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Dust Acoustic Shock Waves in Strongly Coupled Dusty Plasmas with Superthermal Electrons and Ions

Hamid Reza Pakzad
Department of Physics, Bojnourd Branch, Islamic Azad University, Bojnourd, Iran;
email: pakzad@bojnourdiau.ac.ir & ttaranomm83@yahoo.com

Abstract. We use reductive perturbation method and derive the Kordeweg-de Vries-Burgers (KdV-Burgers) equation in coupled dusty plasmas containing ions and electrons obeying superthermal distribution. We discuss the effect of the plasma parameters on the shock wave structure. It is simply shown how soliton profile is converted into shock structure when the coupling force increases. In fact as long as the dispersive term and the dissipative term as well as the nonlinear term are balanced, the shock wave structure forms; otherwise, the soliton forms due to the balance between the dispersive term and the nonlinear term. We show that the effect of superthermal electrons is more influential in comparison with the superthermal ions on the behavior of the shock waves. It is also seen that increasing relative density (μ) decreases the amplitude of shock wave except for very small value of μ. Our investigation is of wide relevance to astronomers and space scientists working on interstellar space plasmas.

Keywords: Shock, Soliton, Coupling, Superthermal electrons, Superthermal ions

1 Introduction

In recent years, the study of nonlinear waves in plasmas has become one of the most important topic in plasma physics. Rao et al. [1] theoretically predicted the existence of dust-acoustic waves (DAWs), in which the inertia is provided by the dust particle mass and the restoring force is provided by the pressures of the inertialess electrons and ions. There has been a rapidly growing interest in understanding the physics of strongly coupled dusty plasma and associated low-frequency dust modes because of their vital role in space and astroplasmas (such as white dwarf matter, interior of heavy planets, etc.) [2–4] and laboratory plasmas (for example, plasma crystals, plasmas produced by laser compression of matter, etc.) [5–7]. It was first pointed out by Ikezi [8] that a classical Coulomb plasma with micron-sized dust particles can readily go into the strongly coupled regime. In fact, a dusty plasma environment becomes strongly (weakly) coupled if the Coulomb interaction energy is comparable to larger (much smaller) than the thermal energy of the charged dust particles. It has been found that in the case of a strongly coupled dusty plasma, the source of dissipation is the electrostatic strong correlation among the highly charged dust particles [9–12], but in a weakly coupled dusty plasma [13,14] the viscous force is the source of dissipation, and is responsible for the formation of shock waves [15]. The laboratory experiments [16–21] as well as a number of theoretical analysis [22–31] conclusively verified the prediction of Ikezi [8] and demonstrated that the dust particles organize themselves into crystalline patterns in such a dusty plasma. It is also observed by experiments that as the coupling is increased, the dust crystals melt and then vaporize so that one encounters the usual weakly coupled ideal Coulomb plasma. Thus, laboratory experiments in such a dusty plasma system provide an excellent opportunity for the study of transitions from the strongly coupled to weakly coupled regimes. A number of authors in the recent years have studied the behavior of dust acoustic shock waves in coupled dusty plasmas [32–39]. Shukla and Mamun [32] have derived Korteweg de Vries-Burgers (KdV-Burgers) equation by reductive perturbation method and they have studied the properties of the solitons and shock waves for strongly coupled unmagnetized dusty plasmas. Also, Mamun et al [33] have studied dusty plasma with a Boltzmann electron distribution, a nonisothermal vortex-like ion distribution and strongly correlated grains in a liquid-like state and discussed about the properties of shock wave structures. Ghosh and Gupta have investigated the nonlinear propagation of shock wave in strongly coupled collisional dusty plasma using the GH model incorporating a charging-delay effect [34]. More recently, it had been found that the electrons and ions distributions play a crucial role in characterizing the physics of the nonlinear waves [40–46]. The nonlinear features of dust acoustic shock waves in a strongly coupled dusty plasma containing nonisothermal has been studied in [43]. The effect of nonthermal ions on dust acoustic shock waves in dusty plasma was investigated in [44]. The kappa function characterized by the spectral index k is found to represent more suitably the particle's velocity distribution observed in number of space and astrophysical environment [47–52]. The observed data are more suitable modeled by k-function that incorporates the superthermal particles in the population Maxwellian distribution. Such function is also thought as a consequence of entropy generalization of plasma system subject to long range interaction and correlation. The shape of k-function may be the possible as an evolution of an initial Maxwellian function due to Landau wave-particle interaction incorporated in Fokker-Planks equation. The regions of occurrence of such distributions along with classification of spectral index k in various space and astrophysical plasma environment representing different temperature regimes are reported in number of investigations. To the best of our knowledge, no attempt has been made on the small and long amplitude of dust-acoustic shock waves in a coupled dusty plasma containing superthermal electrons and ions. The aim of the present paper is to study the effect of superthermal ions and electrons on the dust acoustic shock waves in an unmagnetized, collisionless coupled dusty plasma. The manuscript is organized as follows. The GH equations are presented in Section 2. In Section 3, we derive the Korteweg de Vries Burgers equation using the reductive perturbation method. We study the properties of solitary and shock waves solutions in Section 4. Finally, the main results from this investigation have been given in Section 5.

