Relativistic Kappa Distribution Effects on Dust Charging in Critical Areas of Dusty Plasma

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Abstract. The process of dust grain charging with the help the orbit limited motion theory (OLM) and a kinetic model in the relativistic regime considers in the critical areas of the dust grain density when it is very low and vice versa. A relativistic kappa distribution for currents carried by ion and electron employs and the electrical potential of dust grain by the numerical analyses is calculated by using the relativistic cross-section. In the critical area when the dust grain density is high, it shows that by increasing the electron-to-ion temperature ratio, the relativistic effects increase and the electrical potential of dust grain decreases. Also, it indicates that in the critical areas, the Colombian force between the dust grain and the plasma particles play an important role and have great impact on the process of dust charging. Moreover, it indicates when the density of dust grain is high, as the relativistic effects increased, the dust charging process increases and at low dust grain density, as the relativistic effect increases, the charge of the dust grain decreases. It indicates that the electrical potential of the dust grain is much greater affected by the degree of electron nonextensive relative to the nonextensive degree of ion. Finally, it indicates when the dust grain density is high; the relativistic effects due to of increasing temperature play a more prominent role in the dust charging process, while at low dust grain density; the relativistic effects because of changing the energy of the rest mass play a more prominent role.

Keywords: dust charging, relativistic Kappa distribution, nonextensivity, relativistic effects, orbit limited motion theory (OLM).

1 Introduction

Cosmic, planetary and astrophysical plasmas consist of charged dust particles that have been proven by observation of spacecrafts and they are found in a wide range such as cometary tails, interstellar clouds, the Earth's mesosphere, ionosphere, Saturn's ring, Jupiter's gas ring, as well as in laboratory experiments [1]. These particles exist in different sizes and masses, and may be made of dielectric, metal, and ice particles [2] Investigations of the properties and process of dust grains have received particular attention in the last decade alone, particularly because of their implications for technology [1–5]. The Space plasmas are far from thermal equilibrium that known as Lorentzian dusty plasma and, the suprathermal electron and ion are often found in theirs [1,3,4]. In plasmas that are naturally observed in planetary magnetosphere and in the solar wind, the particle velocity distribution is non-Maxwellian [1,5]. The statistical behavior of space plasma is associated with their nature as collisionless, weakly coupled plasmas and also, particles in a Debye sphere. Weakly coupled plasma is governed by the overall collective electrostatic forces of many particles. The intense interactions between individual particles are relatively rare, thus space plasmas exhibit strong collective behavior that characterizes correlated particles in a Debye sphere, without the phenomenon of localization between individual particles due to interaction or collision. This behavior cannot be understood by classical statistical description of thermal equilibrium [6]. The kappa distributions have been utilized in increasingly numerous studies across the space plasma process, from solar wind and planetary magnetospheres to heliosheath, and beyond to interstellar and intergalactic plasmas, too. Recent observations from the Voyager spacecraft show that the particles in the outer heliosphere are well explained by kappa distribution [7]. Permeating dusty plasmas are far away from the equilibrium and can be created in the inter-penetrating solar/stellar wind that is dominated by electron-ion plasma in surrounding cometary plasma which consists of charged particles other than electrons and ions. Also, between the solar wind and the earths atmosphere, the permeating dusty plasma happens that superthermality effect is modeled by the kappa distribution function [8]. This distribution represents suprathermal deviations from Maxwell's equilibrium and is expected to occur in any low-density plasma in the universe, where the binary collision of charges is sufficiently rare [9]. The dust charging characteristics and the electrical potential of dust grains in Lorentzian plasma are different in terms of their properties in Maxwellian dusty plasma [1]. To understand the physics of this process, it is enough to know only the raw details of all states, statistical mechanics, and distribution functions [10, 11]. The first publication of the Kappa Distribution Studies begins in 1968 and relates to Vasliana's research work [10]. The Boltzmann-Gibbs statistical mechanics cannot adequately describe much space plasma, which is not equilibrium system [11,12]. In contrast, Tsallis's statistical mechanics, that is nonextensive entropy, provide theoretical foundations for the description and analysis of complex off-equilibrium systems [13,14]. The origin of the kappa distribution is in Tsallis statistical mechanics that it has already been examined by several authors and whenever we put q = 1 + 1/k, (The q parameter is as the degree of system nonextensivity in Tsallis and k parameter is as the degree of system nonextensivity in kappa) we get the kappa distribution [15].

