Iranian Journal of Astronomy and Astrophysics
Vol. 8, No. 1, Spring 2021
©Available online at http://journals.du.ac.ir
DOI:10.22128/ijaa.2021.514.1114

Iranian Journal of Astronomy and Astrophysics

Heating and Cooling Studies of Relativistic Accretion Disks Around the Non-Rotating Black with A Simple Model

Mahboobe Moeen Moghaddas
Department of Sciences, Kosar University of Bojnord, Bojnord, Iran;
email: Dr.moeen@kub.ac.ir

Abstract. In this paper, the relativistic accretion disks around the non- rotating black holes are studied. In these disks, we study the fluids with influences of stress viscosity and heat flux in the ignorable magnetic field. In this paper, the reformed conservation equations with the influences of heat flux are used. We use the simple model of heat flux, in which the heat flux is proportional to temperature. Then, the thermodynamic quantities are derived. We use the radial form for the radial component of four velocity in the locally non rotating frame and Keplerian angular momentum. So, the viscous heating and heat flux cooling are derived. Also, the figures of viscous heating and heat flux cooling are derived by various parameters such as the viscous coefficient and the thermal conductivity. Therefore, the energy balance can be study in the relativistic accretion disks. So, we see the region which have the energy balance.

Keywords: Relativistic fluids, influences of relativistic heat flux, energy balance.

1 Introduction

The relativistic accretion disks around the black holes were studied in many works. Various processes have been considered for energy and momentum transfering, some important mechanisms are perfect fluid, viscosity, and magnetic field effects (for example [1], [4], [11], [9] and [7]). But the influences of heat flux were less studied. But, heat flux were studied with the simple models in some papers such as [2].

In this paper, we consider the relativistic fluids with heat flux around the non-rotating black holes. We use the radial model to calculate four velocity, shear tensor, heat flux, etc. Then, viscous heating and heat flux cooling of fluids are derived. So, we can examine the energy balance of the relativistic disks.

2 Metric and scaling

Similar to [4], [11]. we use the M=G=c=1 for scaling (M is the mass of a black hole, G is the gravitational constant and c is the speed of light). This relativistic disk is around the non-rotating black holes, so, we use the Schwarzschild metric in a spherical coordinate system around a non-rotating black hole, wich is shown as

$$-d\tau^2 = g_{tt}dt^2 + g_{rr}dr^2 + g_{\theta\theta}d\theta^2 + g_{\phi\phi}d\phi^2. \tag{1}$$

The non-zero components of Schwarzschild metric in this scaling are as follows

$$g_{tt} = -(1-\frac{2}{r}), \qquad g_{rr} = (1-\frac{2}{r})^{-1}, \qquad g_{\theta\theta} = r^2, \qquad g_{\phi\phi} = r^2 sin^2\theta. \tag{2}$$

3 Relativistic Energy- momentum tensor

The energy-momentum tensor of relativistic non-magnetic viscous fluids with heat flux is given by [10]

$$T^{\mu\nu} = \rho u^{\mu} u^{\nu} + p g^{\mu\nu} + t^{\mu\nu} + q^{\mu} u^{\nu} + u^{\mu} q^{\nu}, \tag{3}$$

where, ρ, is the density, u^{ν} is four-velocity, p is the pressure, $t^{\mu\nu}$ is the shear stress viscosity and q^{ν} is the heat-ux four-vector. The shear stress tensor (by zero bulk viscosity) and the heat flux are given by

$$t^{\mu\nu} = -2\lambda\sigma^{\mu\nu}, \tag{4}$$

$$q^{\mu} = -\kappa h^{\mu\nu}(\partial_{\nu} T + T a_{\nu}), \tag{5}$$

where, in above equation λ, κ, T, and $\sigma^{\mu\nu}$ are the dynamical viscosity coefficient, thermal conductivity, temperature and the shear tensor respectively. Also, the relations of the shear tensor and the other related variables are given by ([6])

$$\begin{aligned}
&\sigma^{\mu\nu} = \frac{g^{\mu\alpha} g^{\nu\beta}}{2}(u_{\alpha;\beta} + u_{\beta;\alpha}) + \frac{1}{2}(a^{\mu} u^{\nu} + a^{\nu} u^{\mu}) - \frac{1}{3}\Theta h^{\mu\nu} \\
&u^{\nu}_{;\mu} = \partial_{\mu} u^{\nu} + \Gamma^{\nu}_{\mu\gamma} u^{\gamma}, \\
&h^{\mu\nu} = h^{\nu\mu} = g^{\mu\nu} + u^{\mu} u^{\nu}, \\
&\Theta = u^{\gamma}_{;\gamma} = \partial_{\gamma} u^{\gamma} + \Gamma^{\nu}_{\gamma\nu} u^{\gamma}, \\
&a^{\mu} = u^{\mu}_{;\gamma} u^{\gamma}, \\
&a_{\nu} = g_{\nu\mu} a^{\mu}.
\end{aligned} \tag{6}$$

