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A Dynamical Stability Study of Triple-Star Systems Using Physical and Geometrical Parameters

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Abstract. The dynamical stability of 141 triple-star systems is investigated. These systems are selected from the updated catalog of multiple stars systems (i.e., the MSC catalog). The distribution of eccentricity for inner and outer orbits is plotted. This diagram shows that the inner orbits are almost circular while the outer ones are oval, indicating higher eccentricity. This confirms that triple-star systems are also hierarchical. The dynamical stability of all systems is investigated using five different criteria. Observational stability parameters and their critical values are calculated using orbital values and the masses of components. In addition, the stability margin against the eccentricity of the outer orbit is plotted. This diagram shows that by increasing the eccentricity of the outer orbit, the distance from the stability limit also increases. Therefore, the higher the eccentricity of the outer orbit is, the more unstable the system becomes. Furthermore, by some investigations, we found that the dependence on the eccentricity of the inner orbit through the factor $1/(1 - e_{in})$ stabilizes many systems in some criteria, and this modifies the corresponding criteria. The results of the investigations show that almost all triple-star systems are stable and have a hierarchical structure. Only five systems (with WDS indexes: 18126-7340, 06467+0822, 02022-2402, 08391-5557, and 00247-2653) are unstable in at least three criteria. The reasons for the instability of these systems are most likely the observational errors or the unreal theoretical criteria. Finally, the introduced five criteria are ordered according to their credibility and precision.

Keywords: multiple star systems, dynamical stability, hierarchical systems.

1 Introduction

The statistical studies of multiple-star systems are usually conducted with different scientific purposes. One of the main purposes is the investigation of the formation mechanisms of binary- and multiple-star systems. In the last few years, much progress has been made with the focus on the formation of multiple-star systems. One of the important achievements in this area is that one must substitute single-process formation scenarios of stellar populations with a collection of long hierarchical formation scenarios[1]. Using Hipparcos catalog, Eggleton and Tokovinin concluded that at least 10 % of all bright stars belong to triple-star (or multiple-star) systems [2]. Before driving the high accuracy kinematic models of triple-star systems, their dynamical stabilities must be determined first. It is obvious that binary star models cannot be used for triple-star systems. This is because the gravitational bound of triple-star systems will be destroyed after some time [3].

Multiple-star systems are in two forms [4,5]: 1) hierarchical systems and 2) non-hierarchical systems. In hierarchical systems, the distances between components are not of the same order of magnitude. As an example, in a triple-star system, the distance between the outer component and the center of mass of the inner components is much higher than the distance between the inner components. However, in non-hierarchical systems, the separation distance between components is of the same order of magnitude. The non-hierarchical systems are usually young and dynamically unstable and their dynamical evolution forms close-binary systems, while the third component escapes. In contrast, a hierarchical system is either stable or unstable. In these systems, if the ellipticity of the outer component is low, the system is usually stable. In general, hierarchical systems keep their hierarchy during dynamical evolution.

Being hierarchical does not guarantee the stability of a multiple-star system. This is because in an unstable hierarchical system, it is possible to throw each component of the system to far distances while the system keeps its hierarchy [6]. However, in the next stages of evolution, the system is perturbed and then becomes unstable. All the known multiple-star systems with determined orbital parameters are hierarchical. However, there exist some hierarchically weak systems in which the size of the inner sub-system is not insignificant in comparison to the distance between the inner sub-system and outer component. The dynamical stability analysis of these systems is very important. In general, the dynamical stability of systems is a function of the mass ratio of the components, as well as the orbital parameters of the outer and inner binaries, etc. [7].

To analyze the dynamical stability of multiple-star systems, there are at least two methods. In the first method, the motion of systems must be determined through observational and analytical methods [8]. Analytical studies of the stability of triple-star systems is a challenging task, but some progress has been made by Szebehely in this respect [9–12]. In the second method, numerical simulations of the dynamical evolution of a multiple-star system are based on the equations of motion [13–16]. In this method, the values of the mass, velocity, and location of all components at a certain time must be determined as initial conditions. However, in the first method, stability criterion parameter and its critical value must be determined using observational data.

