

## Study the Effect of Perturbation in the Evolution of the Filamentary Molecular Clouds

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**Abstract.** Observatory studies indicate that stars are caused by the collapse of dense molecular clouds, and thermal instability can be a factor in creating this collapse. As the formation of stars occurs in molecular clouds, the evolution of molecular clouds is important. In this study, the stability of the magnetized filamentary molecular clouds and their instability growth rate has been investigated. We consider the linear thermal instability of magnetized filament. We showed that the magnetic field makes the filament more stable against thermal instability. Also, increasing the intensity of the magnetic field helps to reduce the growth rate of instability.

*Keywords:* Perturbation, Filamentary molecular clouds, Magnetic field

## 1 Introduction

One of the most important issue in astrophysics is the star formation in molecular clouds. Study of dynamic and evolution of molecular cloud will help a lot to understand star formation. There are various structures in molecular cloud. One of them which many researchers studied is filamentary structures that they are in equilibrium or evolution [1, 2, 3]. Various processes such as gravity, magnetic field and heating govern molecular cloud formation. Thermal instability often occurs in astrophysical medium that considered by different researchers [4, 5, 6, 7].

Parker [8] has suggested that the condensation phenomenon could be a consequence of the instability in the thermal equilibrium of a diffuse medium. Stodolkiewicz [9] studied the gravitational instabilities of MHD systems. The first comprehensive analysis of thermal instability in astrophysical gases has been performed by Field [10]. He showed that in a uniform medium, thermal instability can lead to formation of high density medium. The thermal instability processes have been studied in an expanding gravitational medium by Gomez-Pelaez and Moreno-insertis [11]. It seems that ambipolar diffusion or neutral-ion friction are important mechanisms of energy dissipation in molecular clouds. Nejad-Asghar and Ghanbari studied the above concept and it's effect on molecular cloud thermal instability [12]. They found the conditions of linear thermal instability in small regions leads to the formation of molecular clouds using linear perturbation in MHD equations and comparison with time scale in molecular clouds [13]. Also, they studied synamatic of linear thermal instability of a self-gravity magnetized molecular cloud that unperturbed root is a local expansion or compressional state [14, 15].

Shadmehri [16] studied the magnetized thermal instability in ionized plasma including ambipolar diffusion, Ohm and Hall effect. Also, they studied growth rate of thermal compression state. Thus, thermal instability can be introduced as the most important process in

clump formation in molecular clouds [17, 18]. Recent observations showed that the strength of the magnetic field of molecular clouds is about 10mG [19]. These magnetic fields are strong enough to protect gravitational collapse in molecular cloud. So, we can not neglect the effect of magnetic fields in filamentary molecular clouds evolution. Khesali and Ghoreyshi (2014) studied the instability criterion of a self-gravity system that included Hall and ambipolar-diffusion effect [20].

Hosseinirad et al. (2017) investigated the gravitational instability of a filamentary molecular cloud in non-ideal magnetohydrodynamics [21]. They showed that a more efficient ambipolar diffusion leads to an enhancement of the growth of the most unstable mode, and to the increase of the fragmentation scale of the filament. Also, Hosseinirad et al. (2018) considered non-isothermal filament and showed that in absence of the magnetic field, a softer equation of state cause to more gravitational instability [22].

Understanding how evolution and star formation, requires a thorough examination of the problem of molecular cloud evolution as a place for star formation. According to the observational data, collapsing is possible when instability of the molecular cloud and star formation happens. In this work, we study the perturbation in the filamentary molecular clouds and obtain the regions of stability and instability as well as investigate the growth rate of instability.