2 Basic equations

Let us consider an unmagnetized strongly coupled dusty plasma with negatively charged dust grains and superthermal kappa-distributed electrons and ions of density n_d, n_e and n_i, respectively. Thus, at equilibrium, we have $n_{e0} + Z_d n_{d0} = n_{i0}$, where Z_d is the number of electrons residing on the dust grain and the subscript "0" stands for unperturbed quantities. We assume that the ions are weakly coupled compared to the dust grains. The dynamics of the DAW in our coupled dusty plasma are given by GH equations [53–55] as follows

$$
\begin{aligned}
&\frac{\partial n_d}{\partial t} + \frac{\partial (n_d u_d)}{\partial x} = 0,\\
&\Big[1 + \tau_m \frac{\partial}{\partial t}\Big]\Big[n_d\big(\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} - \frac{\partial \phi}{\partial x}\big)\Big] = \eta_l \frac{\partial^2 u_d}{\partial x^2},\\
&\frac{\partial^2 \phi}{\partial x^2} = n_d + n_e - n_i.
\end{aligned}
\tag{1}
$$

The number densities are normalized by their equilibrium values. u_d is the dust fluid velocity and is normalized by the dust-acoustic speed $C_d = \sqrt{\frac{Z_d T_i}{m_d}}$ (T_i is the ion temperature and m_d is the dust mass), ϕ is the electrostatic wave potential and is normalized by $\frac{T_i}{e}$ (e is the magnitude of the electron charge). The time and space variables are normalized by the dust plasma period $\omega_{pd}^{-1} = \sqrt{\frac{m_d}{4\pi n_{d0} Z_d^2 e^2}}$ and the Debye length $\lambda_D = \sqrt{\frac{T_i}{4\pi n_{d0} Z_d e^2}}$, respectively. We assume that electrons and ions obey the kappa distribution and density associated are given as follows [56–59]

$$\begin{aligned} n_e &= \frac{1}{1-\mu}\Big(1 - \frac{\sigma\phi}{k_e - \frac{3}{2}}\Big)^{-(k_e - \frac{1}{2})}, \\ n_i &= \frac{\mu}{1-\mu}\Big(1 + \frac{\phi}{k_i - \frac{3}{2}}\Big)^{-(k_i - \frac{1}{2})}, \end{aligned} \tag{2}$$

where the normalization has been provided for any value of the spectral index k_i, $k_e > 3/2$. The spectral index is a measure of the slope of the energy spectrum of the suprathermal particles forming the tail of the velocity distribution function; the smaller the value of k_i, k_e the more suprathermal particles in the distribution function tail and the harder the energy spectrum. Kappa distributions approach the Maxwellian as k_i, $k_e \to \infty$. $\mu = \frac{n_{e0}}{n_{i0}}$, where n_{e0} and n_{i0} are the unperturbed number densities of electrons and ions respectively; $\sigma = \frac{T_i}{T_e}$, where T_e is the temperature of electrons; n_i is the ion number density normalized by n_{i0}. η_l is the normalized longitudinal viscosity coefficient and is given by

$$\eta_l = \frac{\tau_d}{m_d n_{d0} \lambda_D^2}\Big[(s + \frac{4}{3}b)\Big], \tag{3}$$

where s and b are the transport coefficients of shear and bulk viscosities, respectively. The viscoelastic relaxation time τ_m is normalized by the dust plasma period τ_d and is given by

$$\tau_m = \eta_l \frac{T_e}{T_d}\Big[1 - \mu_d + \frac{4}{15}u(\Gamma)\Big]^{-1}, \tag{4}$$

where

$$\mu_d = 1 + \frac{1}{3}u(\Gamma) + \frac{\Gamma}{9}\frac{\partial u(\Gamma)}{\partial \Gamma}, \tag{5}$$

is the compressibility [60,61] and $u(\Gamma)$ is a measure of the excess internal energy of the system and is calculated for weakly coupled plasma ($\Gamma < 1$) as $u(\Gamma) = -(\frac{\sqrt{3}}{2})\Gamma^{\frac{3}{2}}$ [62]. To express $u(\Gamma)$ in terms of Γ for a range of $1 < \Gamma < 100$, Slattery et al. [63] analytically derived a relation

$$u(\Gamma) \cong -0.89\Gamma + 0.95\Gamma^4 + 0.19\Gamma^{\frac{-1}{4}} - 0.81, \tag{6}$$

where a small correction term due to finite number particles is neglected.

