showed that in empirical research particle distribution measurements are as well according to the non-Maxwellian distribution function [27]. In a research work, Lima et al. used the Tsallis distribution function for two-dimensional Euler and drift turbulence in a pure-electron plasma column and showed that the Tsallis distribution is fitted well with empirical data [17]. Lavagno et al. have showed that Tsallis distribution function is the result of nonextensive entropy concepts, accounting for the long-range forces in space plasmas and the q parameter is as the degree of system nonextensivity [28]. This distribution function is one of the good examples of nonMaxwellian distribution. Much research has been done to explain the collective interactions in plasmas by using the Tsallis distribution function [12,22,27,28].

On the other hand, it is clear that in laboratory plasmas there are walls for the reactor that help to confine the plasma, so it is important to study the behavior of plasmas near the walls, especially in particle accelerators or fusion reactors [2,3,7]. As a result of the interaction between the walls and plasma, there is usually contamination in the form of dust particles in them or dust particles may also be added to the system for industrial purposes such as PVD or PCVD coating and etc. [2,3]. One of the important features in these plasmas that affects their behavior is the Bohm criterion. The Bohm standard criterion for the plasma stability and the formation of electrostatic sheaths in the walls of these plasmas is described in some standard textbooks, assuming an equilibrium distribution of electrons [2,3]. Furthermore, these sheaths occur in surfaces of space plasmas, antennas or around objects in space such as spacecrafts and satellites. In around of spacecrafts, sheaths are important problems for many plasma instruments and can change and influence the properties of charged particle or dust particles that are released from asteroid or cometary surfaces [14,16]. In a research work, Riemann considered Bohm criterion in the limit of a small Debye length with a two-scale problem of a collision free sheath and of a quasi-neutral presheath and showed that a rigorous kinetic analysis of the vicinity of the sheath edge allows one to generalize Bohm's criterion accounting not only for arbitrary ion and electron distributions, but also for general boundary conditions at the wall [29]. Recently, Breizman and et al had studied one-dimensional evolution of a collisionless plasma next to a solid surface that is immersed into the plasma instantaneously. They presented numerical results that a sufficiently strong reflection eliminates the Debye sheath and changes the wall potential and the plasma flow parameters significantly [30]. Also, the Bohm Criterion for Two-Ion-Species Plasmas is analyzed in numerical method by Azuma and et al. and it shows in detail, the dependence of the condition for two-ion-species plasmas on collisionality and ions mass [31]. Effect of nonextensive distribution of electrons on the plasma sheath floating potential is considered by Sharifian et al and is observed that Debye length and Bohm criterion depend significantly on the non-extensive parameter q [32]. In other research, Asserghine et al. considered a plasma sheath containing primary electrons, cold positive ions, and secondary electrons. They using a onedimensional fluid model with Tsallis distribution and by numerical method the effect of secondary electron emission on q-non-extensive plasma sheath characteristics examined. They showed that significant change fined in the quantities characterizing the nonextensive plasma sheath with the presence of the secondary electrons [33]. In this work, we focus on the effect of dust particle charge fluctuations on the electrical potential in a dusty plasma sheath. To determine the behavior of the electric potential in the sheath, we start with the Poisson equation. Dusty plasma is assumed to consist of thermal electrons, a cold ionic fluid, and immobilized negatively charged dust grains. In walls, we know that electrons and ions combine and disappear, but electrons die faster than ions due to very high thermal velocities, causing the negative boundary potential to be negated by a protective layer called an electrostatic sheath. In order to form this sheath, it is necessary for the ions to be able to reach this region, and therefore, the Bohm criterion determines the minimum velocity of the ions. On the other hand, temporal and spatial changes in dust particle charge lead to new phenomena in these systems. Therefore, the effect of dust particle charge fluctuations on the properties of the electrostatic sheath and the modified Bohm criterion conditions is important, which we will study and the effect of different plasma properties on it. In this work, the velocity of the electrons follows the generalized distribution of Tsallis, because the plasma we are studying is a Lorentzian dusty plasma. It is assumed that the dust potential is negative because the mass of the ion is much larger than the mass of the electron and the electron reaches the dust faster than the ion. The effects of plasma gas type, non-extensive electron degree (qe) and ion to electron density ratio are investigated.