## 2 General Formulation

In this section, we introduce a set of equations for system. We solve these equations by use of linear perturbation method and analyze in the next section. Considering the symmetry axis, the governing equations for molecular clouds of a rotating magnetized string, are as follow

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{V}) = 0, \quad (1)$$

$$\frac{\partial \rho}{\partial t} + (\vec{V} \cdot \vec{\nabla} \vec{V}) = -\vec{\nabla} \psi - \frac{\vec{\nabla} P}{\rho} + \frac{1}{\rho} \frac{\vec{J} \times \vec{B}}{c}, \quad (2)$$

$$\nabla^2 \psi = 4\pi G \rho, \quad (3)$$

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{V} \times \vec{B}), \quad (4)$$

$$\frac{1}{\gamma-1} \left( \frac{\partial P}{\partial t} + V_r \frac{\partial P}{\partial r} \right) + \frac{\gamma}{\gamma-1} P \vec{\nabla} \cdot \vec{V} + \rho \Omega = 0, \quad (5)$$

$$\frac{\partial}{\partial t} (r V_\varphi) + V_r \frac{\partial}{\partial r} (r V_\varphi) = 0, \quad (6)$$

$$P = \frac{R}{\mu} \rho T, \quad (7)$$

$$\begin{aligned} \rho &= \delta \rho + \rho_0, & P &= \delta P + P_0, & \Omega &= \delta \Omega + \Omega_0, & \psi &= \delta \psi + \psi_0, \\ B &= \delta B + B_0, & T &= \delta T + T_0, & V &= \delta V + V_0. \end{aligned} \quad (8)$$

We assume that the environment is homogeneous, so we have

$$\begin{aligned} V_0 &= 0, \quad V_r = U_r, \quad V_\varphi = U_\varphi = r\Omega, \quad \frac{\partial P_0}{\partial t} = 0, \quad \frac{\partial \rho_0}{\partial t} = 0, \\ \frac{\partial B_0}{\partial t} &= 0, \quad \vec{\nabla} P_0 = 0, \quad \vec{\nabla} \rho_0 = 0, \quad \vec{\nabla} T_0 = 0, \quad \vec{\nabla} \times \vec{B}_0 = 0. \end{aligned} \quad (9)$$

We define perturbation as  $\varphi = \varphi_1^{(\omega t + ikr)}$ , then

$$\omega \rho_1 + i\rho_0(\vec{K} \cdot \vec{U}_1) = 0, \quad (10)$$

$$\omega \vec{U}_1 \rho_0 + i\vec{K} P_1 + i\vec{K} \rho_0 \psi_1 - \frac{i}{4\pi}(\vec{K} \cdot \vec{B}_0)\vec{B}_1 - \frac{i}{4\pi}(\vec{B}_1 \cdot \vec{B}_0)\vec{K} = 0, \quad (11)$$

$$r^2 \omega \Omega_1 + 2r \Omega_0 U_1 = 0, \quad (12)$$

$$\psi_1 \left( \frac{iK}{r} - K^2 \right) = 4\pi G \rho_1, \quad (13)$$

$$\vec{B}_1 \omega + i\vec{B}_0(\vec{K} \cdot \vec{U}_1) - i(\vec{K} \cdot \vec{B}_0)\vec{U}_1 = 0, \quad (14)$$

$$\frac{\omega}{\gamma - 1} P_1 + \frac{i\gamma}{\gamma - 1} P_0(\vec{K} \cdot \vec{U}_1) + \rho_0(\Omega_\rho \rho_1 + \Omega_T T_1) = 0, \quad (15)$$

$$\frac{P_1}{P_0} - \frac{\rho_1}{\rho_0} - \frac{T_1}{T_0} = 0, \quad (16)$$

in which  $\Omega_\rho = \left( \frac{\partial \Omega}{\partial \rho} \right)$  and  $\Omega_T = \left( \frac{\partial \Omega}{\partial T} \right)$  are for balanced state. According to the following equations,

$$\vec{B} = B_\varphi \hat{e}_\varphi, \quad \vec{K} = K \hat{e}_r, \quad \vec{U} = u_r \hat{e}_r, \quad (17)$$