3 Derivation of KdV-Burger equation

According to the general method of reductive perturbation theory, we choose the independent variables as [64]

$$\xi = \varepsilon^{\frac{1}{4}}(x - \lambda t), \tau = \varepsilon^{\frac{3}{4}} t, \eta_l = \varepsilon^{\frac{1}{4}}\eta_0, \tau_m = \varepsilon^{\frac{1}{4}}\tau_{m0}, \tag{7}$$

where ε is a small dimensionless expansion parameter which characterizes the strength of nonlinearity in the system and λ is the phase velocity of the wave along the x direction and normalized by dust acoustic velocity. Now, we expand dependent variables as follows

$$\begin{aligned} n_d &= 1 + \varepsilon n_{1d} + \varepsilon^2 n_{2d} + \cdots , \\ u_d &= \varepsilon u_{1d} + \varepsilon^2 u_{2d} + \cdots , \\ \phi &= \varepsilon \phi_1 + \varepsilon^2 \phi_2 + \cdots . \end{aligned} \tag{8}$$

Substituting (8) into (1) and collecting the terms in different powers of ϵ, the following equations can be obtained at the lower order of ϵ

$$n_{1d} = \frac{-\phi_1}{\lambda^2}, \qquad u_{1d} = \frac{-\phi_1}{\lambda}, \qquad \frac{1}{\lambda^2} = \frac{1}{1-\mu}\Big[(\frac{2k_e - 1}{2k_e - 3})\mu\sigma + \frac{2k_i - 1}{2k_i - 3})\Big], \tag{9}$$

At the higher order of ϵ, we have

$$\begin{aligned} &\frac{\partial n_{1d}}{\partial \tau} - \lambda \frac{\partial n_{2d}}{\partial \xi} + \frac{\partial}{\partial \xi}(u_{2d} + n_{1d}u_{1d}) = 0, \\ &\frac{\partial u_{1d}}{\partial \tau} - \lambda \frac{\partial u_{2d}}{\partial \xi} + u_{1d}\frac{\partial u_{1d}}{\partial \xi} - \frac{\partial \phi_2}{\partial \xi} = \eta_0 \frac{\partial^2 u_{1d}}{\partial \xi^2}, \\ &\frac{\partial^2 \phi_1}{\partial \xi^2} = n_{2d} + \frac{1}{1-\mu}[\mu\sigma(\frac{2k_e - 1}{2k_e - 3}) + \frac{2k_i - 1}{2k_i - 3}]\phi_2 \\ &\qquad + \frac{1}{1-\mu}\mu\sigma^2[\frac{4k_e^2 - 1}{(2k_e - 3)^2} - \frac{4k_i^2 - 1}{(2k_i - 3})^2\phi_1^2. \end{aligned} \tag{10}$$

Finally from (9) and (10) KdV-Burger equation yields

$$\frac{\partial \phi_1}{\partial \tau} + A\phi_1 \frac{\partial \phi_1}{\partial \xi} + B\frac{\partial^3 \phi_1}{\partial \xi^3} - C\frac{\partial^2 \phi_1}{\partial \xi^2} = 0, \tag{11}$$

where the coefficients are

$$\begin{aligned} A &= \frac{\lambda^3}{1-\mu}\frac{-\mu\sigma^2(4k_e^2 - 1)}{(2k_e - 3)^2)} + \frac{4k_i^2 - 1}{(2k_i - 3)^2} - \frac{3}{2\lambda}, \\ B &= \frac{\lambda^3}{2}, \\ C &= \frac{\eta_0}{2}. \end{aligned} \tag{12}$$

Equation (11) is a KdV-Burgers equation which includes the effects of ions and electrons following superthermal kappa-distributed (through k_i,k_e) and the dust particles with coupling force between them (through η_0). It is obvious that the dissipative term is due to the effect of η_0, the strong nonlinearity and dispersion term are due to the relative density (μ), relative temperature (σ) and superthermal distribution of ions and electrons.

4 Discussion

Equation (11) is the well known KdV-Burgers equation describing the nonlinear propagation of the dust acoustic shock waves in a coupled dusty plasma with superthermal ions and

superthermal electrons. In this equation, A and B are the nonlinear coefficient and dispersive term and the Burger term (C) arises due to the coupling of dust particles. The KdV-Burgers equation is widely used in plasma physics and theoretical physics. The tangent hyperbolic method is a powerful method for the computation of exact traveling wave solutions. More recently, Asif Shah et al. [65] have derived the monotonic shock waves solution theoretically by employing the tangent hyperbolic method [66].
They used the transformation $\chi = \kappa(\xi - \nu\tau)$ (where κ and ν are wave number and wave velocity, respectively) and presented the solution in terms of independent variable χ as

$$\phi_1 = \frac{12B}{A}\Big[1 - \tanh^2\chi\Big] - \frac{36C}{15A}\tanh\chi. \tag{13}$$

Now, we can use (13) for doing a quantitative analysis on the results. It is clear that the profile of the wave in (13) depends on the plasma parameters (μ, σ, k_i, k_e, η_0, χ). In this analysis, we study the effects of the plasma parameters on the shock waves. The following Figures show the variation of ϕ_1 with respect to different values of plasma parameters. Fig1. Shows the variation in ϕ_1 with κ and η_0 for σ=1, μ=0.2 and k_i, k_e=25. This figure indicates that as η_0 increases, the soliton profile decreases to monotonic shock wave. In fact, if the dissipation term can be ignored in comparison with the nonlinearity and dispersion terms, then solitonic structure will be established by balancing the effects of dispersive and nonlinear terms. On the other hand, if the coupling becomes very strong, the shock waves will appear by balancing the effects of dissipative and dispersive terms. Therefore, the existence of the Burgers term leads to the formation of a shock wave. It is also obvious that both strength and steepness of the shock structure increase by increasing the coupling.