The article is presented in the following fashion: In section 2, the effect of fluctuations in dust particle charge on the properties of the electrostatic sheath and the modified Bohm criterion condition with help OLM theory and Tsallis distribution for electrons are investigated. In section 3, the ion acceleration and electrostatic sheath formation conditions are analyzed and the effects of nonextensive parameter q_e, density ratio of ionto-electron and kind of plasma gas on Bohm criterion are provided. Finally, a summary and conclusions are given in Section 4.

2 Basic equation

One of the results of dust particles in plasmas is the temporal and spatial changes of charge of dust particles, which leads to new phenomena in these systems. In this section, we want to investigate the effect of dust particle charge fluctuations on the properties of the electrostatic sheath and the modified Bohm criterion condition. We start from the Poisson's equation and assume that the dusty plasma consists of thermal electrons, a cold ion fluid and negatively charged immobile dust grains. The electrons and ions are combined and lost because of hitting the wall at the same time. Electrons are lost faster than ions due to very high thermal velocities compared to ions, causing the boundary potential to be negative. This potential is bounded by a layer due to the Debye shielding, which prevents the potential distribution in plasma and is called electrostatic sheath. Also, we assume that at the plate $x = 0$, the ions are entering the sheath region from the main plasma with a drift speed v_{io}. These drifting ions are needed to account for the loss of ions to the wall from the region in which they were created by ionization. In steady state, for the cold ions, we have

$$n_i v_i = n_{i0} v_{i0}, \tag{1}$$

and

$$\frac{1}{2} m_i (v_i^2 - v_{i0}^2) = -e\varphi_s, \tag{2}$$

where e is the charge of electron and n_i, v_i and φ_s are the number density, the velocity of ions and the electrical potential in sheath, respectively. Also, n_{i0} is the number density of single-charged ions at $x = 0$ that there $\varphi_s = 0$ [3]. Due to the cold ions, we have a combination of equations (1) and (2) to distribute density them inside the sheath

$$n_i = n_{i0} (1 - \frac{2e\varphi_s}{m_i v_{i0}^2})^{-\frac{1}{2}}. \tag{3}$$

In non-equilibrium systems with long range interactions that electrons are nonextensive, it has been referred for the following electron number, density is given by

$$n_e = n_{e0} [1 + (1 - q_e) \frac{e\varphi_s}{k_B T_e}]^{\frac{1}{1-q_e}}, \tag{4}$$

where n_{e0} is electron number density in $x = 0$, T_e is electron temperature, k_B is Boltzmann constant and q_e is electron nonextensive parameter [20,34]. Equations (3) and (4) are completed by Poisson's equation

$$\frac{d^2\varphi_s}{dx^2} = 4\pi e (n_e + Z_d n_d - n_i), \tag{5}$$

and the macroscopic charge neutrality condition $Z_d n_d = n_{i0} - n_{e0}$ where n_d is the dust grain number density and Z_d is the number of charges residing on the dust grain surface. We consider in here the dust particle charge fluctuates. The dust particle charge q_d has a constant term q_{d0} and fluctuating term q_{d1} Which means $q_d = q_{d0} + q_{d1}$. Now using equation (3), (4), (5) and $q_{d0} n_{d0} = e n_{e0} - e n_{i0}$ we can express Poisson equation in terms of normalized variables as

$$\frac{d^2\psi_s}{d\xi^2} = -(1 - \alpha\psi_s)^{\frac{1}{\alpha}} - \delta (1 + \frac{2\psi_s}{M^2})^{-\frac{1}{2}} + (\delta - 1) \frac{Q_d}{Q_{d0}}, \tag{6}$$

where in this equation $Q_d = eq_d/k_BT_er_d$ and $Q_{d0} = eq_{d0}/k_BT_er_d$ when charge of dust particles is constant, $Q_d/Q_{d0} = 1$. For this section, charge Q_d is not constant and is determined from the condition $I_e + I_i = 0$. Thus, the cold ion current is as

$$I_i = \pi r_d^2 n_{i0} v_{i0} e(v_{i0}^2 - 2\psi_s c_s^2 - 2Q_d c_s^2)(v_{i0}^2 + 2\psi_s c_s^2)^{-1}. \tag{7}$$

