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Collisional and Radiative Energy Loss in QED and QCD Plasmas

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Abstract. Evaluating the energy loss of an electrically (color) charged particle crossing a high-temperature QED (QCD) plasma at its thermal equilibrium is studied. The average energy loss depends on the particle characteristics, plasma parameters, and QED (QCD) coupling constant α (α_s). All processes through which the energy of a particle changes can be categorized into two main mechanisms: elastic collisions and radiation through bremsstrahlung. We have introduced the final results of collisional and radiation energy loss for an electrically charged particle in a QED plasma, as well as a quark in a QCD plasma. The suppression due to radiation is presented using the Landau-Pomeranchuk-Migdal effect.

Keywords: QED plasma, Quark Gluon Plasma, energy loss, Collision, Radiation.

1 Introduction

Studying the dynamics of charged particles while traveling through a medium is an exciting problem in both theoretical and experimental views. A particle may contain an electric and/or color charge, so we have to apply QED and/or QCD theories to model the particle-medium interaction [1]. The energy of charged particles changes through well-known interactions, divided into collisional and radiation processes. According to the particle specifications (its mass, initial energy, and so on) and medium identifications (density, temperature, ...), some possible interactions become dominant. Thus, theoretical evaluation of the problem depends on the initial conditions of charged particle and the medium. For example, the main contribution to the energy loss for a low energy heavy particle with electric charge in a color singlet state (like protons) is due to collisions with individual atomic electrons, while for light electrically charged particles (like electrons) with similar energies, the energy loss is occurred due to radiation through bremsstrahlung process. The same situations may happen for heavy (light) color-charged particles.

The primary situation in which energy loss plays a crucial role is passing an energetic particle through a hot (ultra)relativistic QED (QCD)plasma. The first studied problem was the energy loss of a heavy muon traveling through a QED plasma. Note that the QED coupling parameter α does not depend on the medium temperature, and it is constant in a vacuum. The electromagnetic waves have only transverse polarization, and the photon is massless in a vacuum. In usual media, the longitudinal component has to be zero to satisfy the gauge invariance of QED. When there are enough fermions in the universe, photons interact with the medium due to the vacuum polarization and acquire a dynamically generated mass because of its interaction with the fermions. In this situation, the QED coupling parameter α becomes a function of temperature due to its interaction with the medium. Calculation of energy loss for a relativistic heavy fermion in a hot QED plasma is an interesting problem that has been widely investigated [2, 3]. A similar problem may happen if a particle carrying color charge passes through a hot QCD medium. If the medium temperature T is very high ($T \gg \lambda_{QCD}$), this medium is a quark-gluon plasma (QGP), i.e., a system of quarks and gluons with a small effective Coulomb-like interaction. It is expected to find the QGP in relativistic heavy-ion collisions or in the core of high-density astronomical objects like neutron stars or quark stars [4] . The most detailed evaluation of energy loss for a fast muon in QED plasma and heavy quark in the QGP has been done by Braaten and Thoma [2, 5]. The first signature of heavy flavor energy loss has been calculated by Bjorken [6], which has now been well-established through RHIC and LHC experiments [7, 8]. A similar effect for low energy cold nuclear matter also has been observed in some other experiments [9, 10]. Calculations show that the dominant energy loss mechanism for a light particle in QCD (QED) plasma is gluon (photon) radiation [11, 12, 13]. For heavy quarks, radiation is further suppressed and the relative contribution of collisional losses is dominant [14]. When the particle mass is not too large, i. e. $M \ll \sqrt{\alpha E T}$ ($M \ll \sqrt{\alpha_s E T}$) in QED (QCD) case, and the travel distance L is not too small, radiative energy loss still dominate over collisional loss.

Studying the energy loss of a colored charge particle produced in a QGP is of phenomenological interest to heavy-ion collisions. However, it is also instructive to discuss the problem of energy loss in QED. Thus, we introduce the results of particle energy loss in QED and QCD plasmas. The collisional contribution to the energy loss in QED and QCD plasmas is studied in section 2. In sections 3 and 4, we discuss the radiative energy loss in high temperature QED and QCD plasmas respectively. The section 5 is devoted to conclusions and remarks.