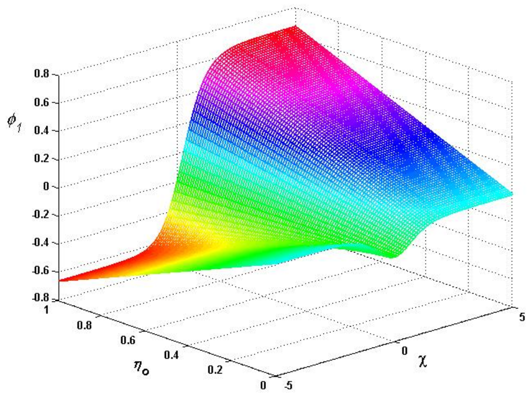

Figure 1: Showing the variation in ϕ_1 with χ and η_0 for μ=0.2,$k_e = k_i$=25, and σ=1.

Figures 2–5, show how the effects of k_e and k_i modify the soliton and shock-like structures which are caused by the electrons and ions following the superthermal distributions. Figurse 2 and 3 indicate that the amplitude of the solitary waves increase by increasing both k_e and k_i values. Nevertheless, it can be said that the effect of the spectral index k_e is more than that of the spectral index k_i on the solitons. As it was mentioned, the distributions approach the Maxwellian as k_i, $k_e \to \infty$. Thus, in the presence of supperthermal electrons and ions, the amplitude of the solitons decreases. Figures 2 and 3 explores the differences in shock wave structure, due to variation of k_e and k_i. Figure 4 show the amplitude of shock wave increases with increasing the values of k_e, but in Figure 5, it is clear the amplitude of shock wave decreases when the spectral index k_i increases. There is an interesting difference between to Figures 2–5. As it is observed in the Figures 2 and 3, the amplitude of the solitons increase sharply when superthermal parameters increase from 1.6 to 3, but the amplitude of the shock wave increases (decreases) when k_e (k_i) changes between 1.6 to 2.5 (1.6 to 2) and it remains almost constant for other values (see Figures 4 and 5).

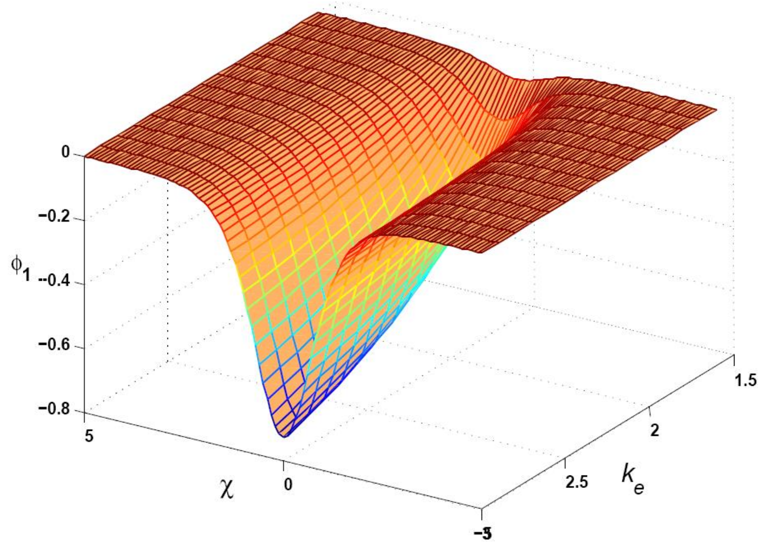

Figure 2: Showing the variation in ϕ_1 with χ and k_e for μ=0.2, k_i=2, σ=1, and η_0=0

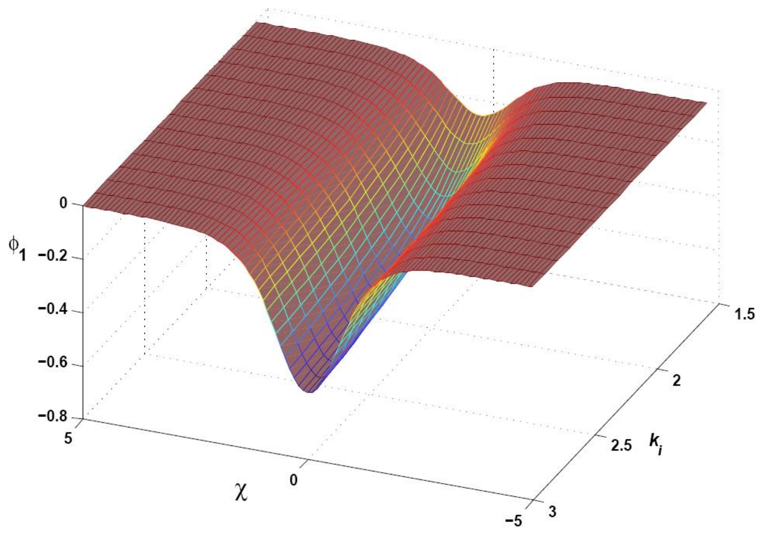

Figure 3: Showing the variation in ϕ_1 with χ and k_i for μ=0.2, k_e=2, σ=1, and η_0=0