In obtaining this equation, $I_i = en_iv_i\sigma$ and fluid motion equations of ion have been used inside the plasma sheath and the cross section of the collision according to the OLM theory is as [3,35],

$$\sigma = \pi r_d^2 (1 - \frac{2eq_d}{m_i v_i^2 r_d}). \tag{8}$$

Based on OLM theory, when dust grains have a negative charge, then the electron current is resulted from incident of plasma particles with the charge e on dust grain as

$$I_e = -e \int_{v_e^{\min}}^{\infty} v_e \sigma f_e(v_e) dv_e. \tag{9}$$

In this equation, $f_e(v_e)$ is the Tsallis distribution that can be written as [35,36]

$$f_e(v_e) = A_{qe} n_e [1 - (1 - q_e)\frac{m_e v_e^2}{2k_BT_e}]^{\frac{1}{1-qe}}, \tag{10}$$

where

$$\left[\begin{array}{c} q_e < 1 \Rightarrow A_{qe} = \frac{(5-3q_e)(3-q_e)\sqrt{1-q_e}}{4} \left[\frac{m_e}{2\pi k_BT_e}\right]^{\frac{3}{2}} \frac{r(\frac{1}{1-q_e}+\frac{1}{2})}{r(\frac{1}{1-q_e})} \\ \\ q_e > 1 \Rightarrow A_{qe} = \frac{(5-3q_e)\sqrt{q_e-1}}{2} \left[\frac{m_e}{2\pi k_BT_e}\right]^{\frac{3}{2}} \frac{r(\frac{1}{q_e-1})}{r(\frac{1}{q_e-1}-\frac{1}{2})} \end{array} \right], \tag{11}$$

and v_e is electron velocity and m_e is electron mass. Also, because of repulsive potential, the plasma particle and the dust grain repel each other and hence, the existence of an initial velocity $v_e^{\min}$ is necessary. Thus, in this case, $v_e^{\min}$ becomes

$$v_e^{\min} = \sqrt{\frac{2e\varphi_d}{m_e}}. \tag{12}$$

The equation (9) can be calculated using, integration by parts, incomplete Beta function and Hyper Geometric functions [37,38] and combination methods. Thus, calculations for current carried by electrons in non-equilibrium plasma are as follows

$$I_e = -\sqrt{8\pi} r_d^2 e B_{qe} n_{e0} [1 - \alpha\psi_s]^{\frac{1}{\alpha}} \sqrt{\frac{k_BT_e}{m_e}} \left[1 + \alpha\frac{e\varphi_d}{k_BT_e}\right]^{\frac{1+2\alpha}{\alpha}}, \tag{13}$$

where

$$B_{qe} = \begin{cases} \frac{(5-3q_e)(3-q_e)\sqrt{1-q_e}}{4(2-q_e)(3-2q_e)} \frac{r(\frac{1}{1-q_e}+\frac{1}{2})}{r(\frac{1}{1-q_e})}, & \text{for } q_e < 1, \\ \\ \frac{(5-3q_e)\sqrt{q_e-1}}{2(2-q_e)(3-2q_e)} \frac{r(\frac{1}{q_e-1})}{r(\frac{1}{q_e-1}-\frac{1}{2})}), & \text{for } q_e > 1. \end{cases} \tag{14}$$

Using equations (7) and (13), we can express the condition $I_e + I_i = 0$ in the form

$$M^2 + 2\psi = \sqrt{\frac{\pi m_e}{8m_i}} B_{qe}^{-1} \delta M [1 - \alpha\psi_s]^{-\frac{1}{\alpha}} [1 + \alpha Q_d]^{-\frac{1+2\alpha}{\alpha}} [M^2 - 2Q_d + 2\psi_s]. \tag{15}$$

By using equation (6) and (15), it is obtained as follow

$$\frac{1}{2}(\frac{d\psi_s}{d\xi})^2 + V(\psi_s) = 0, \tag{16}$$

where the Sagdeev potential $V(Q'_d, \psi_s)$ is given by

$$V(\psi_s) = -\frac{1}{1+\alpha}(1+\alpha\psi_s)^{\frac{1+\alpha}{\alpha}} - \delta M^2\left(1+\frac{2\psi_s}{M^2}\right)^{\frac{1}{2}} + \frac{1}{1+\alpha} + \delta M^2 + V_d, \tag{17}$$