Szebehely and Zare investigated the stability of eight triple-star systems. They concluded that if the inner and outer orbits of these systems become co-rotating and co-planar (i.e., they have the same orbital plane and the rotation is in the same direction), then eight systems must be stable. If the orbital planes are not co-planar, then five systems can be dynamically unstable [10]. In 1981, Fekel investigated a sample of 27 triple-star systems and concluded that 23 systems are stable, while four systems can be unstable if their inner and outer orbits have retrograde motions relative to each other. The orbital motion of these four systems was undetermined at that time [17]. Moreover, Donnison and Mikulskis studied 38 triple-star systems and concluded that all of the systems, irrespective of their mutual orbital inclinations, are stable [18].

At present, the stability of some hierarchical triple-star systems is undetermined. This is because the observational data for these systems do not have enough precision. In addition, there is not a consensus about the conditions of the stability of triple-star systems. In this work, based on the recent observational data and using different observational and analytical criteria, the dynamical stability of 141 triple-star systems is investigated. A comparison between different criteria is also presented.

2 Data Sampling Method

More than thousands of triple-star systems have been detected until now. However, only a small fraction of the inner and outer orbital parameters of these systems are determined. According to the current stability criteria, to determine whether a system is stable or not, one needs to know the orbital period, P, semi-major axis, a, inclination, i, of the inner and outer orbits, and the mass of the system components. In this study, different catalogs were investigated (e.g. CDS, WDS, HIPPARCOS, SB9, MSC), and finally, the MSC catalog, which is up-to-date, free, and comprehensive, was selected. In this catalog, the observational data are available in four different files: "comp", "sys", "orb", and "note"[1]. Each system in the MSC catalog is graded from 0-5 which determines the reliability and precision of the reported parameters for that system. These information are available in the "comp" file. In this work, we just studied triple-star systems with grade values 4 and 5, which have the highest reliability and precision. Moreover, to study the dynamical stability of these systems, only those systems with the known parameters are considered in the final list. Finally, 141 triple-star systems had the necessary conditions for the rest of the study. The mass of the components is reported in the "sys" file and orbital information of the systems is presented in the "orb" file. If more than one value had been reported for a system, the newest data was selected. In Table 1, the information of a few of these 141 triple-star systems is presented.

Table 1: Photometric properties of the sample data.

WDS	e_{ex}	e_{in}	$a_{ex}('')$	$a_{in}('')$	$i_{ex}(^\circ)$	$i_{in}(^\circ)$	$M_1(M_\odot)$	$M_2(M_\odot)$	$M_3(M_\odot)$
02022-2402	0.338	0.058	0.414	0.102	180	152.5	0.63	0.62	1.15
18126-7340	0.628	0.65	1.522	0.35	15.9	82.8	1.46	0.73	0.82
15313-3349	0.269	0.523	1.6	0.415	141.2	92.16	1.71	1.64	1.26
08391-5557	0.644	0.38	1.4	0.13	128.1	149	1.45	1.06	1.86
00247-2653	0.125	0.076	1.808	0.491	65.5	13.6	0.08	0.08	0.11
...	...	...	...	...	...	...	...	...	...

3 Dynamical Stability Criteria of Triple-star Systems

It is usually useful to use the "three-body problem" to obtain a special criterion for the dynamical stability of multiple-body systems. The three-body problem was introduced about 200 years ago by Lagrange and he solved this problem for the first time. This problem also has been studied by many other researchers until now (e.g., see [19,20]).

The dynamical stability of three-body systems with circular orbits are usually determined by calculating the real ratio of the semi-major axis of the outer orbit, a_{ex}, to the semi-major axis of the inner orbit, a_{in}, and then comparing the result with a critical value. This is because, in observational data, only the projection of the orbits can be obtained. If the value of this ratio becomes more than a defined critical value, then the system is stable. For non-circular orbits, the periastron distance of the tertiary of the outer orbit, q, can be used instead of a_{ex} [21]. Then, the main problem is to determine the value of this critical value (theoretically or observationally) by using orbital or physical parameters.