therefore

$$\omega \rho_1 + iK \rho_0 u_{1r} = 0, \quad (18)$$

$$\omega u_{1r} + \frac{ik}{\rho_0} P_1 + iK \psi_1 - 2r \Omega_0 \Omega_1 + \frac{1}{4\pi \rho_0} B_{0\varphi} B_{1\varphi} \left( \frac{2}{r} + iK \right) = 0, \quad (19)$$

$$r^2 \omega \Omega_1 + 2r \Omega_0 u_{1r} = 0, \quad (20)$$

$$\psi_1 \left( \frac{iK}{r} - K^2 \right) = 4\pi G \rho_1, \quad (21)$$

$$B_{1\varphi} \omega + iK B_{0\varphi} u_{1r} = 0, \quad (22)$$

$$(\rho_0 \Omega_\rho - \frac{\gamma \omega}{\gamma - 1} \frac{P_0}{\rho_0}) \rho_1 + \frac{\omega}{\gamma - 1} P_1 + \rho_0 \Omega_T T_1 = 0, \quad (23)$$

$$\frac{P_1}{P_0} = \frac{\rho_1}{\rho_0} + \frac{T_1}{T_0}. \quad (24)$$

By use of the following equations

$$\begin{aligned} \rho_1 &\rightarrow \frac{\rho_1}{\rho_0}, & P_1 &\rightarrow \frac{P_1}{P_0}, & T_1 &\rightarrow \frac{T_1}{T_0}, & u_{1r} &\rightarrow \frac{u_{1r}}{r\Omega_0}, & \Omega_1 &\rightarrow \frac{\Omega_1}{\Omega_0}, & \omega_1 &\rightarrow \frac{\omega_1}{\Omega_0}, \\ B_{1\varphi} &\rightarrow \frac{B_{1\varphi}}{B_{0\varphi}}, & \psi_1 &\rightarrow \frac{\psi_1}{\psi_0}. \end{aligned} \quad (25)$$

We extract dimensionless forms of the above equation, in which

$$y \frac{\rho_1}{\rho_0} + im \frac{u_{1r}}{r\Omega_0} = 0, \quad (26)$$

$$ym \frac{u_{1r}}{r\Omega_0} + \frac{i}{\gamma} \frac{P_1}{P_0} + a \frac{\psi_1}{\psi_0} - \frac{2m^2}{\varepsilon} \frac{\omega_1}{\Omega_0} + \alpha \left( \frac{2}{\varepsilon} + i \right) \frac{B_{1\varphi}}{B_{0\varphi}} = 0, \quad (27)$$

$$y \frac{\omega_1}{\Omega_0} + \frac{2m}{\varepsilon} \frac{u_{1r}}{r\Omega_0} = 0, \quad (28)$$

$$\left( \frac{i}{\varepsilon} - 1 \right) \frac{\psi_1}{\psi_0} = \frac{1}{\varepsilon^2} \frac{\rho_1}{\rho_0}, \quad (29)$$

$$y \frac{B_{1\varphi}}{B_{0\varphi}} + im \frac{u_{1r}}{r\Omega_0} = 0, \quad (30)$$

$$\frac{P_1}{P_0} = \frac{\rho_1}{\rho_0} + \frac{T_1}{T_0}. \quad (31)$$

We substitute the following equations in equations (26)-(31)

$$\begin{aligned} y &= \frac{\omega}{kc_s}, & m &= \frac{r\Omega_0}{c_s}, & \varepsilon &= kr, & c_s^2 &= \gamma \frac{P_0}{\rho_0}, & c_a^2 &= \frac{B_{0\varphi}^2}{4\pi\rho_0}, & \tilde{c}_a^2 &= \frac{c_a^2}{r^2\Omega_0^2}, \\ \tilde{c}_s^2 &= \frac{c_s^2}{r^2\Omega_0^2}, & \sigma_\rho &= \frac{k_\rho}{k}, & \sigma_T &= \frac{k_T}{k}, & a &= \frac{k\psi_0}{c_s^2} = 4\pi G \left( \frac{m}{\Omega_0} \right)^2 \rho_0, \\ \alpha &= \frac{c_a^2}{c_s^2}, & k_\rho &= \frac{\mu(\gamma-1)\Omega_\rho\rho_0}{Rc_sT_0}, & k_T &= \frac{\mu(\gamma-1)\Omega_T}{Rc_s}. \end{aligned} \quad (32)$$

