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Simulation of Electron Acoustic Wave Propagation in Non-Uniform Magnetized Plasma

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Abstract. The Propagation of electron acoustic solitary waves is studied in a plasma consisting of hot and cold electrons and stationary ions and in the presence of nonuniform external magnetic field. Our simulation results in this paper show that the electron acoustic solitary waves radiate some amount of energy during their traveling through the varying magnetic field. The important point is that we do not have other dissipative sources like particle interaction or viscosity effects, in this model. We also find that the electron density wave moves with a small wavelets around the central electron density even if the external magnetic field is constant. We propose to perform a laboratory experiment which will be able to identify the special new features of the electron acoustic waves propagation in a magnetized plasma with external varying magnetic field in which have been predicted in this investigation. Furthermore, our theoretical analysis brings a possibility to develop more refined theories of nonlinear acoustic waves that may occur in astrophysical nonuniform magnetized plasmas.

Keywords: Electron-acoustic waves, Solitary waves, Modified KdV-Burgers equation, Varying Magnetic Field

1 Introduction

The study of nonlinear localized waves in plasmas as a reach nonlinear media is an attractive subject in theoretical physics as well as experimental and laboratory considerations. There are several important phenomena in space environments and astrophysical situations which can be understood only through nonlinear behaviors, such as the cusp region of the terrestrial magnetosphere [1, 2], geomagnetic tail [3] and description of dayside auroral acceleration region [4, 5], beside experimental applications [6-10]. Watanabe et al. [11] used a linear electrostatic Vlasov dispersion equation to show that electron acoustic waves can be destabilized in such a plasma. Later on, Yu and Shukla [12] and also Gary et al. [13] obtained a dispersion relation for EAWs in a two (electron-ion) and three (two-temperature electrons and ions) components plasmas, respectively. Electron acoustic (EA) waves is a special kind of plasma wave fluctuations which may occur in media with two distinct electron populations referred to cold and hot electrons. The propagation of EA solitary waves in different plasma systems has been studied by several authors in unmagnetized two electron plasmas [14-17] as well as in magnetized plasmas [18-22]. The properties of obliquely propagating EASWs in magnetized plasmas have been studied by Mace and Hellberg [19]. They showed that negative potential EASWs corresponding to compression of the cold electron density can be created in such media. Ergun et al. [23-24] observed that BEN bursts in the dayside auroral zone have three-dimensional wave structure by including the magnetic field effects. The external magnetic field and the wave obliqueness are found to change the properties of the EA waves significantly. In all mentioned researches, external magnetic field has been considered as a constant vector throughout the medium, but we know that in a realistic situation, magnetic field is not a constant vector at all. In this work, we have tried to treat the problem using numerical solutions, beside an analytical evaluation using the small amplitude perturbation technique. In order to find a physical sense, we present a crude estimation for the evolution of EA solitary wave in the earth atmosphere where the magnetic field clearly has spatial variation [25].

Outlines of this paper are as follows: The basic dynamical equations governing our plasma model is presented in Section 2. In Section 3, the effect of varying magnetic field on the harmonic waves is investigated as simulationally. The last section is devoted to some concluding.

2 Basic equations

We consider homogeneous plasmas consisting of a cold electron fluid, hot electrons obeying a Maxwellian distribution and stationary ions in the presence of a space dependent external magnetic field $\overrightarrow{B} = B(r)\widehat{z}$. The nonlinear dynamics of electron acoustic solitary waves is extracted from the continuity and motion equations for cold electrons, in addition to the Poisson's equation [26] as

$$\frac{\partial n_c}{\partial t} + \overrightarrow{\nabla}.(n_c\overrightarrow{u_c}) = 0, \frac{\partial u_c}{\partial t} + (\overrightarrow{u_c}\overrightarrow{\nabla}), \tag{1}$$

$$\overrightarrow{u_c} = a\overrightarrow{\nabla}\varphi - b(\overrightarrow{u_c} \times \widehat{z}), \tag{2}$$