Furthermore, whenever $k \to \infty$, then the Kappa distribution is transformed to the Maxwellian distribution [13,16]. Lubner has shown that Kappa distribution function is the result of nonextensive entropy concepts, accounting for the long-range forces in space plasmas and the k parameter is as the degree of system nonextensivity. This distribution function is one of the good examples of non-Maxwellian distribution [5]. Two definitions of kappa distributions currently dominate that are different in their kappa indices and temperaturelike parameters and one is used less than the other [15]. When some or all of the plasma particles move faster than their thermal velocities, Maxwellian's usual distribution becomes inadequate for the study of superthermal particles. Under such conditions, the kappa distribution can be used to model plasma behavior. Spacecraft observations have confirmed the existence of superthermal particles in near-Earth and laboratory plasma [17]. The hot and non-collisional particles with energies of hundreds of keV or above have been found to play an important role in the dynamics of plasmas. High-energy electrons have been found in the Earth outer radiation belt, damaging the functioning of the spacecraft [18,19]. Much research has been done to explain the collective interactions in plasmas by using the kappa distribution function [20-22]. In a research work, the Debye length in plasma is obtained with the kappa distribution function and the results are compared with the Debye length with the Maxwell distribution. The results show that the Debye length in plasma with kappa distribution is shorter than the Debye length with Maxwell distribution and the difference between these two Debye lengths is strongly dependent on k parameter [23]. In another research, G. Nicolaou et al. worked on the interpretation of the kappa distribution function in space plasmas and came to good agreement by examining various parameters such as temperature and k parameter and comparing the results with experimental values [3]. Also I. Lourek and M. Tribeche worked on the process of dust grain charging in non-equilibrium plasma starting from the kappa distribution function and generalizing to Kaniadakis statistics for electrons and the Maxwellian distribution function for ions [24]. In this study, we focus on the consequence the process of dust grains charging and calculating the electrical potential of dust grains by considering Orbital-Limited Motion Theory (OLM). This theory (OLM) is the most commonly used theory of dust charging that is important point in understanding dusty plasmas and can deduce the surface potential of a dust grain [2]. It was initially proposed by Langmuir and Mat Smith [25], and completed in the 1960s [26,27]. In 1994, Barkan et al. considered the theory of empirical dust charging phenomena resulting from the bombardment of Maxwellian electrons and positive ions to theoretically and empirically investigate the dust grain surface potential [11]. Although this theory uses simple concepts such as angular motion and collision cross-section, it predicts the dust electrical potential with acceptable accuracy for a wide range of grain sizes [28–34]. We shall explore the effects of different values of on various characteristics of the plasma. Here, electron and ion velocities are assumed to follow generalized Kappa distribution as plasma under our consideration is a Lorentzian dusty plasma. It is assumed that the dust potential is negative because the mass of the ion is very larger than the mass of the electron and the electron reaches the dusty dust faster than the ion. It has been realized that the particle velocity distribution function can change from the non relativistic to relativistic behavior in some space and laboratory plasmas. In solar physics, and around of the earth orbit it is observed that velocities distribution functions are isotropic approximately from thermal energies to modestly relativistic energies [35,36]. In the range velocities v/c > 0.1 a plasma can be defined as a relativistic plasma [36]. Thus, the relativistic Kappa distribution for dust grain charging currents in plasma is considered. The general relativistic kappa distribution is introduced and developed by Xiao that modeled the highly energetic particles in plasmas where magnetic mirror geometries occur [37]. The first, We obtain dust grain electrical potential using a kinetic model and Quasi-neutrality condition of plasma and as a result investigate the effects of the types plasma gas and degree of nonextensive $(k_e \text{ and } k_i)$ in relativistic regime on dust grain charging process. Due to the complex behavior of dusty plasmas, the results have been analyzed and compared in two critical areas with very high and very low dust densities. Numerical study in this case shows that in Lorentzian dusty plasma, the electrical potential of dust grains behave very differently at very low and very high densities. This is a complex phenomenon and the key is to understand different phenomena in astrophysical plasmas. some situations such as in the Van Allen radiation belts, in interstellar medium, space plasma, in earth magnetosphere, where the velocity of ion and electron species becomes approximately similar to the speed of light, the consideration of relativistic behavior of plasma has significant role in modifying the nonlinear structures. In addition, in the Kappa statistic, another important issue is determining the amount of k that is non-extensivity degree of system and one of the open issues in Kappa statistics [38,39]. The article is presented in the following fashion: In section 2, the dust grain charging process by relativistic Kappa distribution function and the relativistic cross-section are formulated on OLM theory and the relativistic electron and ion currents on dust grain are obtained. In section 3, the numerical solved of nonlinear equation is presented and results are discussed. The influence of relativistic effects on the dust grain electrical potential by the changing of parameters such as degree of nonextensive k and types gas plasma are investigated. The results are analyzed in two very critical zones; very high and very low of dust densities. Finally, a summary and conclusions are given in Section 4.