2 Collisional energy loss

At first, we consider an electrically charged relativistic particle with mass M in a QED plasma consisting of electrons, positrons and photons. The particle may lose its energy through Coulomb scattering with electrons and positrons, or Compton collisions with photons. It is shown that the energy loss due to Coulomb and Compton scattering are of the same order. It may be noted that, Compton scattering is rare but, it is very efficient in energy transfer between particle and plasma environment. Summing the Coulomb and Compton contributions to the collisional energy loss becomes [3]

$$-\frac{dE}{dx} = \frac{e^4 T^2}{48\pi}\left[\ln\frac{eE}{e^2 T} + \frac{1}{2}\ln\frac{ET}{M^2} + A\right] \tag{1}$$

where $A = \ln 48 + \frac{3}{2}\left(\frac{\zeta'(2)}{\zeta(2)} - \gamma\right) - \frac{7}{6} \approx 0.984$. The $\zeta(x)$ and $\zeta'(x)$ are the Riemann zeta function and its derivative. For light particle energy loss due to Compton scattering (the term $\ln\frac{ET}{M^2}$) is negligible.

The first calculation of quark collisional energy loss has been done by Bjorken [6]. He calculated the energy loss of a massless quark due to elastic scattering off the QGP constituents by averaging the cross section for elastic scattering times the mean energy transfer over the thermal distribution. The result is as follows

$$-\frac{dE}{dx} = \frac{16\pi}{9}\alpha_s^2 T^2 \ln\left(\frac{4pT}{k_D^2}\right)\left[exp\left(-\frac{k_D}{T}\right)\left(1+\frac{k_D}{T}\right)\right] \tag{2}$$

where p is the particle momentum, T is the QGP temperature and $k_D = \sqrt{3}m_g$ while $m_g^2 = \frac{4\pi\alpha_s T^2}{3}(1 + \frac{n_f}{6})$.

The energy loss also has been calculated by combining techniques of plasma physics and high temperature QCD. The induced chromoelectric field in the wake of a high energy quark is used to calculate $\frac{dE}{dx}$. That induced field is related to the longitudinal and transverse dielectric functions, which can be expressed in turn in terms of the gluon self-energy. Through this method, we have

$$-\frac{dE}{dx} = \frac{16\pi}{9}\alpha_s^2 T^2 \ln\left(\frac{k_{max}}{k_D}\right)\frac{1}{v^2}\left(v + \frac{v^2-1}{2}\ln\left(\frac{1+v}{1-v}\right)\right) \tag{3}$$

in which $k_{max} \approx \frac{4pT}{\sqrt{p^2+M^2}-p+4T}$ while p is the particle momentum.

In another method, the energy loss of a quark with energy E has been calculated for two different limits: $E \ll \frac{M^2}{T}$ and $E \gg \frac{M^2}{T}$. The heavy quark has a kinetic energy much greater than QGP temperature T. In the limit $E \ll \frac{M^2}{T}$, contributions to the energy loss has been obtained from the corresponding QED calculation [2]. For converting results into QCD problem, "e" in QED calculations is replaced by the $g_s = \frac{4}{3}\sqrt{4\pi\alpha_s}$ [15]. The thermal photon mass $m = eT/3$ also is replaced by the thermal gluon mass $m_g = g_s T\sqrt{\frac{1+n_f/6}{3}}$ where n_f is the number of active flavors in the QGP. In this condition, we have

$$-\frac{dE}{dx} = \frac{8\pi\alpha_s^2 T^2}{3}\left(1 + \frac{n_f}{6}\right)\left[\frac{1}{v} - \frac{1-v^2}{2v^2}\ln\frac{1+v}{1-v}\right]\ln\left(2^{\frac{n_f}{6+n_f}}B(v)\frac{ET}{m_g M}\right) \tag{4}$$

where $B(v)$ is a smooth function starts from $B(0) = 0.604$, increases up to $B(0.88) = 0.731$ and then decreases to $B(1) = 0.629$ [2].

In the limit $E \gg \frac{M^2}{T}$ soft and hard contributions in the energy loss have been calculated separately and then added to each other to find

$$-\frac{dE}{dx} = \frac{8\pi\alpha_s^2 T^2}{3}\left(1 + \frac{n_f}{6}\right)\ln\left(2^{\frac{n_f}{12+2n_f}}0.920\frac{\sqrt{ET}}{m_g}\right) \tag{5}$$

For intermediate region $E \approx M^2/T$ we have to use two limits which connected smoothly in E_{cross}. Calculations show that we can use (4) up to $E_{cross} = 1.8\frac{M^2}{T}$ and then switch to (5).