$$\nabla^2\varphi = \frac{1}{a}n_c + n_h - (1 + \frac{1}{a}). \tag{3}$$

In the above equations, n_c (n_h) is the cold (hot) electron number density normalized by its equilibrium values n_{c0} (n_{h0}). u_c is the cold electron fluid velocity normalized by the phase speed of electron acoustic $C_e = \sqrt{\frac{k_B T_h}{a m_e}}$ in which k_B is the Boltzmann's constant, e is the electron charge, m_e electron mass and $a = \frac{n_{h0}}{n_{c0}}$. The important parameter in our model is the space dependent parameter $b = \frac{\frac{eB(r)}{mc}}{\omega_{pc}}$ normalized by the cold electron plasma frequency ω_{pc} and φ is the electrostatic wave potential normalized by $\frac{k_B T_h}{e}$. The time and space variables are in units of the cold electron plasma period ω_{pc}^{-1} and the hot electron Debye radius λ_{Dh}, respectively. The basic set of equations (1)-(3) can be expanded as

$$\frac{\partial n_c}{\partial t} + \frac{\partial}{\partial x}(n_c u_{cx}) + \frac{\partial}{\partial y}(n_c u_{cy}) + \frac{\partial}{\partial z}(n_c u_{cy}) = 0, \tag{4}$$

$$\frac{\partial u_{cx}}{\partial t} + (u_{cx}\frac{\partial}{\partial x} + u_{cy}\frac{\partial}{\partial y} + u_{cz}\frac{\partial}{\partial z})u_{cx} = a\frac{\partial\varphi}{\partial x} - bu_{cy}, \tag{5}$$

$$\frac{\partial u_{cy}}{\partial t} + (u_{cx}\frac{\partial}{\partial x} + u_{cy}\frac{\partial}{\partial y} + u_{cz}\frac{\partial}{\partial z})u_{cy} = a\frac{\partial\varphi}{\partial y} + bu_{cx}, \tag{6}$$

$$\frac{\partial u_{cz}}{\partial t} + (u_{cx}\frac{\partial}{\partial x} + u_{cy}\frac{\partial}{\partial y} + u_{cz}\frac{\partial}{\partial z})u_{cz} = a\frac{\partial\varphi}{\partial z}, \tag{7}$$

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2} = \frac{1}{a}n_c + n_h - (1 + \frac{1}{a}). \tag{8}$$

As mentioned before, the Maxwellian distribution for hot electrons is considered as follows

$$n_h = e^{\varphi} \tag{9}$$

3 Study by simulation method

We have solved equations (4)–(8) simulationally to observe the time evolution of the electron density dealing with a varying magnetic field. Time derivations have been solved using the second order Runge-Kutta method, while space derivatives have been expanded by centered two point finite difference approximation. The grid spacing has been chosen as δx= 0.05, 0.11 and the time steps have been taken as δt=0.25δx. Since we have considered boundary conditions to solve the problem, the simulation is valid as long as the wave reaches the boundaries. Also, for simplicity, we have considered the charge neutrality condition in the above equations. Therefore, the Poisson equation (8) is out of the question. As equations (4)–(7) and condition (9) clearly show, finding the exact solution of the system parameters is difficult. So, we have studied time evolution of a lump of electron density in this environment which is not the exact solution of the system. These governing equations are nonlinear. Thus, plasma parameters (particle density, velocity, and electric potential) are not constant and evolve nonlinearly. We have selected the initial harmonic shape for the density of electrons as

$$n_e(X = x - x_0, Z = z - z_0, t) = \mathrm{sech}^2 \frac{X^2 + Z^2}{W^2},$$

where x_0, z_0, and W^2 control the initial position and the width of electron density distribution, respectively. It can be noted that we have considered normalized values for all parameters in this model. We have set up several simulations for fixed and unstable values of the external magnetic field. Figure 1 presents the time evolution of electron density in a fixed magnetic field B=2, while $u_x = u_z = 0.4$ and $u_y = 0.2$. Figure 1 shows that two lateral small amplitude wavelets are created on both sides of electron density in the direction of propagation. This figure also demonstrates that initial velocities change during the evolution. It is clear that the chosen initial conditions are not the solution of equations, and it is the reason for small distortions which will grow in the time. In another simulations, we have examined the interaction of electron density with the non-uniform magnetic field $B = 2+2e^{-0.5(x-5)^2}$, where $X = x-5$, as presented in Figure 2, and initial velocity is $u_x = u_z = 0.4$ and $u_y = 0.2$. Taking a Gaussian perturbation for the magnetic field which contains both increasing and decreasing parts seems to be a good selection for examining the effects of a varying magnetic field. Moreover, most of naturally created perturbations of magnetic fields are Gaussian [27]. As perturbation is not a function of \z" we do not see such phenomena in the \z" direction. These figures clearly show that backward propagating waves are created when electron density reaches the region with a varying magnetic field. Please note that the initial function of electron density is a symmetric function of variables \x" and \z", while the magnetic field perturbation is a function of \x" only. Therefore, the derivatives of electric potential in the right-hand side of equations (5) and (7) are not symmetric. On the other hand, the magnetic field affects the x and y components of wave velocity directly and that's why we see that the velocity in the direction of perturbation changes. The right panel of Figure 2 clearly shows asymmetry in the \x" and \z" directions. The profile of created shock waves is a complicated function of initial conditions of plasma parameters (initial electron density and velocity components). Comparison of Figures 1 and 2 shows that these two lateral waves let grow rapidly and amplify when localized electron density reaches the varying magnetic field. But what happens for the electron density when it travels among a varying magnetic field? It may be argued that a non-uniform magnetic field does not exert the same force on electron particles, and therefore wave propagation due to particle oscillation will not be stable. In general, it can be said that a non-uniform magnetic field can be introduced as a new factor in causing shock-like perturbation in behavior of the wave.