Then, we solve the equations (26)-(31) and we put matrix coefficient equal zero which leads to the following third order equation

$$b_0\omega^3 + b_1\omega^2 + b_2\omega + b_3 = 0. \quad (33)$$

In this equation,  $b_i$  is the function in terms of equilibrium values. We used Routh-Harwitz method for the study of the stability and instability region. We form  $\Delta_1$  and  $\Delta_2$  and  $\Delta_3$  in this method

$$\Delta_1 = |b_1|, \quad \Delta_2 = \begin{vmatrix} b_1 & b_0 \\ b_3 & b_2 \end{vmatrix}, \quad \Delta_3 = \begin{vmatrix} b_1 & b_0 & 0 \\ b_3 & b_2 & b_1 \\ 0 & 0 & b_3 \end{vmatrix}. \quad (34)$$

According to this method, if the determinism of these  $\Delta$  is positive the favorite region will be stable ( $\Delta_1 > 0, \Delta_2 > 0$  and  $\Delta_3 > 0$  region has stability) and if the determinism of one of these  $\Delta$  is negative ( $\Delta_i < 0$ ) we will have an instability. These coefficients are obtained based on a set of  $\alpha$  and  $\varepsilon$ , in which  $\alpha$  is dependent on the magnetic field and  $\varepsilon$  is dependent on the distance obtained by giving these coefficients the plot of the stability regions and the growth rate.

### 3 Diagram Analysis

Crutcher et al. (2010) find that  $n < 300 \text{ cm}^{-3}$  (in the diffuse interstellar medium). They also find strong evidence for B in molecular clouds being  $n > 300 \text{ cm}^{-3}$  to  $10^5 - 10^6 \text{ cm}^{-3}$  [19]. In typical molecular cloud  $T \sim 10^\circ \text{ K}$ , the sound speed is  $0.19 \text{ kms}^{-1}$  (e.g. [23, 24]). The Alfven speed can also be defined as  $V_A \equiv \frac{B_0}{\sqrt{4\pi\rho_0}} = 0.5 \text{ kms}^{-1} \left(\frac{B_0}{10\mu\text{G}}\right) \left(\frac{n_{\text{H}_2}}{10^3 \text{ cm}^{-3}}\right)^{-\frac{1}{2}}$  [20]. Opher et al. (2009) found that the magnetic field strength in the local interstellar medium is  $3.7 - 5.5 \mu\text{G}$  [25]. Also, their model (2006) using Voyager1 and 2 data showed an interstellar magnetic field of about  $2 \mu\text{G}$  [26]. Pogorelov et al (2006) showed that the cause of the deflection of the interstellar neutral hydrogen flow from the direction of propagation of neutral helium in the inner heliosheath is the presence of a magnetic field equal to  $4 \mu\text{G}$  [27].

Since the sound speed is obtained from the  $V_S = \sqrt{\frac{RT}{\mu}}$  ( $R = 8.314 * 10^7 \text{ erg g}^{-1} \text{ k}^{-1}$  and  $\mu = 2$  for typical molecular cloud), so we use typical values:  $B \approx 4 \mu\text{G}$ ,  $T \sim 10^\circ \text{ K}$  and  $n = 10^3 \text{ cm}^{-3}$ . In this case, the value of  $\alpha$  is equal to 1. For stronger magnetic fields, we typically consider  $\alpha$  equal to 10.