2 Basic equations and charging process

In this section, to determine the dust charging process, we start from the collision crosssection of the dust grain in relativistic dusty plasma. We assume that plasma is relativistic non-magnetic and it is containing electrons, ions and dust particles as a spherical dust grain of radius r_d and charge q_d . The potential of dust grain is $\varphi_d = q_d/r_d$ and the Debye length is very larger than dust grain of radius. The relativistic cross-section is as $\sigma = \pi b_i^2$ that b_j is impact parameter between the dust and the plasma particle j which j is electron or ion (j = e or i). In the Ref [40], we have considered the relativistic CLM theory and shown that by the conservation of momentum and energy in relativistic regime, the cross section is

$$\sigma = \pi r_d^2 \left(1 - \frac{\sqrt{1 - \beta_j^2}}{\beta_j} u_j\right)^2,\tag{1}$$

$$u_j = \frac{q_j \varphi_d}{m_j c^2},\tag{2}$$

where u_j means ratio of potential energy to rest energy, $\beta_j = v_j/c$ is relativistic factor, v_j is speed of the plasma particle before its grazing collision with the dust grain, c is velocity of light and mj and q_j are rest mass and charge of plasma particle, respectively. Based on OLM theory, when both the ion and the electron currents are equal and dust grains have a negative charge, then the currents are resulted from incident of plasma particles with the charge q_j on dust grain as

$$I_j = q_j \int\limits_{v_j^{\min}}^c v_j \sigma_j^d f_{\kappa_j}(v_j) dv_j,$$
(3)

In this equation, $f_{\kappa_i}(v_j)$ is the relativistic Kappa distribution that can be written as [5,15,41]

$$f_{\kappa_j}(v_j) = \frac{n_j}{4\pi m_j^3 \Lambda} \left[1 + \frac{1}{\kappa_j} \frac{\varepsilon_j}{k_B T_j}\right]^{-k_j},\tag{4}$$

and

$$\Lambda_j = \int_0^{v_j} \left[1 + \frac{1}{k_j} \frac{\varepsilon_j}{k_B T_j}\right]^{-\kappa_j} v_j^2 dv_j,\tag{5}$$

where k_j is nonextensive parameter, $\varepsilon = \gamma m_j c^2$ is relativistic energy, $\gamma = (1 - \beta_j^2)^{-1/2}$ is relativistic factors, $\alpha_j = m_j c^2 k_B T_j$ is temperature parameter, k_B is Boltzmann constant, T_j and n_{0j} denote respectively temperature and density of plasma particle. Also, in relativistic regime $v_j^{\min} \leq v_j \leq c$ and there are two situations for considering to approximate v_j^{\min} , attractive potential and repulsive potential. In an attractive potential, the plasma particle and the dust grain attract each other; then, the initial velocity is not required for the incident particles but repulsive potential the plasma particle and the dust grain repel each other and hence the existence of an initial velocity v_j^{\min} is necessary. Thus, in this case, v_j^{\min} becomes

$$v_j^{\min} = c_v \sqrt{1 - \frac{1}{u_j^2}}.$$
 (6)