that sentence V_d is the effect of fluctuations in the dust particle charge

$$V_d = \frac{\delta - 1}{Q_{d0}}\int_0^{\psi_s} Q_d d\psi_s. \tag{18}$$

In order to derive equations (15), we have used the appropriate boundary conditions $\psi_s = d\psi_s/d\zeta = 0$ at $\zeta = 0$. The equation (16) indicates that the Sagdeev potential is modified by the charge of dust grain fluctuation and thus ψ_s in electrostatic sheath can by Sagdeev potential analytic or this equation must had solved by numerical method. By estimating the Sagdeev potential in a dusty plasma with a dust grain charge in fluctuation and in small amplitudes of $\psi_s \ll 1$, it is obtained

$$V(\psi_s) = \frac{1}{2}\psi_s^2\left[\left(\frac{\delta}{M^2} - 1\right) + \left.\frac{d^2V_d}{d\psi_s^2}\right|_{\psi_s=0}\right], \tag{19}$$

which in this equation must be $V(Q_d, \psi_s) < 0$ means

$$\frac{\delta}{M^2} + \left.\frac{d^2V_d}{d\psi_s^2}\right|_{\psi_s=0} < 1. \tag{20}$$

This inequality is the condition of the modified Bohemian criterion in a dusty plasma with a the Tsallis distribution function for electrons and the fluctuating charge of dust grains. Using equations (15) and (18), the inequality (20) is as follow

$$y = [AM^4 + BM^2 + CM + D] < 0, \tag{21}$$

where

$$\begin{aligned}
&A = \tfrac{\delta-1}{Q_{d0}} - \tfrac{1+2\alpha}{1+\alpha Q_{d0}},\\
&B = \tfrac{1+2\alpha}{1+\alpha Q_{d0}}\delta - 2(\delta-1) + 2\tfrac{\delta-1}{Q_{d0}} + 2Q_{d0}\tfrac{1+2\alpha}{1+\alpha Q_{d0}} - 2,\\
&C = -2\tfrac{\delta-1}{Q_{d0}}\sqrt{\tfrac{8m_i}{\pi m_e}}B_{qe}\delta^{-1}[1+\alpha Q_d]^{\frac{1+2\alpha}{\alpha}},\\
&D = 2\delta\left(-Q_{d0}\tfrac{1+2\alpha}{1+\alpha Q_{d0}} + 1\right).
\end{aligned} \tag{22}$$

When the dust grain charge is constant then $Q_d = Q_{d0}$ and therefore $V_d = 0$ and as a result, we come to $M > \sqrt{\delta}$. The equation (21) indicates that the condition for the existence of an electrostatic sheath and the acceleration of ions into this region is that $y(M)$ function be negative. $y(M)$ is a nonlinear function of Mach number and is very complex because it depends on various parameters of plasma. These parameters limit the initial velocity of the ions entering the electrostatic sheath region and the Mach number.