3 Radiative energy loss

Moving particle in a QED/QCD plasma radiates energy through bremsstrahlung process. In QED case, we consider an energetic charged particle entering the plasma with its shell. Then, we evaluate the radiative energy loss of an on-shell charged particle traveling through hot QED plasma. We will assume that particle energy is greater than the plasma temperature, i. e. $E \gg T$. Also, we assume that particle energy is greater than the photon thermal energy: $\mu \approx eT \gg M$, where M is the particle mass. The charged particle is scattered by the plasma constituents, changes the direction of its motion and emits bremsstrahlung photons. We consider a thin plasma layer L which is very smaller than the particle mean free path λ i. e. $L \ll \lambda$. In this situation, the probability that the charged particle undergoes a Coulomb scattering, ($\frac{L}{\lambda} \ll 1$) is negligible, and thus, the particle will lose its energy only

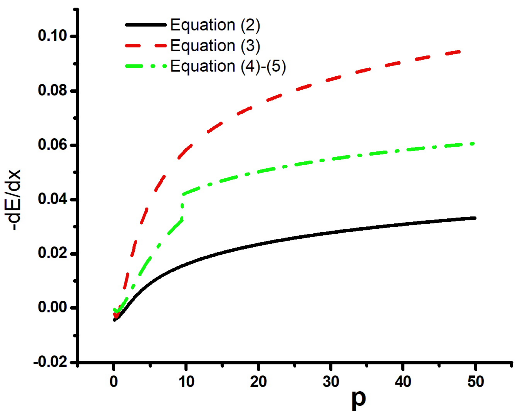

Figure 1: Collisional energy loss as functions of particle momentum in a QGP at thermal equilibrium.

through the radiation process. For a light particle, and considering the Bethe-Heitler (BH) approximation, the radiation energy loss becomes

$$-\frac{dE}{dx} \approx \alpha^2 ET \ln \frac{\mu^2}{M^2} \tag{6}$$

For a massive particle ($M \gg \mu \approx eT$), the intensity of the BH radiation is suppressed by the factor $\frac{\mu^2}{M^2}$. Thus radiation energy loss for a heavy quark in a thin layer of QGP becomes

$$-\frac{dE}{dx} \approx \alpha^3 T^3 \frac{E}{M^2} \tag{7}$$

The radiative energy loss for a quark also can be calculated using the reaction operator formalism (DGLV) [16, 17] and employing the generalized dead cone approach [18]. The DGLV formalism is based on siutably expansion of the quark energy loss in terms of the number of the scatterings experienced by the propagating quark. In the single hard scattering limit, only the leading order term is considered and the radiative energy loss of a heavy quark in a QGP is calculated as

$$\begin{aligned}
-\tfrac{dE}{dx} &= 24\alpha_s^3 \rho_{QCD} \tfrac{1}{\mu_g}(1-\beta_1)\left(\sqrt{\tfrac{1}{1-\beta_1}\ln\tfrac{1}{\beta_1}}-1\right)F(\delta) \\
F(\delta) &= 2\delta - \tfrac{1}{2}\ln\left(\frac{1+\frac{M^2}{s}e^{2\delta}}{1+\frac{M^2}{s}e^{-2\delta}}\right) - \left(\frac{\frac{M^2}{s}\sinh(2\delta)}{1+2\frac{M^2}{s}\cosh(2\delta)+\frac{M^4}{s^2}}\right) \\
\delta &= \tfrac{1}{2}\ln\left[\tfrac{1}{1-\beta_1}\ln\left(\tfrac{1}{\beta_1}\right)\left(1+\sqrt{1-\frac{1-\beta_1}{\ln\left(\frac{1}{\beta_1}\right)}}\right)^2\right] \\
C &= \tfrac{3}{2} - \tfrac{M^2}{48E^2T^2\beta_0}\ln\left[\frac{M^2+6ET(1+\beta_0)}{M^2+6ET(1-\beta_0)}\right]
\end{aligned} \tag{8}$$

where $\beta_1 = \frac{\mu_g^2}{CET}$, $\beta_0 = \sqrt{1-\frac{M^2}{E^2}}$, $\mu_g^2 = 4\pi\alpha_s T^2\left(1+\frac{n_f}{6}\right)$, $s = 2E^2 + 2E\sqrt{E^2-M^2} - M^2$, $\rho_{QGP} = \rho_q + \frac{9}{4}\rho_g$, $\rho_q = 16T^3\frac{1.202}{\pi^2}$ and $\rho_g = 9n_f T^3\frac{1.202}{\pi^2}$. It may be noted that strong coupling constant itself is a function of plasma temperature as

$$\alpha_s(T) = \frac{6\pi}{(33 - 2n_f)\ln\left(\frac{19T}{\Lambda_{\bar{m}s}}\right)} \tag{9}$$

while $\Lambda_{\bar{m}s}$ can be taken as $0.08GeV$.