The dynamical stability of three-body systems can be determined without using simulations [7,8]. Some of these methods are obtained by analytical considerations and using

[1] http://www.ctio.noao.edu/ atokovin/stars

special assumptions or using small approximations, etc. A summary of some of these criteria is presented in the following subsections.

3.1 Z Criterion

Mardling and Aarseth claimed that the Z criterion is valid for a wide range of mass ratios, eccentricities, and mutual orbital inclinations, Φ (except for $\Phi = 90°$) [7]. These researchers used the relation between the variables in the binary-tides problem [22] and the hierarchical three-body problem to propose a criterion for hierarchical triple-star systems as follows [23]

$$R_p^{ex} = C\left[(1+q_{ex})\frac{(1+e_{ex})}{(1-e_{ex})^{1/2}}\right]^{2/5} a_{in}, \tag{1}$$

where R_p^{ex} is the separation of the third component when it is in periastron, $C = 2.8$, and $q_{ex} = m_3/(m_1+m_2)$. The drawback of this relation is that the relatively weak dependence between dynamical stability and e_{in} as well as the mass ratio m_1/m_2 are ignored. Further considerations suggest an upper limit for the mutual orbital inclination Φ with the coefficient $f = 1 - 0.3\Phi/180$, which is useful in numerical simulations. This can also be used for the dynamical stability of retrograde orbits [13]. In the end, they suggested the following empirical relation for the Z criterion [7]

$$Z = \frac{a_{ex}(1-e_{ex})}{a_{in}(1-e_{in})} > Z_c = 2.6\frac{(1+e_{ex})^{0.4}(1+q)^{0.4}}{(1-e_{ex})^{0.0728}(1+e_{in})^{1.2}}\left(1-0.3\frac{\Phi}{\pi}\right). \tag{2}$$

3.2 F criterion

Harrington studied a wide range of triple-star systems to obtain a dynamical stability criterion for these systems. At first, he studied systems with equal masses and then systems with mass ratios of comparable magnitude. Finally, he proposed the following criterion for these systems [13,24]

$$F = \frac{a_{ex}(1-e_{ex})}{a_{in}} > F_c = A\left[1 + B\log\left(\frac{1+\frac{M_3}{M_1+M_2}}{3/2}\right)\right] + 2. \tag{3}$$

In equation 3, for co-planar and prograde motions ($\Phi = 0°$), we have $A = 3.5$ and $B = 0.7$, while for retrograde motions ($\Phi = 180°$), we have $A = 2.75$ and $B = 0.64$.
Harrington reported that this equation is valid for angles in the range $\phi = 0° - 180°$ except for $\Phi = 90°$.

3.3 X criterion

Eggleton and Kiseleva found that the period ratio or X is not uniform for different mass ratios and have a complex relation [25,26]. They proposed approximate stability criteria for hierarchical triple-star systems that are applicable for a wide range of inner and outer eccentricities, inclinations, mass ratios. This criterion is formulated as follows [27]

$$X = \frac{P_{ex}}{P_{in}} > X_c = (\frac{q_{ex}}{1+q_{ex}})^{1/2}(\frac{1+e_{in}}{1-e_{ex}})^{3/2}Y_c^{3/2}, \tag{4}$$

where Y_c is given by

$$Y_c \approx 1 + \frac{3.7}{q_{ex}^{1/3}} + \frac{2.2}{1+q_{ex}^{1/3}} + \frac{1.4}{q_{in}^{1/3}}\frac{q_{ex}^{1/3}-1}{q_{ex}^{1/3}+1}. \tag{5}$$

In equations 4 and 5, $q_{in} = m_1/m_2$, $q_{ex} = (m_1 + m_2)/m_3$ and m_3 is the mass of the heavier component in the inner binary system.