3 Results and discussion

From equations (21) and (22), it is cleared that the analysis of the ion acceleration condition and the formation of electrostatic sheaths is complex. These equations show that with Tsallis distribution for electrons and the fluctuating charge of dust grains, the modified Bohm criterion is heavily dependent on nonextensive degree of electron (q_e) and also, the characteristics of the plasma such as the ion density ratio to electron and kind of plasma gases. In present section, the analytical of nonlinear equations (21) and (22) are provided with the effects of nonextensive parameter q_e, density ratio of ion-to-electron and kind of plasma gas on $y(M)$ function and modified Bohm. Figure 1 shows the variations of $y(M)$ function vs. The Mach number for the different density ratios of ion-to-electron with nonextensive parameters $q_e = 1.25$ in hydrogen plasma gas. In these Figures, the curves are plotted for the different density ratios ion-to-electron that in Figure 1(a), $\delta = 1$ and it is ordinary plasma and in Figure 1(b), $\delta = 3$ (triangular line), $\delta = 5$ (plural line), $\delta = 7$ (circular line) and $\delta = 9$ (dotted line) and they are dusty plasma. In $\delta = 1$, we have an ordinary electron-ion plasma and figure 1(a) is drawn for it. Figure 1(a) shows that for Mach numbers greater than one, $y(M)$ function is negative and by increasing the Mach number, the faster the $y(M)$ function becomes negative. This means that in ordinary plasmas the condition of the electrostatic sheath is that the initial velocity of the ions entering this region be at least equal to or greater than the velocity of the ion-acoustic. Furthermore, Figure 1 (b) is for when the initial densities of ions and electrons are not equal and therefore we have a dust plasma. Figure 1 (b) shows that as δ increases, the $y(M)$ function becomes negative for values greater than Mach number. In other words, as the δ increases, the lower bound for the Mach number becomes larger so that it can negate the y function. Due to the quasi-neutrality condition in plasma, an increase in δ means that the ion to electron density ratio has increased, which means that the electric charge or dust particle density has increased ($Z_d n_d$). In other words, as the electric charge or density of dust particles in the plasma increases, the possible values for the Mach number minimum become larger, thus modifying the Bohm criterion condition and the electrostatic sheath formation. Therefore, increasing the minimum ion velocity to accelerate into the sheath region. The physical reason for this modify is that as the electric charge or density of the dust particles increases, the Coulomb attraction between the ions and the dust particles increases. Therefore, in order for the ions to be able to overcome this attraction and separate from the plasma body and accelerate towards the wall and the sheath area, they need a higher initial velocity. These results also show the great importance of dust particle charge fluctuations in changing the Coulomb attraction between ions and dust particles and thus on the modification of the Bohm criterion and the formation of electrostatic sheath. The influence of kinds of plasma gasses on $y(M)$ function and thus modified Bohm criterion condition in the formation of electrostatic sheaths is presented as a function of Mach number in Figures 2 that the curves are plotted for four of plasma gases include of hydrogen plasma (triangular line), helium plasma (plural line), oxygen plasma (circular line) and argon plasma (dotted line). The results are obtained for a dusty plasma with nonextensivity of degree $q_e = 1.25$ for electron and density ratio of ion-to-electron $\delta = 5$. Figure 2 shows that as the mass of the ions increases, the $y(M)$ function becomes negative for larger Mach numbers. It is also clear from Figure 2 that there is no linear relationship between increasing the mass ratio of ions to increasing the Mach minimum number. The results indicate that as the mass of the ions increases, more initial velocity is required to separate the ions from the main body of the plasma and accelerate them toward the plasma wall and sheath area. As the mass of the ions increases, the velocity of the acoustic ion decreases and therefore, the Mach number increases. In Figure 3, the effect nonextensive degree of electron on $y(M)$ function