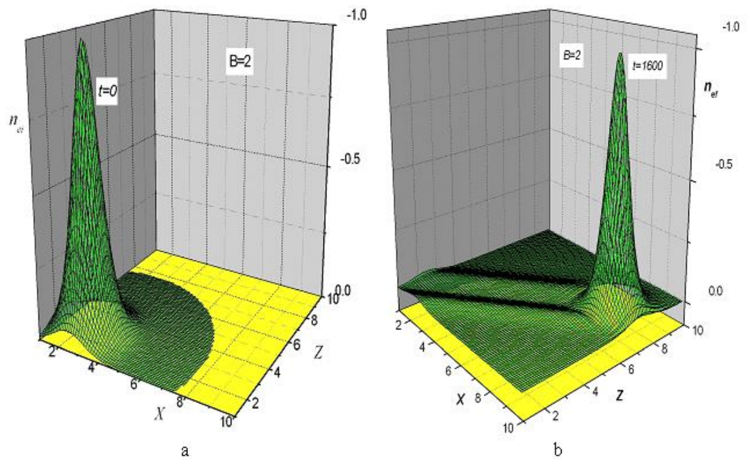

Figure 1: Time evolution of electron density at $t = 0$ (left) and $t = 1600$ (right) when $u_x(t=0) = u_z(t=0) = 0.5$, $u_z(t=0) = 0.1$, and $B = 2$.

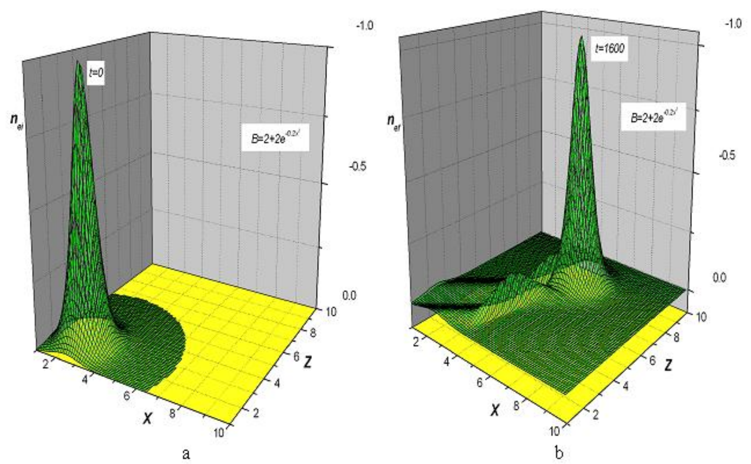

Figure 2: Time evolution of electron density at $t = 0$ (left) and $t = 1600$ (right) when $u_x(t=0) = u_z(t=0) = 0.5$, $u_z(t=0) = 0.1$, and $B = 2 + 2e^{-0.5(x-5)^2}$.

4 Conclusions

The present study studies the behavior of electron acoustic solitary waves in the plasmas containing a cold electron fluid, hot thermal electrons and stationary ions under the influence of a varying magnetic field. Our results in this paper show that the electron acoustic solitary waves radiate some amount of energy during their traveling through the varying magnetic field. Radiated energy emerges as backward moving oscillatory shock profiles. It is interesting that we have not considered any dissipative sources like particle interaction or viscosity effects. In fact, the space dependent magnetic field can be introduced as a new source of dissipation. We also find that the electron density waves move with a small wavelet around the central electron density even if the external magnetic field is constant. Our theoretical study confirms the existence of collisionless shocks driven by a laser-produced magnetic piston [28].

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