3.4 T criterion

The criterion proposed by Mardling and Aarseth for co-planar and co-rotating orbits is in the form [28]

$$\left(\frac{P_{out}}{P_{in}}\right)^{2/3} \geq 2.8(1+q_{ex})^{1/15} \times (1+e_{ex})^{0.4}(1-e_{ex})^{-s}, \tag{6}$$

where s is constant and equal to 1.2. Tokovinin investigated the dynamical stability in two situations. First, after analyzing multiple-star systems with two known orbits and using the MSC catalog (version 2002), he found that $s = 0.9$ has better conformity in this criterion [1,29]. Second, if the mass ratio is omitted in equation 6, then this equation can be rewritten as follows

$$\frac{P_{ex}}{P_{in}}(1-e_{ex})^{1.8} \geq 4.7(1+e_{ex})^{0.4}. \tag{7}$$

He also removed two groups of systems from his data. The removed systems included uncertain observational orbits (i.e., systems with $P_{ex} > 300yr$) and spectroscopic orbits (with $P_{ex} < 10d$), which their orbital parameters is modified during a period because of tidal forces. Then, he concluded that the following empirical criterion can be used for the remaining systems [30]

$$\frac{P_{ex}}{P_{in}}(1-e_{ex})^{3} \geq 5. \tag{8}$$

3.5 Q Criteria

Valtonen and Karttunen used the perturbation theory for the three-body problem and obtained a new criterion as follows [20]

$$Q = \frac{a_{ex}(1-e_{ex})}{a_{in}}, \tag{9}$$

$$Q_c = 3\left(1+\frac{m_3}{m_B}\right)^{1/3} \times \left(\frac{7}{4}+\frac{1}{2}\cos\Phi - \cos^2\Phi\right)^{1/3}\left(1-e_{ex}\right)^{-1/6}. \tag{10}$$

In equation 10, $\Phi = 0°$ corresponds to prograde coplanar motions; $\Phi = 180°$ to retrograde coplanar motions; and $\Phi = \pi/2$ to mutually orthogonal orbits for the outer and inner binaries.

4 Investigating Stability of Triple-Star Systems

Based on the five stability criteria introduced in section 3, the dynamical stability of 141 triple-star systems in the MSC catalog is analyzed. The values of the stability parameter Z, X, Q, F, and T and their critical values Z_c, X_c, Q_c, F_c, and T_c are calculated. The values of criteria can be calculated using observational data. In addition, Orlov and Petrova defined the stability margin of the triple-star systems as the following ratios

$$\delta_F = \frac{F-F_c}{F_c}, \quad \delta_X = \frac{X-X_c}{X_c}, \quad \delta_T = \frac{T-T_c}{T_c}, \quad \delta_Z = \frac{Z-Z_c}{Z_c}, \quad \delta_Q = \frac{Q-Q_c}{Q_c}. \tag{11}$$

For a larger δ, the stability is stronger, while for unstable systems, we have $\delta < 0$.
The eccentricity of inner and outer orbits has an important role in determining the stability of triple-star systems. The distribution of these parameters is presented in Figure 1.

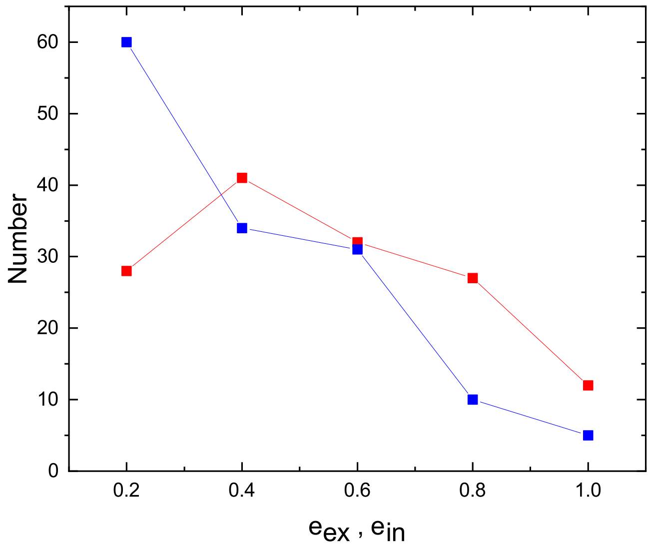

Figure 1: The distribution of outer and inner eccentricities of the 141 systems selected from the MSC catalog. The blue line shows e_{in} and the red line shows e_{ex}.