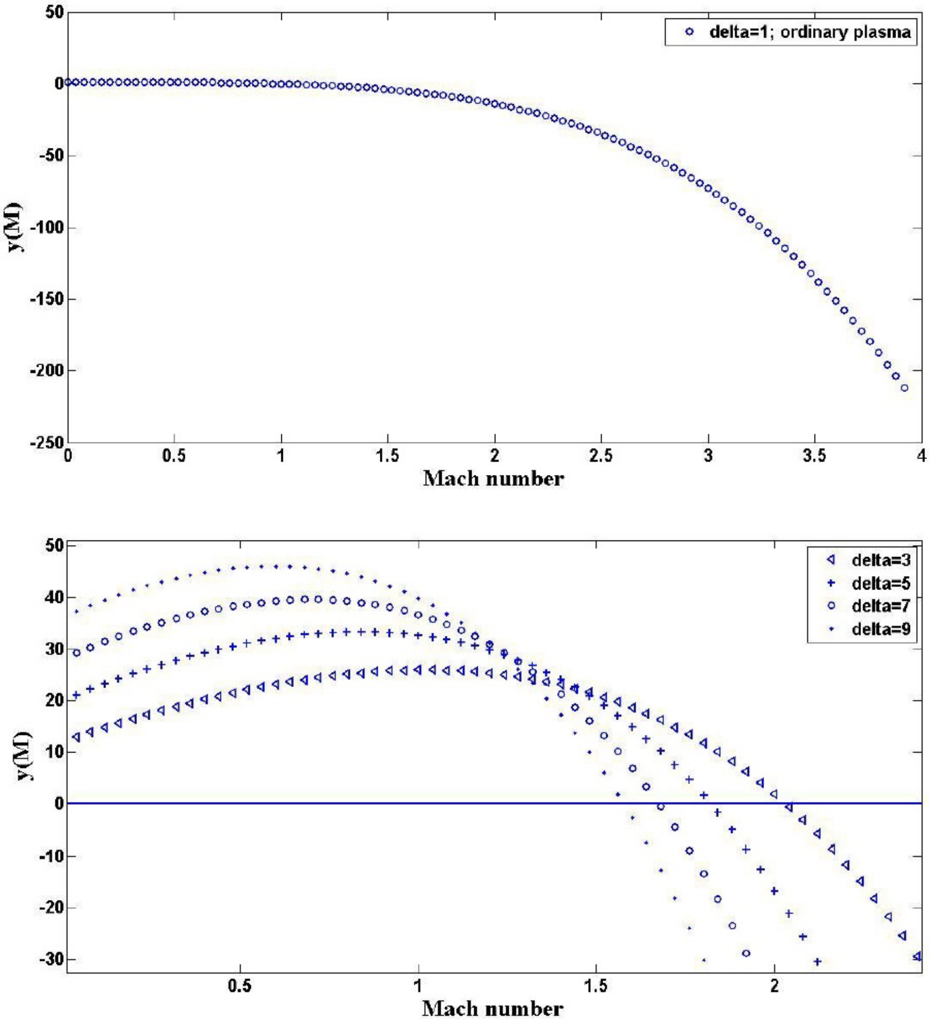

Figure 1: The variations of $y(M)$ function vs. Mach number with different density ratios of ion-to-electron when (a) $\delta = 1$ (ordinary plasma) and (b) $\delta = 3$ (triangular line), $\delta = 5$ (plural line), $\delta = 7$ (circular line) and $\delta = 9$ (dotted line) with $q_e = 1.25$ in the hydrogen plasma.

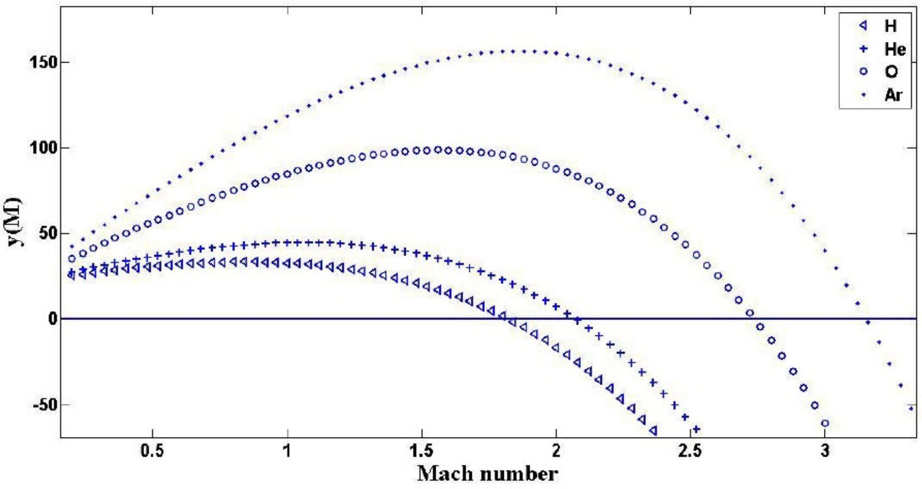

Figure 2: The variations of $y(M)$ function vs. Mach number for different kinds of plasma that hydrogen plasma (triangular line), helium plasma (plural line), oxygen plasma (circular line) and argon plasma (dotted line) with $q_e = 1.25$ and $\delta = 5$.

and modified Bohm criterion is presented as a function of Mach number. These curves are plotted for the different values of $q_e = 1.15$ (triangular line), $q_e = 1.2$ (plural line), $q_e = 1.25$ (circular line) and $q_e = 1.3$ (dotted line) when density ratio ion-to-electron $\delta = 5$ and the plasma gas is hydrogen. Figure 3 shows that as the degree of electron nonextensive increases, the y function becomes negative in values greater than Mach number. It is clear from Figure 3 that the farther we go from the Maxwell equilibrium distribution function where $q_e = 1$, the ions need a higher initial velocity to be able to separate from the main body of the plasma and move towards the plasma sheath and the wall. This figure shows the large effect of the degree of electron nonextensive on the formation of the electrostatic sheath and the modify of the Bohm criterion. It is also clear from Figure 3 that the rate of increase of the minimum Mach number is faster for higher degrees of electron nonextensivity. On the other hand, according to the result of the second section, which showed the independence of the Bohm criterion from the distribution function and the nonextensivity degree of electron, it is clear from the results of this figure that considering the dust particle charge fluctuations, the electron nonextensivity degree and its distribution function plays an important role on electrostatic sheath formation and modify Bohm criterion.