The amount of energy per unit area decreases by increasing  $\alpha$  in this case we expect the instability to be lower. The instability region in an environment without a magnetic field decreases, because of the effect of the temperature gradient on the instability of the system compared with the density gradient, as shown in Fig. 1. In Fig. 2, we investigate the effect of the magnetic field on the stability of the system. The effect of the magnetic field is more stabilizing the system which the system becomes more stable, which is consistent with the results of previous works such as Khesali et al. (2012) [28]. In fact, the magnetic field has the effect of freezing stabilizing the system.

In Fig. 3, we plot the stability region for two different magnetic fields. The diagrams show that the system is more stable by increasing the strength of the magnetic field. Also a magnetic brake is observed in the collapse of the system [29, 30, 31]. Comparing the Fig. 3 and Fig. 4, shows that the increase of  $\varepsilon$  decreases the stability area, which is similar to the Fig. 1.

By considering the real part  $\frac{\omega}{kc_s}$  and its plotting in  $\frac{k}{k_p}$ , we examine the growth rate of the system. We first consider the system without a magnetic field. Increasing  $\varepsilon$  reduces the energy per unit area and as expected, the growth rate of the system instability decreases as shown in Fig. 5. We consider the effect of increasing  $\varepsilon$  on the magnetized system. In this case, by increasing the growth rate decreases system instability as shown in Fig. 6. In Fig.

7, we compare the effect of the presence and absence of a magnetic field in the system. As expected, the presence of the magnetic field causes the system to become more stable, and the growth rate of the system with a magnetic field is much less than the system without magnetic field which is in accordance with previous work [32].

Also, in Fig. 8, in accordance with the preceding explanations, we consider two magnetized environments with different intensity of the magnetic field in which increasing the effect of the magnetic field reduces the growth rate of the instability.

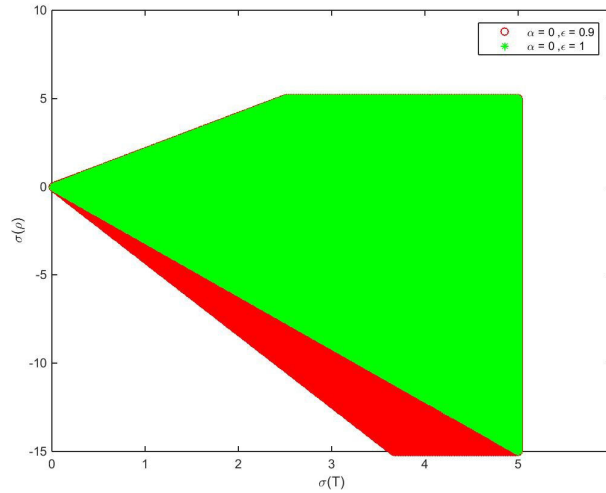


Figure 1: Regions of stability in the  $\sigma_T - \sigma_\rho$  plane when the magnetic field is absent ( $\alpha = 0$ ).

## 4 Conclusion

In this paper, we studied the stability of the magnetized filamentary molecular clouds. We concluded that:

1. When  $\varepsilon$  increases the energy of the system decreases, as a result, the instability region also decreases.
2. Since the wave vector is perpendicular to the magnetic field, the area of stability region increases with increasing  $\alpha$  (increase of the magnetic field intensity).
3. When  $\varepsilon$  increases the growth rate of system instability decreases.
4. Increase of the magnetic field in the system decreases the growth rate of instability.