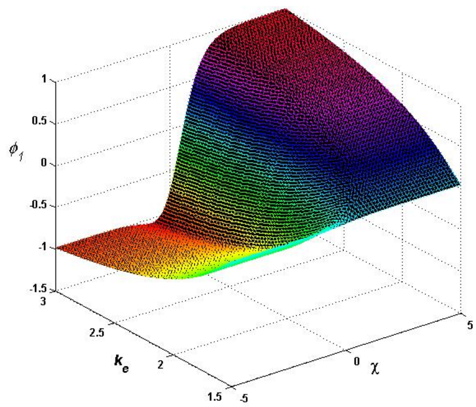

Figure 4: Showing the variation in ϕ_1 with χ and k_e for μ=0.2, k_i=2, σ=1, and η_0=0.5

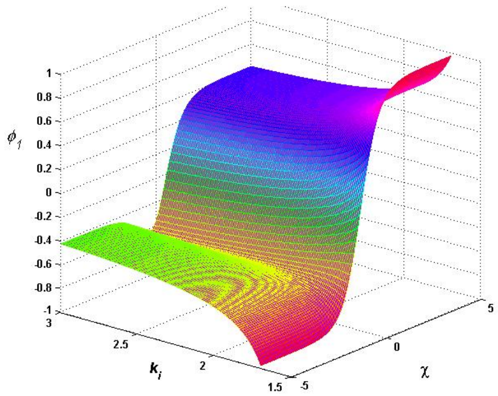

Figure 5: Showing the variation in ϕ_1 with χ and k_i for μ=0.2, k_e=2, σ=1, and η_0=0.5

In Figures 6 and 7, keeping k_i, k_e, σ, and η_0 at constant values, the effect of the μ on the shock waves are investigated. Interestingly, it can be found that the effect of increasing μ is to shift from a kink wave structure to an anti-kink type, as can be seen on Figure 6. Therefore, we can conclude that there is a critical value for μ (while the other parameters are constant) in which the amplitude of shock wave is zero. According to equation (12) the structure of monotonic shock wave depends on the sign of the nonlinear term. When $A > 0$ ($A < 0$) a kink (an anti-kink) wave structure is formed [64]. Figure 7 show the effect of relative density (μ) on the shock wave when k_i=k_e=25. It is found that for large values of k_i and k_e, increasing μ reduces wave power.

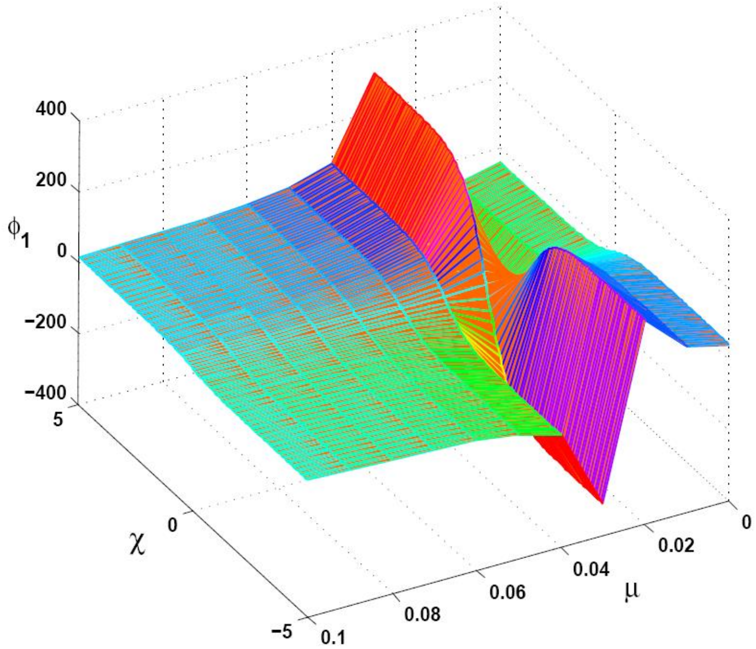

Figure 6: Showing the variation in ϕ_1 with χ and μ for k_e=k_i=2, σ=1, and η_0=0.5