4 Summary and Conclusion

In this work, with the help OLM theory and assumes a Tsallis distribution for current carried by electrons, a cold ion fluid, negatively charged immobile dust grains and dust particle charge fluctuations, the Poisson's equation solved and the behavior of the electrical potential in the plasma sheath determined. This potential bounded by a layer due to the Debye shielding, which prevents the potential distribution in plasma and called electrostatic sheath. It indicated that the nature of electrical potential in plasma sheath dependent to the Sagdeev potential and the properties of electrostatic sheath strongly affected by fluctuations of dust grain charge that modified the Bohm criterion condition. By solving

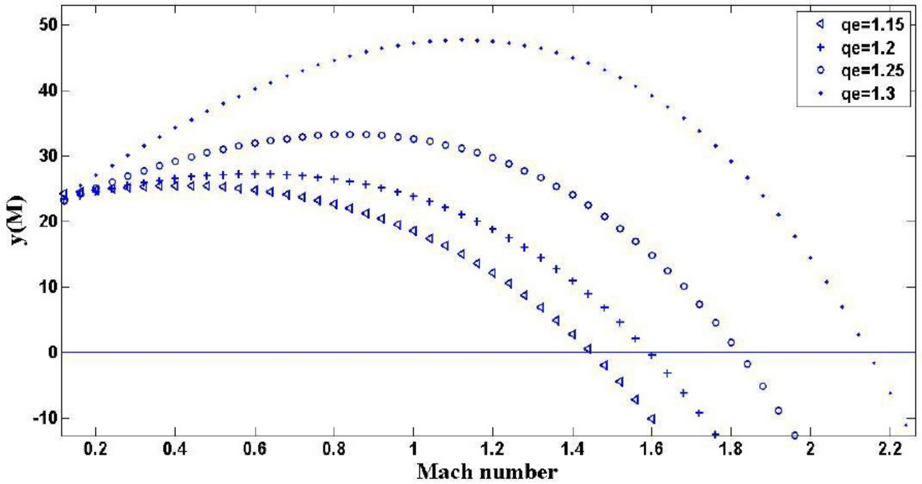

Figure 3: The variations of $y(M)$ function vs. Mach number with different q_e nonextensive parameter of electron, that $q_e = 1.15$ (triangular line), $q_e = 1.2$ (plural line), $q_e = 1.25$ (circular line) and $q_e = 1.3$ (dotted line) with $\delta = 5$ in the hydrogen plasma.

the Sagdeev potential, the necessity of negative the y function obtained, which leaded to the modified of the Bohm criterion condition and acceleration of ions into sheath region and the formation of an electrostatic sheath. The y function was a nonlinear function of Mach number and parameters of plasma. These parameters limited the initial velocity of the ions entering the electrostatic sheath region and the Mach number. This function showed that the modified Bohm criterion was heavily dependent on nonextensive degree of electron (q_e) and the characteristics of the plasma such as the ion density ratio to electron and kind of plasma gases. It showed that in ordinary plasmas, the condition of the electrostatic sheath is that the initial velocity of the ions entering this region be at least equal to or greater than the velocity of the ion-acoustic. Furthermore, it observed that in dusty plasmas, by increasing in the dust particle electric charge or density, the y function becomes negative for values greater than Mach number and the Mach number minimum become larger; thus, the Bohm criterion condition and the electrostatic sheath formation modified and the minimum ion velocity to accelerate into the sheath region increased. The results also showed that role of dust particle charge fluctuations was very important in changing the Coulomb attraction between ions and dust particles and thus those modified the Bohm criterion and the formation of electrostatic sheath. It indicated that as the mass of the ions increases, the y function becomes negative for larger Mach numbers and there is no linear relationship between increasing the mass ratio of ions to increasing the Mach minimum number. The results showed that as the mass of the ions increased, the velocity of the acoustic ion decreased and therefore the Mach number increased. On the other hand, we showed the large effect of the degree of electron nonextensive on the formation of the electrostatic sheath and the modification of the Bohm criterion. It is shown that the farther we go from the Maxwell equilibrium distribution function, the ions need a higher initial velocity to be able to separate from the main body of the plasma and move towards the plasma sheath and the wall and by increasing of the electron nonextensive degree, the y function becomes negative in values greater than Mach number. It also indicated that the rate of increase of the minimum Mach number was faster for higher degrees of electron nonextensivity. Finally, the results indicated that considering the dust particle charge fluctuations and the electron nonextensivity degree plays an important role on electrostatic sheath formation and modifies Bohm criterion.