According to Figure 1, the eccentricity of most of the inner binary systems (more than 67% of all systems) is lower than 0.4 and these systems have nearly circular orbits. For outer orbits, the majority of systems have a eccentricity of about 0.4, and the eccentricity of more than half of the systems is higher than 0.6. This indicates that these systems are hierarchical, and the orbital size of their outer component is relatively large in comparison to the size of the inner orbit.
In the following subsections, the dynamical stability of all 141 triple-star systems is investigated in different stability criteria.

4.1 Z Criterion

This criterion is a function of inner eccentricity e_{in} and mutual orbital inclination Φ (see equation 2). According to this criterion, only four systems with WDS indexes 08391-5557, 15313-3349, 02022-2402, and 18126-7340 are dynamically unstable. Considering the systems with updated data, system 02022-2402 with the observational data presented by Tokovinin in 2014 for inner orbit, and system 18126-7340 with the observational data presented by Tokovinin in 2021, the stability of these systems is concluded. Comparing the inclination of the inner binary of system 15313-3349 in the MSC catalog with previously reported data, we speculate that the inclination may be incorrectly recorded as 921.6 due to typographical mistake (this value is 90.9 in the VB6 catalog in 2011). After correcting the inclination value to 92.16, this system became stable. Therefore, the only unstable system in this criterion is system 08391-5557. It is worth mentioning that, the period of the outer component of this system is reported to be $P_{ex} = 876.8599yr$. This value probably has not been measured with enough accuracy.

4.2 F criterion

In this criterion, we need mutual orbital inclination Φ, which can be calculated as follows

$$\cos\Phi = \cos i_{wide} \cos i_{close} \pm \sin i_{wide} \sin i_{close} \cos(\Omega_{wide} - \Omega_{close}), \quad (12)$$

where i is the inclination, Φ is the mutual orbital inclination, and Ω is the position angles of the lines of nodes. Equation 12 gives two values for Φ if the value of Ω is not determined precisely. Noting that the value of Φ can be calculated only for a small fraction of all systems, we used both values of Φ to study the dynamical stability of the systems. The dynamically unstable systems are listed in Table 2.

Table 2: Dynamically unstable systems in F criterion.

sign	Range of $\Phi(^\circ)$	Dynamically unstable systems
Positive	$\Phi < 90$	18126-7340, 06467+0822, 02022-2402, 15313-3349, 00247-2653, 08391-5557
Positive	$\Phi > 90$	All systems are dynamically stable
Negative	$\Phi < 90$	06467+0822
Negative	$\Phi > 90$	18126-7340, 22288-0001, 12108+3953, 22388+4419, 20396+0458, 00057+4549

Using Table 2, systems that are stable for both cases of minus and plus signs are considered to be stable, and systems that are unstable in two cases are considered to be unstable. Systems that are stable in one state and unstable in other state are considered to be stable. Consequently, dynamically unstable systems in this criterion are 06467+0822 and 18126-7340. Mutual orbital inclination for both of these systems and for both signs are in the range $50^\circ - 90^\circ$. Thus, the instability of these systems can be attributed to the fact that the orbital plane of these systems are almost perpendicular. However, this criterion is defined for co-planar systems.

4.3 X criterion

Stability criterion in this case is a function of only the ratio of the periods. Periods of the systems, especially the periods of the outer orbits, which are longer than those of the inner orbits, are usually measured with high uncertainty. In addition, this criterion is not a function of inclination, i. The majority of systems (21 systems) are unstable according to this criterion.