The equation (3) can be calculated using, integration by parts, incomplete Beta function and Hyper Geometric functions [42,43] and combination methods. Thus, calculations for currents carried by ions and electrons in non-equilibrium plasma are as follows

$$I_j = -r_d^2 n_j L_j [1 + G_j u_j^2], (7)$$

where L_j and G_j are respectively (which j is electron or ion (j = e or i), as

$$L_j = \frac{(-1)^{-k_j}}{k_j^4} \left(\frac{\alpha_j^2 \pi e x}{m_j \Lambda_j}\right) (k_j^2 F_j - F_j''),\tag{8}$$

$$G_j = -\frac{2F'_j - F''_j}{k_j^2 F_j - F''_j} k_j,$$
(9)

and in these

$$\Lambda_{j} = -\frac{k_{j}}{2\alpha_{j}} \left[-\frac{2}{3} \left(\frac{k_{j}}{\alpha_{j}} \right)^{2} + (1+k_{j}) \left(1 + \left((\frac{4}{3} \left(\frac{k_{j}}{\alpha_{j}} \right)^{2} - 1) + (2+k_{j}) (\frac{10}{3} \left(\frac{k_{j}}{\alpha_{j}} \right)^{2} - 1 \right) \right) \right]$$
(10)

$$F_j = \left[\left(\frac{\pi k_j (1+k_j)}{2\sin(\pi k_j)} \right) - \frac{1}{1-k_j} \left(\sum_{n=0}^{\infty} \frac{1-k_j}{1-k_j+n} \frac{(-k_j+n-2)!}{(-k_j-2)!} \right) \right],\tag{11}$$

$$F'_{j} = \alpha_{j} \left[\left(\frac{\pi k_{j} (1+k_{j})(2+k_{j})}{6\sin(\pi k_{j})} \right) - \frac{1}{1-k_{j}} \left(\sum_{n=0}^{\infty} \frac{1-k_{j}}{1-k_{j}+n} \frac{(-k_{j}+n-3)!}{(-k_{j}-3)!} \right) \right],$$
(12)

$$F_j'' = \alpha_j^2 \left[\left(\frac{\pi k_j (1+k_j)(2+k_j)(3+k_j)}{24\sin(\pi k_j)} \right) - \frac{1}{1-k_j} \left(\sum_{n=0}^{\infty} \frac{1-k_j}{1-k_j+n} \frac{(-k_j+n-4)!}{(-k_j-4)!} \frac{1}{n!} \right) \right].$$
(13)

In the equations (7)-(13) when j = e those are in the case a repulsive potential $(q_j \varphi_d > 0)$ and j = i those are in the case an attractive potential $(q_j \varphi_d < 0)$. We assume that the system is neutral from almost an electrical point of view. The quasi-neutrality condition of the system as follows

$$\frac{n_e}{n_i} = 1 - Z_d \frac{n_d}{n_i},\tag{14}$$

where n_d and Z_d are the density and the number of charges of the dust grain and it is common in literature that the parameter is defined to consider this ratio; it's in relativistic regime as

$$P = \frac{m_e c^2 r_d n_d}{e^2 n_i}.$$
(15)

Now, as the final step for evaluating the dust grain charging process and the electrical potential of dust grain, we have to use equation (7) for electrons and ions with owing to the fact that $I_i + I_e = 0$, we can obtain

$$u_e^3 + \frac{1}{P} \left(1 - \frac{L_i G_i m_e^2}{L_e G_e m_i^2} \right) u_e^2 + \frac{1}{G_e} u_e + \frac{1}{G_e P} \left(1 - \frac{L_i}{L_e} \right) = 0.$$
(16)

This equation shows that the electrical potential of the dust grains depends on the P parameter in relativistic regime. From equation (16), it is clear that this equation is very nonlinear and has no analytical solutions. Thus, we analyze the above equation using two critical areas when the P parameter is too large the equation (16) will be as follow

$$u_e = -\sqrt{\frac{1}{G_e}},\tag{17}$$

and this equation will be as

$$u_{e} = -\sqrt{\frac{L_{e} - L_{i}}{L_{e}G_{e} - \frac{m_{e}^{2}}{m_{i}^{2}}L_{i}G_{i}}},$$
(18)

when P parameter is very small. Equations (17) and (18) show that when the P parameter is very large or very small, the electric potential of dust grain is independent of the P parameter, and is a function of the electron and ion temperatures as well as the k nonextensive parameter in relativistic regime. Now, using boundary conditions and numerical methods, the electric potential of the dust grain in plasma can be obtained in terms temperature and mass of plasma particles.