References

[1] Chang, T. T. S. 2015, An introduction to space plasma complexity, Cambridge University Press.

[2] Fortov, V. E., & Morfill, G. E. 2009, Complex and dusty plasmas: from laboratory to space, CRC Press.

[3] Shukla, P. K., & Mamun, A. 2015, Introduction to dusty plasma physics, CRC press.

[4] Pierrard, V., & Lazar, M. 2010, Solar physics, 267, 153.

[5] Vasyliunas, V. M. 1968, J. Geophysical Research, 73, 2839.

[6] Swisdak, M. 2013, Physics of Plasmas, 20, 062110.

[7] Mott-Smith, H. M., & Langmuir, I. 1926, Physical review, 28, 727.

[8] Al'Pert, Y. L. 1966, American J. Physics, 34, 544.

[9] Rostoker, G. 1980, J. geomagnetism and geoelectricity, 32, 431.

[10] Willis, C., Coppins, M., Bacharis, M., & Allen, J. 2010, Plasma Sources Science and Technology, 19, 065022.

[11] Denra, R., Paul, S., & Sarkar, S. 2016, AIP Advances, 6, 125045.

[12] Abid, A., Ali, S., & Muhammad, R. 2013, J. Plasma Physics, 79, 1117.

[13] Nicolaou, G., *et al.* 2018, ApJ., 864, 3.

[14] Robertson, S. 2013, Plasma Physics and Controlled Fusion, 55, 093001.

[15] Leubner, M. 2004, Physics of Plasmas, 11, 1308.

[16] Myra, J. 2021, J. Plasma Physics, 87, 1.

[17] Lima, J., Silva Jr, R., & Santos, J. 2000, Phys. Rev. E, 61, 3260.

[18] Plastino, A., & Plastino, A. 1993, Physics Letters A, 174, 384.

[19] Tsallis, C. 1988, J. statistical physics, 52, 479.

[20] Tsallis, C., Prato, D., & Plastino, A. R. 2004, Ap & SS., 290, 259.

[21] Guo, L., & Du, J. 2011, Physica A: Statistical Mechanics and its Applications, 390, 183.

[22] Liu, S.-Q., & Chen, X.-C. 2011, J. Plasma Physics, 77, 653.

[23] Guo, L., & Du, J. 2011, Physica A: Statistical Mechanics and its Applications, 390, 183.

[24] Livadiotis, G., & McComas, D. 2009, J. Geophysical Research: Space Physics, 114, 1.

[25] Huang, K. 2009, Introduction to statistical physics, Chapman and Hall/CRC.

[26] Beck, C., Benedek, G., Rapisarda, A., & Tsallis, C. 2005, Complexity, metastability and nonextensivity, World scientific Perss.

[27] Silva, R., Alcaniz, J., & Lima, J. 2005, Physica A: Statistical Mechanics and its Applications, 356, 509.

[28] Lavagno, A., *et al.* 1996, arXiv preprint astro-ph/9607147.

[29] Riemann, K.-U. 1991, J. Physics D: Applied Physics, 24, 493.

[30] Breizman, B. N., & Kiramov, D. I. 2021, J. Plasma Physics, 87, 1.

[31] Azuma, S., Fukuyama, A., & Takizuka, T. 2012, Contributions to Plasma Physics, 52, 512.

[32] Sharifian, M., Sharifinejad, H., Zarandi, M. B., & Niknam, A. 2014, J. Plasma Physics, 80, 607.

[33] Asserghine, A., Missaoui, A., El Kaouini, M., & Chatei, H. 2022, Contributions to Plasma Physics, 62, e202100084.

[34] Taherimoghadam, M., Motie, I., Bakhshayeshi, A., & Mirzaye, T. 2019, Iranian J. Astronomy and Astrophysics, 6, 71.

[35] Gong, J., & Du, J. 2012, Physics of Plasmas, 19, 023704.

[36] Liu, S.-Q., & Chen, X.-C. 2011, J. Plasma Physics, 77, 653.

[37] Zwillinger, D., & Jeffrey, A. 2007, Table of integrals, series, and products, Elsevier.

[38] Gradshteyn, I. S., & Ryzhik, I. M. 2014, Table of integrals, series, and products, Academic press.