4.4 T criterion

Considering $s = 0.9$ in equation 6 for 10 systems, the value of δ is negative ($\delta < 0$), so these systems are unstable. Figure 2 shows the eccentricity of outer orbit versus the orbital period of the triple-star systems with two known orbits for $s = 0.9$ and 1.2. The value of s proposed in [1] has better compatibility with new observational data. Nevertheless, nine systems are located in the forbidden region. However, six of nine systems have a long orbital period (i.e., $P_{ex} > 300\, yr$) and the measured values have low precision.

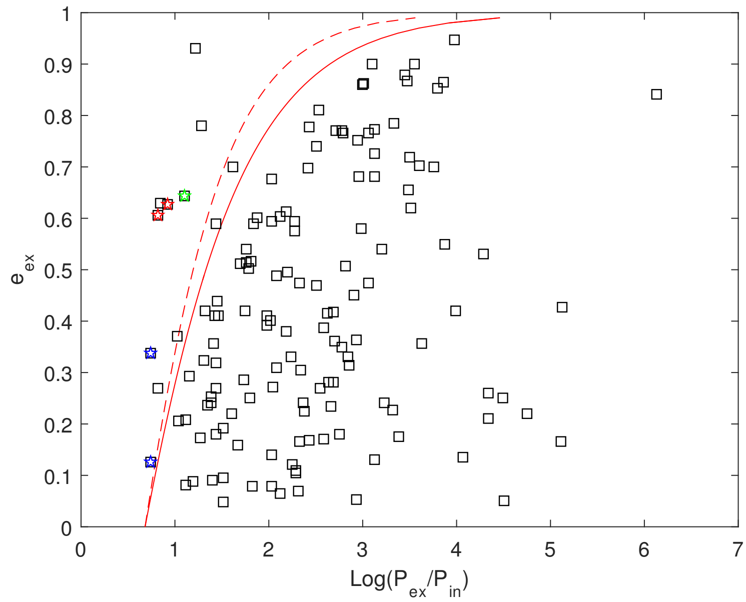

Figure 2: Eccentricity of outer orbit versus the ratio of orbital periods. The solid line corresponds to $s = 1.2$ and the dashed line to $s = 0.9$. The blue, red, and green squares are unstable in Q, F, and Z criteria.

By investigating the parameters of the unstable systems, especially the eccentricity of outer orbits e_{ex}, it is found that for values of e_{ex} that are closer to 1, the systems are more unstable. For instance, system 04220+1932 with $e_{ex} = 0.93$ and $\delta = -0.84$ has the highest departure from the stability limit (Figure 3). Therefore, the higher the eccentricity of the outer orbit is, the more unstable the system will be. It is interesting to note that, for all of these unstable systems except system 15313-3349, the value of e_{ex} is greater than 0.6, and the reason for their instability may be the high value of e_{ex}. Thus, this criterion is not suitable for systems with a higher value of outer eccentricity.

The Tokovinin's empirical relation, equation 8, was used, and after removing unreliable systems with $P_{ex} > 300yr$ and spectroscopic orbits with $P_{ex} < 10d$, only 100 systems remained. Under this condition, 15 systems have $\delta < 0$ and are unstable. To show this, e_{ex} is plotted versus the logarithm of the stability parameter in Figure 4. To consider the dependence of stability on inner eccentricity e_{in}, we considered the factor $1/(1 - e_{in})$ in equation 7. In this way, the number of unstable systems would be three. Therefore, equation 8 is corrected and the new empirical criterion can be written as follows

$$T' = \frac{P_{ex}}{P_{in}} \frac{(1 - e_{ex})^{1.8}}{(1 + e_{ex})^{0.4}(1 - e_{in})} \geq 4.7. \tag{13}$$

Figure 5 shows e_{ex} versus the logarithm of T' and Figure 5 shows that the three systems (06467+0822 with the highest departure; 02022-2402 and 00247-2653 with the lowest departure) are outside the stability region. The most probable reason for the instability of these