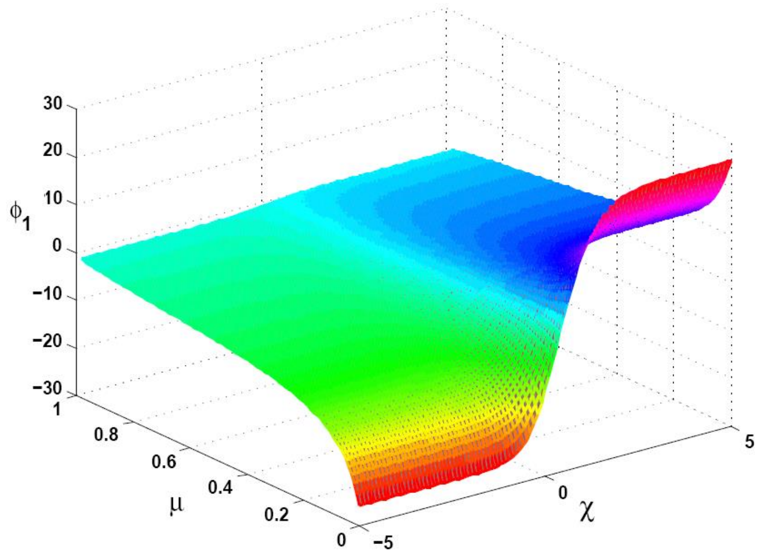

Figure 7: Showing the variation in ϕ_1 with χ and μ for k_e=k_i=25, σ=1, η_0=0.5

Figures 8 and 9 explores the differences in shock waves structure due to the variation of σ for small and large values of k_i and k_e. It is obvious that increasing σ decreases the amplitude of shock wave. Also, it is seen that the strength of shock wave in Figure 8 (for small values of k_i and k_e) is more than that of in Figure 9 (for large values of k_i and k_e). It is also seen the rate of decrease of ϕ_1 with respect to σ is more in Figure 9. So in the presence of supperthermal electrons and ions, the amplitude of the shock waves decreases sharply, when σ increases.

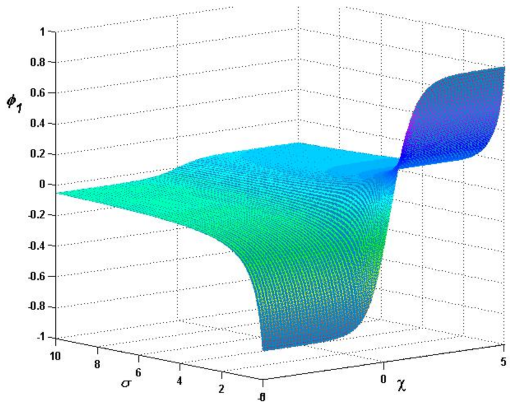

Figure 8: Showing the variation in ϕ_1 with χ and σ for k_e=k_i=2, μ=0.2, and η_0=0.5

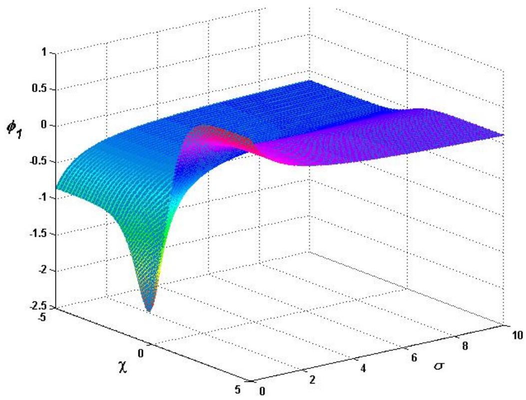

Figure 9: Showing the variation in ϕ_1 with χ and σ for k_e=k_i=25, μ=0.2, and η_0=0.5

5 Conclusions

We have considered an unmagnetized dusty plasma system consisting of ions and electrons obeying superthermal distribution and strongly coupled negatively charged dust. We have investigated the properties of dust acoustic solitary and shock waves by deriving the KdV-Burgers equation and used the given solution in [65,66]. The physical mechanism of shock wave formation in this work is somewhat similar to that of Ghosh et al. [34] and Shukla [67], in which a shock wave appeared in the collisional coupled dusty plasmas. As long as the dispersive term and the dissipative term as well as the nonlinear term are balanced, the shock wave structure forms; otherwise, the soliton forms due to the balance between the dispersive term and the nonlinear term. We have shown that if the coupling force is negligible, the solitary waves associated with negative potential can exist. We have also shown how soliton profile is converted into shock structure when the coupling force increases. The dissipation caused by the dust-dust correlation also has an effect on the amplitude of the shock wave.