3 Results and discussion

From equations (17) and (18), it is cleared that the electrical potential of dust grain can change under the action of charging. These equations show that in relativistic regime, the dust grain electrical potential is heavily dependent on nonextensive degree of system (k_i) and also, the characteristics of the plasma such as the ions and electrons temperatures and kind of plasma gases. In present section, the analytical solution of nonlinear equations (17) and (18) are provided with the effects of nonextensive parameters k_i and k_e , temperature ratio of electron-to-ion and kind of plasma gas on dust grain electrical potential in relativistic regime for in two critical areas. In Figure 1 (a) and (b), the effect nonextensive degree of electron in relativistic regime on the dust grain electrical potential for in two critical areas $P \to \infty$ and $P \rightarrow 0$ are presented as a function of the ratio of temperature of electron-to-ion, respectively. These curves are plotted for the different values of $k_e = 4$ (solid line), $k_e = 5$ (dashed line), $k_e = 6$ (dotted line) and $k_e = 7$ (dash dotted line) in relativistic regime when nonextensive degree of ion $k_i = 4$ and the plasma gas is helium. It is shown from Figure 1 (a) that by increasing electron temperature to ion in high dust grain density or low ion density which shows that the relativistic effects increase, the electrical potential of dust grain is decreased. As the ratio temperature of electron-to-ion increases, the velocity of the electrons relative to the ions increases as a result the electron current increase and on the other hand because of high dust grain density, the repulsive force between dust grain and electrons is increased and thus low electrons attach to dust grain and electrical potential of dust grain decrease. Furthermore, it is indicated from Figure 1 (b) that by increasing electron temperature to ion in low dust grain density or high ion density, the electrical potential of dust grain is increased. But as the temperature ratio of electron-to-ion increases further, the electrical potential of dust grain will remain constant and independent of temperature. By increasing in temperature ratio of electron-to-ion, as the electron current increases due to the decrease in the density of the dust grain, the repulsive force between the dust grain and the electrons also decreases, so that more electrons reach the dust grain, thereby increasing the electrical potential. Also, Figure 1 (a) shown that in high dust grain density or low ion density by increasing temperature ratio of electron-to-ion, the effect nonextensive parameter k_e on the dust grain electrical potential is becomes more prominent. On the other hand, the dust grain electrical potential is not any affected from nonextensive parameter k_e by increasing temperature ratio of electron-to-ion in low dust grain density or high ion density. It is also consistent with Figure 1 (b) that as nonextensive degree of electron k_e increases, the impact of the dust grain electrical potential from temperature changes decreases rapidly. So, when the density of dust grains is high, as the relativistic effects increase, the dust charging process and also the electrical potential of dust grain increase. And conversely, at low dust densities, as the relativistic effects increase, the electric potential and charge of the dust grain decrease. Figure 2 shows the variations of the electrical potential of dust grain vs. The ratio of temperature of electron-to-ion in two critical areas $P \to \infty$ in Figure 2 (a) and $P \rightarrow 0$ in Figure 2 (b) with the different values of nonextensive parameters k_i in helium plasma gas. In these figures, the curves are plotted for the different nonextensivity of ion

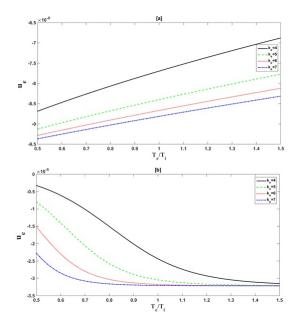
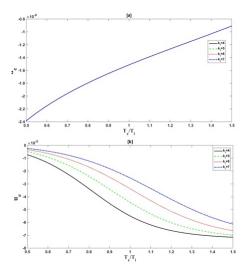


Figure 1: The variations of the dust electrical potential vs. temperature ratio of electron-toion with different e nonextensive parameter of electron when (a) P parameter is very large and (b) P parameter is very small, that $k_e = 4$ (solid line), $k_e = 5$ (dashed line), $k_e = 6$ (dotted line) and $k_e = 7$ (dash dotted line) with $k_i = 4$ in the helium plasma.