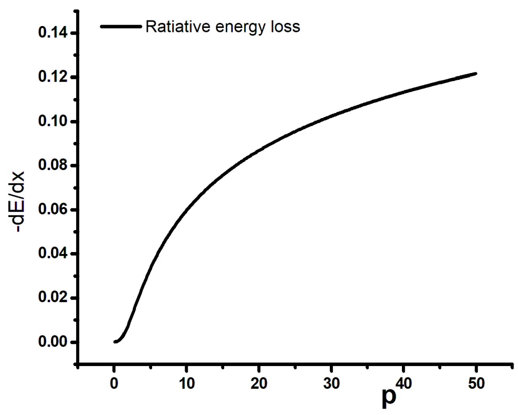

Figure 2: Radiative energy loss as a function of particle momentum in a QGP at thermal equilibrium.

For light charged particles, the difference in the behavior of energy loss in QED and QCD is mostly due to the different problem setting and initial conditions. In QED, we study the energy losses of a charged particle (like electron) coming from infinity. In QCD case, the quantity of physical interest is the medium-induced energy loss of a parton produced within the QCD plasma. In the case of an electrically charged particle produced within a QED plasma, the medium-induced radiative energy loss behaves similar to the radiation energy loss in the QCD plasma, despite the photon and gluon radiation spectra being drastically different because the bremsstrahlung cones for soft gluons are broader than for soft photons. On the other hand, the average radiative loss of an 'asymptotic light parton' crossing a QCD plasma is similar to that of an asymptotic charged particle crossing a QED plasma. For heavy particles, the difference between radiative energy loss in QED and in QCD is more pronounced, even when the same physical situation is considered.

4 Time evolution of particle spectra

In some astrophysical phenomena, particles with initial distribution function $f_i(p)$ traveling through a QED/QCD plasma in its thermal equilibrium state with temperature T. It is possible that plasma temperature change in time because of expansion due to its high internal pressure. At the same time, the momentum distribution function $f(p)$ of incident particles evolves because of their Brownian motion in the expanding plasma. we can look at this process as an interaction between equilibrium and nonequilibrium degrees of freedom. For example, consider heavy quarks (HQ) passing through expanding quark-gluon plasma. We can study the time evolution of HQ distribution function by solving the Fokker-Planck

(FP) equation [19]. For a uniform plasma the FP equation becomes

$$\frac{\partial f(p)}{\partial t} = \frac{\partial}{\partial p}\left[p\mathfrak{A}(p)f(p) + \frac{\partial}{\partial p}\mathfrak{D}(p)f(p)\right] \tag{10}$$

where $\mathfrak{A}$ and $\mathfrak{D}$ are drag and diffusion coefficients. The drag coefficient can be calculated as follows

$$\mathfrak{A}(p) = -\frac{1}{p}\frac{dE}{dx} \tag{11}$$

The diffusion parameter is calculated by the Einstein relation as $\mathfrak{D}(p) = ET\mathfrak{A}(p)$ [20].

Let us study the time evolution of particle distribution functions while traveling in a thermally equilibrated dense QGP. We have used expressions (2), (4) and (5) to calculate drag and diffusion coefficients. The Gaussian distribution has been chosen as HQ initial condition. Figure 3 demonstrates variation of HQ distribution function using (2) for calculating drag and diffusion coefficients. Profile of HQ distribution function uniformly expands in time, while changing rate of distribution function for HQs with higher momentum is greater as compared with probability of finding low momentum HQs. It is interesting that the peak of distribution function shifts toward higher momentum values. Figure 4 demonstrates time evolution of Gaussian initial distribution function if we calculate the drag and diffusion coefficient using (4) and (5). Similar to previous calculations, the distribution function expands in time, but changing rate is not uniform. According to this figure, the population of HQs with higher momenta is rapidly damped while changing rate of population for particles with lower energy is slower. It may be noted that, energy loss for high and low momentum HQs in this approach is defined with different relations.