With the strong dissipation, the shock wave structure becomes steeper. We have seen that the amplitude of both the solitary and shock waves increase with increasing the value of k_e, but the amplitude of the solitary (shock) waves increases (decreases) with increasing the value of k_i. Also, the numerical analysis shows that the effect of superthermal electrons is more influence in comparison with that of the superthermal ions on the waves in our model. We have observed that the amplitude of shock waves decreases when (σ) increases. This means that the amplitude of shock waves in the medium increases when the temperature of electrons increases or the temperature of ions decreases. It can be seen that increasing μ decreases the amplitude of shock wave except for very small value of μ. In fact, one can observe the existence of kink and anti kink shock waves for the small values of μ. The ranges of different plasma parameters used in this investigations are very wide ($0 < \sigma < 10$, $0 < \mu < 1$, $0 < \eta_0 < 0.5$, $1.6 < k_e, k_i < 25$) and are relevant to [33,44,56]. Strongly coupled plasma is of great interest in science, because of its applications in the interior of heavy planets, plasmas produced by laser compression of matter and nonideal plasmas for industrial applications. Our present investigation would also be useful to study the localized nonlinear structures in protoneutron stars [68], stellar polytropes [69], dark-matter halos [70] and Saturn's magnetosphere [71].

References

[1] Rao, N. N., Shukla, P. K., & Yu, M. Y. 1990, Planet. Space Sci. 38, 546.

[2] Eslami, P., Mottaghizadeh, M. & Pakzad, H. R. 2013, IEEE Trans. Plasma Sci., 41, 3589.

[3] Saini, N. S., & Singh, K. 2016, Phys. Plasmas, 23, 103701.

[4] Denra, R., Paul, S., Ghosh, U., & Sarkar, S. 2018, J. Plasma Phys., 84, 5.

[5] Shukla, P. K., & Mamun, A. A. 2002, Institute of Physics: Bristol, UK.

[6] Boufendi, L., & Bouchoule, A. 2002, Plasma Sources Sci. Technol., 11, 211.

[7] Zaman, D. M. S., Mannan, A., Chowdhury, N. A., & Mamun, A. A. 2020, High Temp., 58, 789.

[8] Ikezi, H. 1986, Phys. Fluids, 29, 1764.

[9] Mamun, A. A., & Ashrafi, K. S. 2010, Phys. Rev. E, 82, 026405.

[10] Thomas, H., Morfill, G. E., & Tsytovich, V. N. 2003, Plasma Phys. Rep., 29, 895.

[11] EL-Shamy, E. F., & Al-Asbali, A. M. 2014, Phys. Plasmas, 21, 093701.

[12] Sharma, S. K., Borugh, A., Nakamura, Y., & Bailung, H. 2006, Phys. Plasmas, 23, 053702.

[13] Paul, S. K., Mandal, G., Mamun, A. A., & Amin, M. R. 2009, IEEE Trans. Plasma Sci., 37, 627.

[14] Mamun, A. A., Shukla, P. K., & Eliasson, B. 2009, Phys. Plasmas, 16, 114503.

[15] Rahman, A., Mamun, A. A., & Kurshed Alam, S. M. 2008, Ap&SS, 315, 243.

[16] Hayashi, Y., & Tachibana, K. 1994, Jpn. J. Appl. Phys., Part 1, 33, L804.

[17] Chu, J. H., & Lin, I. 1994, Phys. Rev. Lett., 72, 4009.

[18] Thomas, H., Morfill, G. E., Demmel, V., Goree, J., Feuerbacher, B., & Mohlmann, D. 1994, Phys. Rev. Lett., 73, 652.

[19] Saini, N., & Shalini, S. 2013, Ap&SS, 346, 155.

[20] Baluku, T. K., & Hellberg, M. A. 2012, Phys. Plasmas, 19, 012106.

[21] Sultana, S., Kourakis, I., & Hellberg, M. A. 2012, Plasma Phys. Controlled Fusion, 54, 105016.

[22] Rosenberg, M., & Kalman, G. 1997, Phys. Rev. E, 56, 7166.

[23] Totsuji, H., Kishimoto, T., & Totsuji, C. 1997, Phys. Rev. Lett., 78, 3113.

[24] Hamaguchi, S., Farouki, R. T., & Dubin, D. H. E. 1997, Phys. Rev. E, 56, 4671.

[25] Lee, H. C., & Chen, D. Y. 1997, Phys. Rev. E, 56, 4596.

[26] Rahim, Z., Adnan, M., Qamar A., & Saha, A. 2018, Phys. Plasmas, 25, 083706.

[27] El-Taibany, W. F., El-Labany, S. K., Behery E. E., & Abdelghany, A. M. 2019, Eur. Phys. J. Plus, 134.

[28] Yaroshenko, V. V. 2020, Phys. Rev. E, 102, 023201.

[29] Jones, W. D., Lee, A., Gleman, S. M., & Douce, H. J. 1975, Phys. Rev. Lett., 35, 1349.

[30] Sharma, S. K., Borugh, A., Nakamura, Y., & Bailung, H. 2006, Phys. Plasmas, 23, 053702.

[31] Schippers, P., Blanc, M., & et al. 2008, Res., 113, A07208.

[32] Shukla, P. K., & Mamun, A. A. 2001, IEEE Trans. Plasma Sci., 29, 221.

[33] Mamun, A. A., Eliasson, B., & Shukla, P. K. 2004, Phys. Lett. A, 332, 412.