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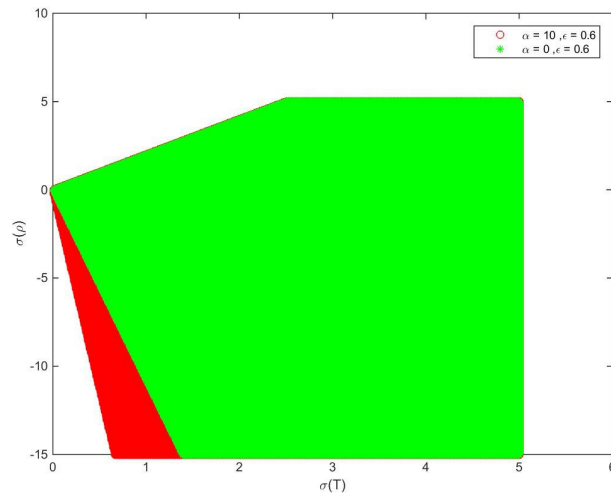


Figure 2: Regions of stability in the  $\sigma_T - \sigma_\rho$  plane with the effect of the magnetic field ( $\alpha = 0$  and 10).

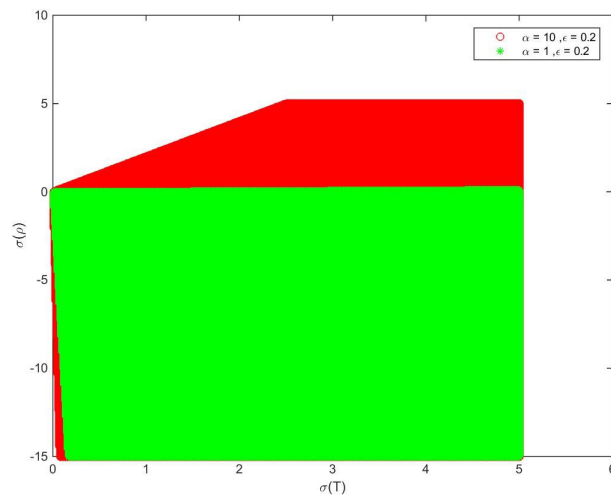


Figure 3: Regions of stability in the  $\sigma_T - \sigma_\rho$  plane with different values of the magnetic field ( $\alpha = 1$  and 10).

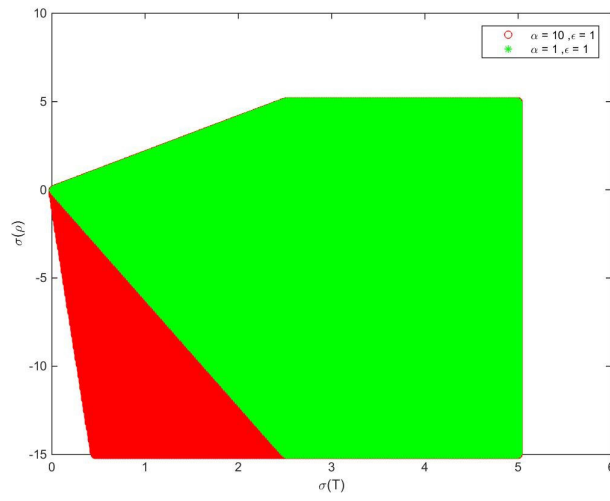


Figure 4: Same as the Fig. 3 but the  $\varepsilon = 1$ .

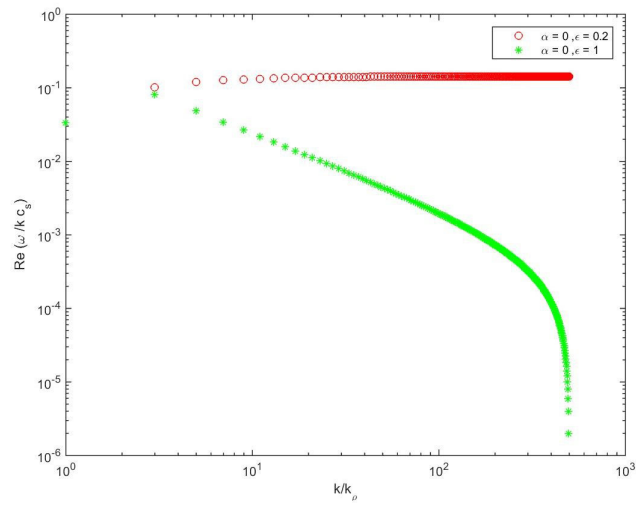


Figure 5: The growth rate of the system instability when the magnetic field is absent ( $\alpha = 0$ ).