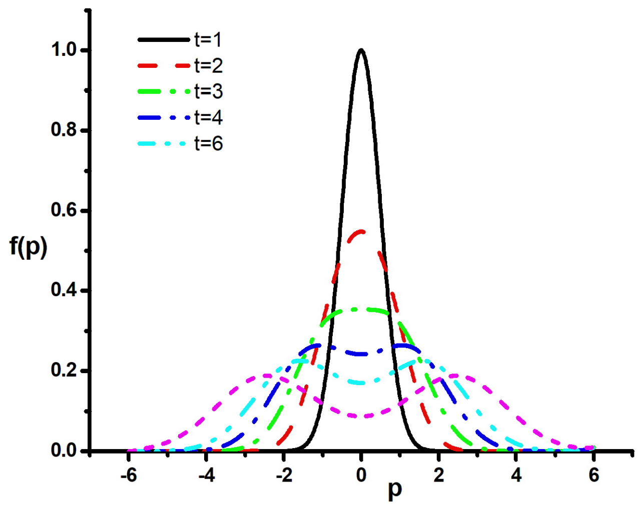

Figure 3: Time evolution of HQ distribution function if we calculate the drag and diffusion coefficient by (2). Initial distribution is the Gaussian function.

Figure 3 shows that rate of energy loss respect to the particle momentum calculating by the equation (3) is uniform. But the rate of energy loss calculated by equations (4) and (5)

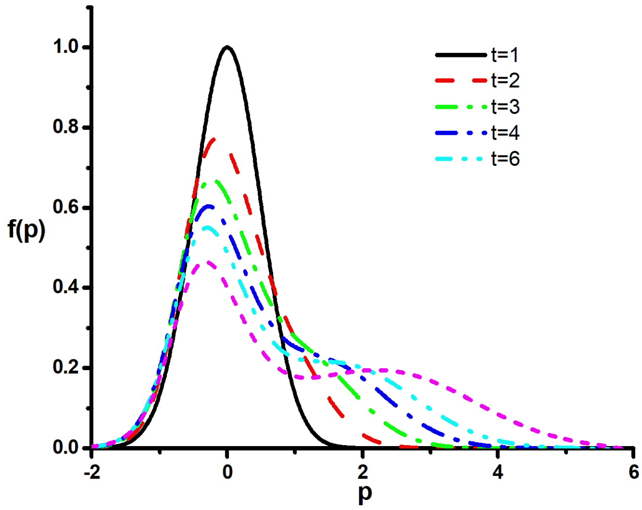

Figure 4: Time evolution of HQ distribution function if we calculate the drag and diffusion coefficient by (4) and (5). Initial distribution is the Gaussian function.

is not uniform. Energy loss for low momentum particles is not equal to the energy loss for fast particles.

5 Conclusion and Remarks

Collisional and radiation energy loss of charged particle while passing through QED/QCD dense plasmas is an important phenomenon in high energy physics and astrophysics. We have introduced presented relations for these issues using different approaches. A straight forward method to calculate energy loss of a massless charged particle due to elastic scattering off the QED/QGP plasma is averaging the cross section for elastic scattering times the mean energy transfer over the thermal distribution, which has been introduced by Bjorken. Another method is presented by employing the induced chromoelectric field in the wake of a high energy particle to calculate the energy loss. Induced field is related to the longitudinal and transverse dielectric functions. Radiative energy loss for charged particle is calculated using the reaction operator formalism and considering the generalized dead cone approach.

We have used these relations to calculate the drag and diffusion coefficient for numerically solving the time evolution of particle distribution function as traveling through a thermal bath using the Fokker-Planck equation. We showed that the Bjorken approach provides a uniform changing distribution function rate while its peak shifts toward higher momentum values. We cannot find a straight and exact relation for the particle energy loss in the chromoelectric field approach. Thus, it calculated for low energy and high energy limits. Using these relations provide different profile for the time evolution of particle distribution function.

Several problems could be investigated in further studies. It is interesting to apply these results in more realistic situations in QED/QCD plasmas. Time evolution of cosmic rays' distribution function while passing through the earth atmosphere is an interesting problem.

Explaining the creation of shock profiles in the core of super dense astrophysical objects can be investigated using the quark energy loss in a QCD plasma. These issues can be investigated in further works.

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