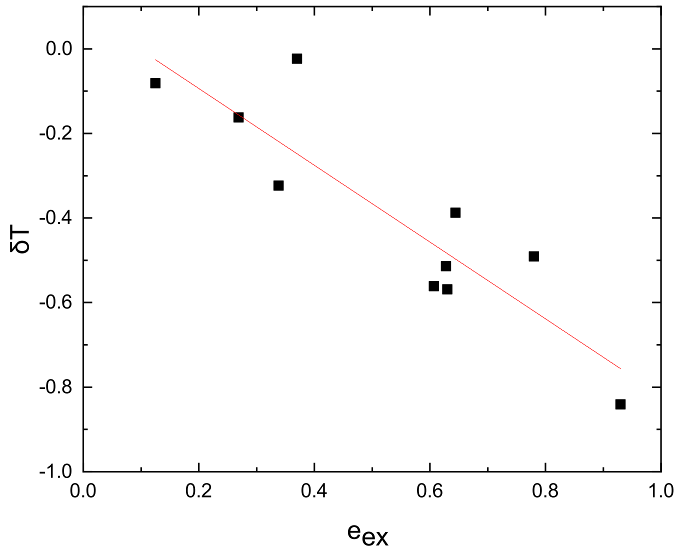

Figure 3: Dynamical stability margin versus e_{ex}. The red line, which is fitted to data, indicates that for higher values of e_{ex}, the systems are more departed from the stability limit.

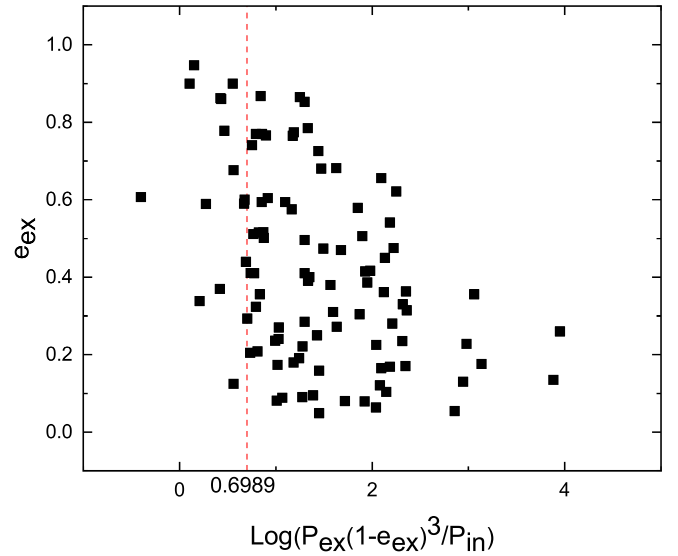

Figure 4: Outer eccentricity versus the logarithm of stability parameter. The red dashed line determines the stability limit.

systems is that the mass ratios have not been considered in this criterion. In addition, this criterion is defined for co-planar systems, while the orbits of these systems are not co-planar.

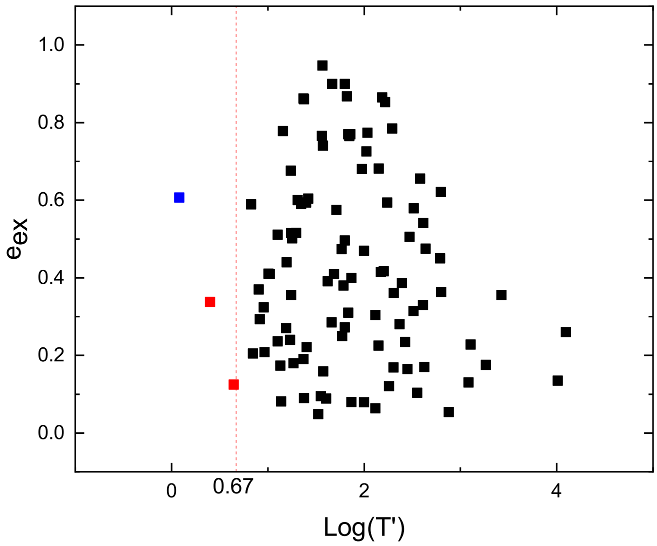

Figure 5: Eccentricity of outer orbit versus the logarithm of the corrected stability parameter. The red dashed line shows the new stability limit. The red squares are unstable systems in Q, X, and T criteria. The blue squares represent the unstable systems in F, X, and T criteria.