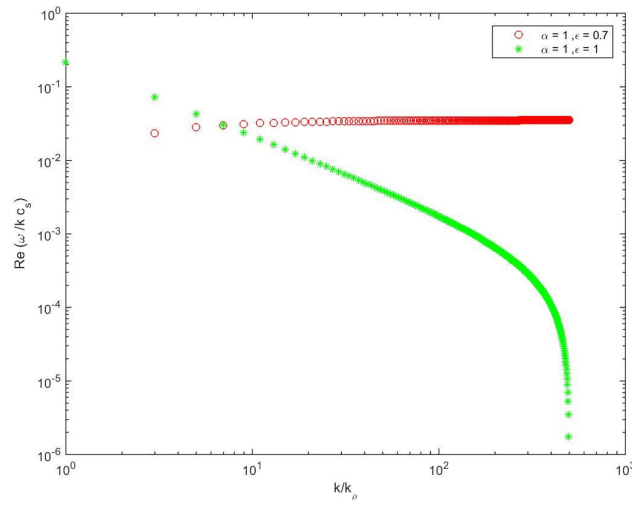


Figure 6: The effect of the increase of  $\varepsilon$  on the growth rate of the instability of the magnetized system ( $\alpha = 1$ ).

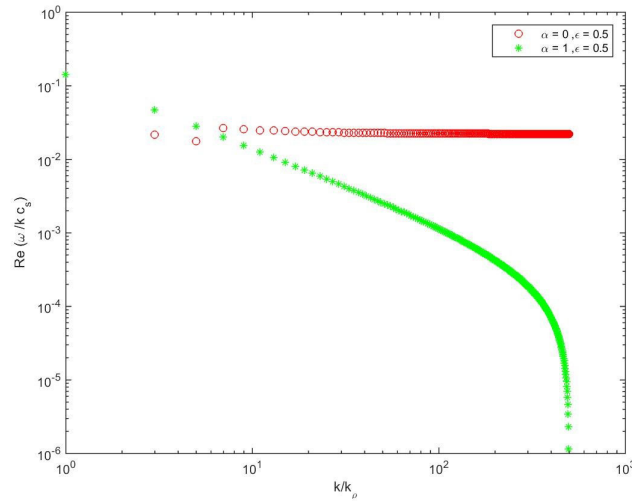


Figure 7: Compare the effect of the presence and absence of a magnetic field in the growth rate of the system instability ( $\alpha = 0$  and 1).

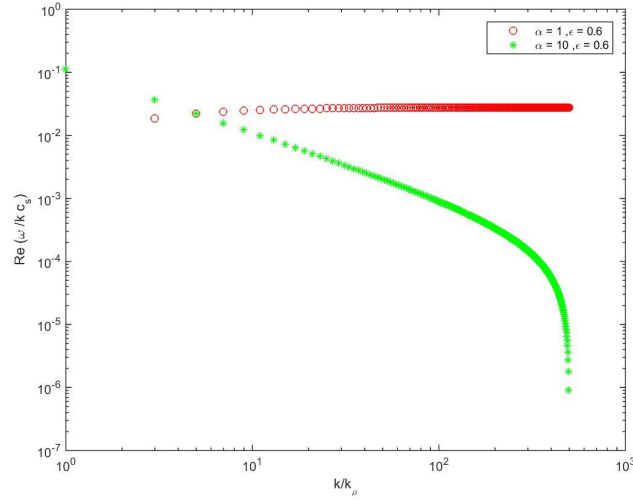


Figure 8: Same as the Fig. 7 with different intensity of the magnetic field ( $\alpha = 1$  and  $10$ ).

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