[34] Ghosh, S., & Gupta, M. R. 2005, Phys. Plasmas, 12, 092306.

[35] Cousens, S. E., Yaroshenko, V. V., Sultana, S., Hellberg, M. A., Verheest, F., & Kourakis, I. 2014, Phys. Rev. E, 89, 043103.

[36] Emamuddin, M., & Mamun, A. A. 2018, Physics of Plasmas, 25, 013708.

[37] Ferdousi, M., Sultana, S., Hossen, M. M., Miah, M. R., & Mamun, A. A. 2017, Eur. Phys. J. D, 71, 102.

[38] Mishra, R. & Dey, M. 2018, Phys. Scr., 93, 085601.

[39] Almutalk, S. A., El-Tantawy, S. A., El-Awady, E. I., & El-Labany, S. K. 2019, Phys. Lett. A, 16, 1937.

[40] Zhang, LP, & Xue, J. 2005, Phys. Plasmas, 12, 042304.

[41] Mamun, A. A, Russell, S. M., Mendoza-Briceno, Cesar A., Alam, M. N., Datta, T. K., & Das, A. K. 2000, Planet. Space Sci., 48, 163.

[42] Carins, R. A., Mammun, A. A., Bingham, Ro Dendy, R., Bostrom R., & Shukla, P. K. 1995, Geophys. Res. Lett., 22, 2709.

[43] Pakzad, H. R., & Nobahar, D. 2022, New Astronomy, 93, 101752.

[44] Lin, M. M., & Duan, W. S. 2007, Chaos Soliton. Fract., 33, 1189.

[45] Denra, R., Paul, S., Ghosh, U., & Sarkar, S. 2018, J. Plasma Phys., 84, 5.

[46] El-Taibany, W. F., El-Labany, S. K., Behery, E. E., & Abdelghany, A. M. 2019, Eur. Phys. J. Plus, 134, 457.

[47] Feldman, W. C., Asbridge, J. R., Bame, S. J., & Montgomery, M. D. 1973, J. Geophys. Res., 78, 2017.

[48] Formisano, V., Moreno, G., & Palmiotto, F. 1973, Geophys. Res., 78, 3714.

[49] Scudder, J. D., Sittler, E. C., & Bridge, H. S. 1981, J. Geophys. Res., 86, 8157.

[50] Saini, N. S., Ghai, Y., & Kohli, R. 2016, J. Geophys. Res., 121,5944.

[51] Pakzad, H. R., & Nobahar, D. 2019, IJAA, 6, 95–106.

[52] Gaelzer, R., Ziebell, L. F., Vinas, A. F., Yoon, P. H., & Ryu, C. M. 2008, ApJ, 677, 676.

[53] Shukla, P. K., & Mamun, A. A. 2003, New J. Phys., 5, 17.

[54] Kaw, P. K., & Sen, A. 1998, Phys. Plasmas, 5, 3552.

[55] Kaw, P. K. 2001, Phys. Plasmas, 8, 1870.

[56] Baluku, T. K., & Hellberg, M. A. 2008, Phys. Plasmas, 15, 123705.

[57] Baluku, T. K., Hellberg, M. A., & Verheest, F. 2010, Europhys. Lett., 91, 15001.

[58] El-Tantawy, S. A., El-Bedwehy, N. A., & Moslem, W. M. 2011, Phys. Plasmas, 18, 052113.

[59] Lazar, M., Kourakis, I., Poedts, S., & Fichtner, H. 2018, Planet. Space Sci., 156, 1308.

[60] Sumi, R. A., Tasni, I., Anowar, M. G. M., & Mamun, A. A. 2019, J. Plasma Phys., 85, 905850611.

[61] Ichimaru, S., & Tanaka, S. 1086, Phys. Rev. Lett., 56, 2815.

[62] Ichimaru, S., Iyctomi, H., & Tanaka, S. 1987, Phys. Rep., 149, 91.

[63] Slattery, W. L., Doolen, G. D., & Dewitt, H. E. 1980, Phys. Rev. A, 21, 2087.

[64] Khondaker, S., Mannan, A., Chowdhury, N. A., & Mamun, A. A. 2019, Contrib. Plasma Phys., 59, e201800125.

[65] Shah, A., & Saeed, R. 2009, Physics Letters A, 373, 4164.

[66] Wazwaz, A. M. 2008, Nonlinear Sci. Numer. Simul., 13, 584.

[67] Shukla, P. K. 2000, Phys. Plasmas, 7, 1044.

[68] Lavagno, A., & Pigato, D. 2011, Eur. Phys. J. A, 47, 52.

[69] Plastino, A. R., & Plastino, A. 1993, Phys. Lett. A, 174, 384.

[70] Feron C., & Hjorth, J. 2008, Phys. Rev. E, 77, 022106.

[71] Krimigis, S. M., Carbary, J. F., Keath, E. P., Armstrong, T. P., Lanzerotti, L. J., & Gloeckler, G. 1983, J. Geophys. Res., 88, 8871.