4 Basic conservation equations

The basic conservation equations of relativistic fluids are included, mass conservation equation, momentum conservation equation and energy conservation equation. The mass conservation equation is as follows

$$-4\pi H_{\theta} \rho u^{r} r^{2} = \dot{M}, \tag{7}$$

where H_{θ} and $\dot{M}$ are the half thickness and mass-accretion rate. Also, the momentum conservation equations and the energy equation are derived by inserting the influences of heat flux in the conservation equations of [9] and [11] as

$$\begin{aligned}
&\dot{M}\eta u_{\phi} - 4\pi H_{\theta} r^{2}(t^{r}_{\phi} + q^{r} u_{\phi} + u^{r} q_{\phi}) = \dot{M} j, \\
&4\pi H_{\theta} r^{2}((p + \rho + u) u_{t} u^{r} + t^{r}_{t} + q^{r} u_{t} + u^{r} q_{t}) = \dot{E},
\end{aligned} \tag{8}$$

$$u^{r}\left(\frac{du}{dr} - \frac{u + p}{\rho}\frac{d\rho}{dr}\right) = q^{+}_{vis} - q^{-}_{rad}. \tag{9}$$

In above equations, $\eta = \frac{\rho + P + u}{\rho}$, $\dot{M} j$, and $\dot{E}$ are the relativistic enthalpy (u is the internal energy), the total inward flux of the angular momentum and actual rate of change of the black hole mass respectively. We use the $\dot{M} = \dot{E} = 1$ in the above equations ([4] and [11]) Also, q^{+}_{vis} is viscous heating rate and q^{-}_{rad} is radiative cooling rate which are as follows

$$\begin{aligned}
&q^{+}_{vis} = t^{\mu\nu}\sigma_{\mu\nu} = 2\lambda\sigma^{\mu\nu}\sigma_{\mu\nu}, \\
&q^{-}_{rad} = -q^{\mu}_{;\mu} - q^{\mu} a_{\mu}.
\end{aligned} \tag{10}$$

5 Solving conservation equations

We use the relativistic state equation $p = \rho T$. For inertial energy, $u = \rho T\frac{15T+6}{5T+4}$ is used ([4]); so, we assume $u \simeq 3\rho T$. By inserting $H_\theta = -\frac{1}{4\pi\rho u^r r^2}$ (from equation (7)) in equation (8) we have

$$\begin{aligned}(1+4T)u_\phi + \frac{1}{\rho u^r}(t^r_\phi + q^r u_\phi + q_\phi u^r) &= j,\\ \rho(1+4T)u_t u^r + t^r_t + q^r u_t + q_t u^r &= -\rho u^r. \end{aligned} \tag{11}$$

5.1 Four velocity

Similar to [7], the radial form is used for the radial component of velocity in the locally non-rotating frame which is given by

$$u^r_{LNRF} = -\frac{\beta}{r^n}, \tag{12}$$

where β and n are positive and constant quantity. By using the Keplerian angular momentum ($\Omega = \frac{u^\phi}{u^t} = r^{-3/2}$), the component of four velocity in LNRF is derived as

$$(u^t, u^r, u^\theta, u^\phi)_{LNRF} = \left(\frac{r^{3/2}}{r^n}\sqrt{\frac{r^{2n}+\beta^2}{r^3-1}}, -\frac{\beta}{r^n}, 0, \frac{r^3}{r^n}\sqrt{\frac{r^{2n}+\beta^2}{r^3-1}}\right). \tag{13}$$

The Components of four-velocity by the transformation equation ([9]) are derived as([7])

$$(u^t, u^r, u^\theta, u^\phi) = \left(\frac{\sqrt{r(r^{2n}+\beta^2)}}{r^n\sqrt{r-3}}, -\frac{\beta\sqrt{r^2-2r}}{r^{2n+1}}, 0, \frac{\sqrt{r^{2n}+\beta^2}}{r^{n+1}\sqrt{r-3}}\right). \tag{14}$$

The components of four acceleration, projection tensor, and shear tensor have been calculated with above four-velocity.
We assume $q^\mu = -\kappa h^{\mu\nu} T a_\nu$ and $q_\mu = -\kappa h^\nu_\mu T a_\nu$ in equation (11); then, the thermodynamic variables such as density, pressure, temperature, and etc. are derived.

6 Heating and cooling

The critical dynamical coefficient of viscosity ([11]) is used in equation (10) to calculate the viscous heating of relativistic disks. So, the Figure 1 shows the heat generated by viscosity.