4.5 Q criterion

According to observational data, ten systems are unstable in this criterion. After some trial and error and considering the eccentricity of inner orbit with factor $1/(1-e_{in})$, seven of ten systems became stable and only three systems (00247-2653, 02022-2402, and 18126-7340) remained unstable. Then, the stability parameter in this criterion (i.e., equation 9) can be corrected in the following form

$$Q = \frac{a_{ex}(1-e_{ex})}{a_{in}(1-e_{in})}. \tag{14}$$

Most of the 141 triple-star systems are dynamically stable in the introduced five stability criteria. However, the number of unstable systems is different in each criterion. After analyzing, re-investigating, and revision, Table 3 presents the number of unstable systems in each criterion.

Table 3: Number of unstable systems in each criterion.

Criterion	X	Q	F	T'	T	Z
Number of unstable systems	21	3	2	3	4	1

Table 4: Triple-star systems that are dynamically unstable in at least three criteria.

WDS index	criterion
08391-5557	Z, T, X
06467+0822	F, X, T, T'
18126-7340	F, X, T, Q
02022-2402	X, T, Q, T'
00247-2653	Q, T, X, T'

Triple-star systems that are dynamically unstable in at least three criteria are listed in Table 4. It seems that the observational data of these systems must be modified.

5 Discussion and Conclusion

The age of stars selected in this study is several order of magnitude larger than the dynamical time scale. Therefore, if these systems were unstable, they would not exist for such a long time. Then, it is logical to suppose that all of the selected triple-star systems are dynamically stable. After analyzing all of the 141 triple-star systems, 120 systems were found to be stable in all of the five introduced stability criteria. However, five systems (with WDS indexes: 18126-7340, 06467+0822, 02022-2402, 08391-5557, and 00247-2653) were found to be unstable in at least three stability criteria. Some systems were unstable according to the old observational data, while using updated observational data (which are more precise and reliable), they turned out to be stable. The inefficiency of a stability criterion in a certain range of orbital and physical parameters, such as inclination and mass ratio, can be the reason for the inconsistency of the results for some systems(whether these systems are stable or unstable). In addition, the uncertainty of the observational data can be considered as another reason for the incompatibility of the results in different stability criteria.

The e_{in} versus e_{ex} distribution diagram shows that inner orbits are usually circular, while outer orbits are usually more eccentric. This emphasizes that these systems are hierarchical. In addition, from stability margin versus outer eccentricity e_{ex} diagram it is conceivable that by increasing the outer eccentricity e_{ex}, the departure from the stability limit increases too. In other words, the more e_{ex} is, the more unstable systems become.

The dependence of the dynamical stability on the eccentricity of the inner orbits e_{in} has not been considered in almost all stability criteria. In the present study, and after some trial and error, this dependence was considered as $1/(1-e_{in})$ in T and Q criteria, so some systems became stable in these criteria. It is worth mentioning that δ determines the stability margin and not the exact stability limit. Consequently, since some unstable systems are close to the stability limit, we cannot conclude with certainty that these systems are unstable. According to the results of this study, the Z criterion has the highest compatibility with observational data. This can be attributed to the fact that the Z criterion is a function of more orbital and physical parameters compared to other stability criteria. Then, the introduced five stability criteria can be graded according to their credibility. The order of the credibility of the stability criteria from high to low is Z, F, T, Q, and X.

The mass ratio q_{ex} for all unstable systems (except 01543-4230 in X criterion) is more than 0.3. Accordingly, we suggest that the dependence of the stability criterion on mass ratio must be considered, or at least each criterion has to be defined in a specific range of this parameter.

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