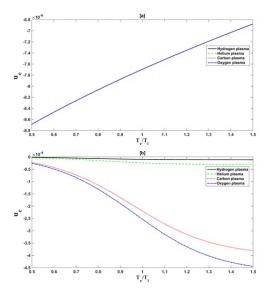
in relativistic regime that $k_i = 4$ (solid line), $k_i = 5$ (dashed line), $k_i = 6$ (dotted line) and $k_i = 7$ (dash dotted line) in relativistic regime when nonextensive degree of electron $k_e = 4$. Figure 2 (a) shows that in high dust grain density as the ratio of temperature of electronto-ion increases, the dust grain electrical potential is increased, similar Figure 1 (a) and in contrast to it, the nonextensive degree of ion k_i not any effect on the dust grain electrical potential. Moreover, from Figure 2 (b) indicated that by increasing ratio of temperature of electron-to-ion in low dust grain density, the dust grain electrical potential is increased. Also, it is found from Figure 2 (b) that as ion nonextensivity degree rises, bigger electrical potential for dust grain occurs at plasmas with higher ratio of temperature electron-to-ion. It is also indicated from this figure that as nonextensive degree of ion k_i decreases, the impact of the dust grain electrical potential from temperature changes decreases rapidly. In addition, from Figure 2(b) it is evident that within the confines of isothermal plasma (means of $T_e \approx T_i$), the effect of the degree of nonextensive of ion on the electrical potential of the dust grains is much more severe in low dust grain density. From the comparison of Figures 1 and 2, it is indicated that the electrical potential of the dust grains is much greater affected by the degree of electron nonextensive relative to the nonextensive degree of ion. We also know that as the kappa increases, the kappa distribution function approaches the Maxwell distribution function. According to the Figures 1 and 2, it is cleared that when the ion degree of extensive is constant, as the electron nonextensive parameter increases, the electrons move closer to the Maxwell equilibrium distribution and the electrical potential of the dust grains is increased in low dust grain density and it is decreased in high dust grain density. Conversely, when the degree of electron nonextensive is constant, with increasing ion degree of nonextensive, the ions move closer to the Maxwell equilibrium distribution



and the electrical potential of the dust grains is reduced in high dust grain density but it is unchanged at low dust grain density.

Figure 2: The variations of the dust electrical potential vs. temperature ratio of electronto-ion with different k_e nonextensive parameter of electron when (a) P parameter is very large and (b) P parameter is very small that $k_e = 4$ (solid line), $k_i = 5$ (dashed line), $k_i = 6$ (dotted line) and $k_i = 7$ (dash dotted line) with $k_e = 4$ in the helium plasma.

The influence of kinds of plasma gasses on the dust electrical potential is presented as a function of temperature ratio of electron-to-ion in Figures 3 that the curves are plotted for four of plasma gases include of hydrogen plasma (solid line), helium plasma (dashed line), carbon plasma (dash dotted line) and oxygen plasma (dotted line). The results are obtained for a dusty plasma in relativistic regime with nonextensivity of degree $k_e = 4$ and $k_i = 3$ for electron and ion in two critical areas of dusty plasma. Figure 2 (a) and Figure 3 (b) are presented in two positions of the P parameter very large and very small, respectively. It is cleared from the Figure 3 (a) that in relativistic regime and at high dust grain densities, the mass and type of plasma gas have no effect on the dust charge processing and the electrical potential of the dust grain. Furthermore, from Figure 3 (b) it is indicated that by increasing mass of ions, the electrical potential of dust grain is increased as ratio of temperature electron-to-ion is enhanced. As the mass of the ion increases, the energy of rest mass in relativistic regime increases and according to equations (1) and (2), the cross-section of the ion and dust grain increases, resulting in an increase in ion current. On the other hand, when the density of dust grain is high, the repulsion force between the dust particles and the electrons is also high, resulting in a decrease in the electron current. As a result, the effect of the Colombian repulsion and the mass of the plasma ions in the grain processing neutralize each other. Also, when the dust grain density is low, the repulsion force between the dust particles and the electrons also decreases, resulting in an increased electron current. As a result, the effect of the Colombian repulsion and the mass of the plasma ions in the grain processing enhance the charge of dust grain. That is, the higher the mass of the ions, the greater the charge and thus the potential for dust grains. Thus, when the dust grain density is high, the relativistic effects of increasing temperature play a more prominent role in the dust charging process, while at low dust grain densities; the relativistic effects because