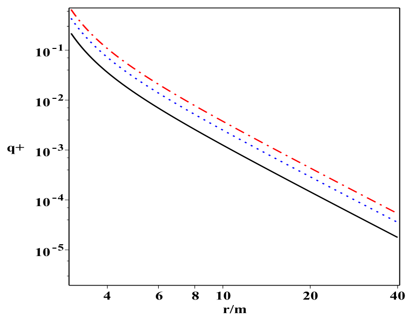

Figure 1: Viscous heating for $n = -1/2$, $j = 3$, and $k = 1$. Solid black curve is for $\lambda_{crit} = 1$, doted blue curve is for $\lambda_{crit} = 2$ and dash-dot red curve is for $\lambda_{crit} = 3$.

To calculate the cooling, in equation (10), we first calculate the covariant derivative of heat flux; so, in Schwarzschild metric, we have

$$q^{\mu}_{;\mu} = \frac{dq}{dr} + q^r(\Gamma^{\theta}_{t\theta} + \Gamma^{\phi}_{t\phi}) = \frac{dq^r}{dr} + q^r\frac{2}{r}. \tag{15}$$

Similarly, $q^{\mu}a_{\mu}$ has been calculated. Then, the figure of heat flux cooling is shown in Figure 2.

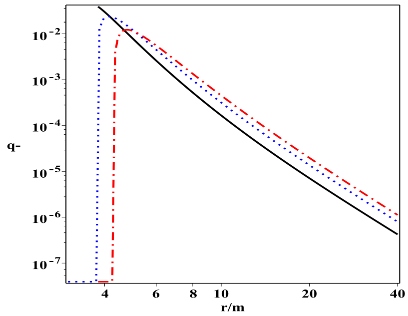

Figure 2: Heat flux cooling for $n = -1/2$, $j = 3$, and $\lambda_{crit} = 1$. Solid black curve is for $k = 1$, doted blue curve is for $k = 2$ and dash-dot red curve is for $k = 3$.

Finally, the energy balance can be studied by checking the net heat energy of fluid from the equation (9). So, we calculate the pure energy by $q = q^{+}_{vis} - q^{-}_{rad}$, the Figure 3 shows the net energy of fluids in three values of thermal conductivity.

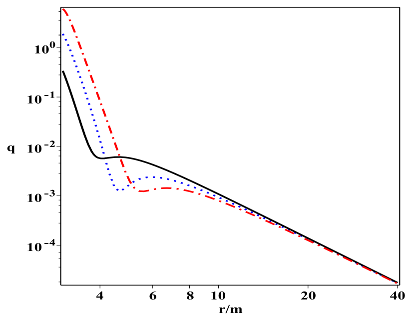

Figure 3: Pure energy for $n = -1/2$, $j = 3$, and $\lambda_{crit} = 1$. Solid black curve is for $k = 1$, doted blue curve is for $k = 2$ and dash-dot red curve is for $k = 3$.

7 Conclusion

In this paper, we calculate the viscous heating and heat flux cooling of relativistic accretion disks. So, we used the radial model for the radial component of velocity in the locally non-rotating frame and the Keplerian form for the angular momentum. Therefore, the calculations show that heating and cooling are much more effective near the black hole. Also, we study the energy balance of viscous fluids with the influences of the heat flux around the non-rotating black holes. We see that, there is no equilibrium with the heat generating and cooling around the event horizon. But, in the outer layers of these relativistic fluids, there is the thermal equilibrium, so the assumption of thermal balcnce for the outer layers of the fluids around the non-rotating black holes will be a good assumption.

Acknowledgments

This research was supported by the grant of Kosar University of Bojnord with the grant number of NO.0008151345".

References

[1] Abramowicz, M. A., Chen, X., Granath, M., & Lasota, J. P. 1997, ApJ., 471, 762.

[2] Compere, G., & Oliveri, R. 2017, Mon. Not. R. Astron. Soc., 468, 4351.

[3] Eckart, C. 1940, Phys. Rev., 58, 919.

[4] Gammie, C. F., & Popham, R. 1998, ApJ., 498, 313.

[5] Mihalas, D., & Mihalas, BW. 1984, Foundations of Radiation Hydrodynamics, Oxford University Press.

[6] Misner, C. W., Thorne, K. S., & Wheeler, J. A. 1973, Gravitation, Freeman, San Francisco.

[7] Moeen, M. 2018, Acta Physica Polonica B, 49, 1445.

[8] Moeen, M. 2017, IJAA., 4, 205, arXiv:1712.02493 [astro-ph.HE].

[9] Moeen, M. M., Ghanbari, J., & Ghodsi, A. 2012, PASJ., 64, 137.

[10] Pimentel, O. M., Lora-Clavijo, F. D., & Gonzalez, G. A. 2016, Gen Relativ Gravit.

[11] Takahashi, R. 2007, MNRAS., 382, 567.

[12] Weinberg, S. 1972, Gravitation and Cosmology (John Wiley & Sons).