of changing the energy of the rest mass will play a more prominent role.

Figure 3: The variations of the dust electrical potential vs. temperature ratio of electron-toion for different kinds of plasma gases in temperature ratios $T_i/T_e = 1$ in relativistic regime that hydrogen plasma (solid line), helium plasma (dashed line), carbon plasma (dash dotted line) and oxygen plasma (dotted line) with nonextensivity degree when (a) P parameter is very large and (b) P parameter is very small with $k_e = 4$ and $k_i = 3$.

4 Summary and Conclusion

In this work, with the help OLM theory and a kinetic model in relativistic regime, the process of dust grain charging considered in critical areas of dust grain density when it was very low and vice versa. It assumed that the currents carried by electrons and ions using the relativistic cross-section are causes dust grain charging process. A relativistic kappa distribution for currents carried by ions and electrons was employed and the electrical potential of dust grain by numerical analyses was calculated. In critical area of high dust grain density, it shown that by increasing ratio of temperature electron-to-ion, the relativistic effects increased and the electrical potential of dust grain decreased. Also, it indicated that in critical areas, the Colombian force between the dust grain and the plasma particles played an important role and had a great impact on the process of dust charging. Furthermore, in critical area of low dust grain density indicated when the electron temperature was bigger than ion temperature, the electrical potential of dust grain was will independent of temperature. Also it shown that the effect nonextensive parameter k_e on the dust grain electrical potential became more prominent in high dust grain density by increasing temperature ratio of electron-to-ion. Although, by increasing temperature ratio of electron-to-ion in low dust grain density, the dust grain electrical potential was not any affected from nonextensive parameter k_e . Moreover, it was indicated when the density of dust grains is high, as the relativistic effects increased, the dust charging process and also the electrical potential of dust grain increased. And conversely, at low dust densities, as the relativistic effects increased,

the electric potential and charge of the dust grain decreased. It shown that in high dust grain density, the nonextensive degree of ion k_i not any effect on the dust grain electrical potential. Also, it found that as ion nonextensivity degree ridded, bigger electrical potential for dust grain occurred at plasmas with higher ratio of temperature electron-to-ion. In addition, it is evident that within the confines of isothermal plasma, the effect of the degree of nonextensive of ion on the electrical potential of the dust grains was much more severe in low dust grain density. It indicated that the electrical potential of the dust grains was much greater affected by the degree of electron nonextensive relative to the nonextensive degree of ion. It was found that by increasing the degree of electron nonextensive, the electrons moved closer to the Maxwell equilibrium distribution and the electrical potential of the dust grains increased in low dust grain density and vice versa in high dust grain density. On the other hand, with increasing ion degree of nonextensive, the ions moved closer to the Maxwell equilibrium distribution and the electrical potential of the dust grains reduced in high dust grain density but it unchanged at low dust grain density. It was made clear that in relativistic regime and at high dust grain densities, the mass and type of plasma gas had no effect on the dust charge processing and the electrical potential of the dust grain. Furthermore, it shown that by increasing mass of ions, the electrical potential of dust grain increased as ratio of temperature electron-to-ion was enhanced. Finally, it was indicated when the dust grain density was high, the relativistic effects of increasing temperature played a more prominent role in the dust charging process, while at low dust grain densities the relativistic effects because of changing the energy of the rest mass played a more prominent role.

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