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The Radiative Transfer in Accretion Discs with Linear Plank Function: Role of Scattering Effect

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Abstract. Radiative transfer in a geometrically thin accretion disc with finite optical depth is considered under the plane-parallel approximation. The Eddington factor that is defined as the ratio of the mean intensity to radiation stress tensor, is assumed be constant. We have focused our attention on the scattering effect and the optical depth. The emergent intensity as well as other radiative quantities are analytically obtained related to the vertical structure of disc while a linear Planck function is applied. The effect of scattering on the radiative quantities is considered for two cases: (i) isothermal and (ii) temperature gradient, and both cases are assumed to be in local thermodynamical equilibrium (LTE) too. Our results show that scattering for an isothermal atmosphere is more significant than an atmosphere with temperature gradient. Moreover, the emergent intensity is changed by the disc optical depth. We also explore the limb-darkening effect for the both thick and thin optically discs, separately.

Keywords: accretion, accretion discs, opacity, radiative transfer, scattering

1 Introduction

Accretion disc around a central object is an accepted paradigm in order to describe several energetic phenomena either on small scales, like discs around Young stellar object (YSO), or Cataclysmic variable stars (CV) or on large scales, like AGNs or quasars. The main idea of accretion discs lies in the fact that falling matter releasing gravitational potential energy, heats the gas, and generates radiation. Moreover, the accreting masses possess a considerable amount of angular momentum per mass unit which has to be removed in order to be accreted into the central object. What causes the lost of angular momentum is the friction caused by turbulent viscosity working between adjacent gas layers in the disc. Since the process of the angular momentum removal occurs on slower timescales compared to the free-fall time, infalling gas with sufficiently high angular momentum can form a disc-lice structure around central body, which can be thin or thick depending upon the geometrical shape. The different models of accretion discs depending on the geometry of the disc (thin or thick), process of energy emission and outward angular momentum transfer have been extensively studied during the past four decades (see [10] for a review).

The study of radiation hydrodynamics is followed by several excellent monographs (e.g., [13, 16, 4]). In accretion discs, matter emits or absorbs radiation, while radiation gives (or remove) energy and momentum to (or from) matter. The behavior of radiation interacting with matter is known as radiative transfer. Radiative transfer in the accretion disc has been investigated in relation to the structure of a static disc atmosphere and the spectral energy distribution from the disc surface by several authors([14, 3, 15, 5]). The accretion-disc atmosphere differs from the stellar atmosphere in some aspects ([17, 1, 20, 10]): (i) Viscous heating in the atmosphere may exist as an energy source for radiation. (ii) The gravitational acceleration is quite different from that of a star. (iii) The optical depth of the disc is finite. (iv) The scattering may be dominant in some cases.

The radiative-transfer equations are given in many textbooks ([12, 13, 19, 11]). In order to solve these equations, some approximations are employed. In many cases the diffusion or Eddington approximation provides a satisfactory description in an optically thin regime as well as in an optically thick one. Moreover, in the accretion disc the physical quantities have gradients in the horizontal (radial) direction. However, the ratio of the radial gradient of the physical quantities to the vertical gradient is generally on the order of Z/r. Hence, the plane-parallel approximation, which is usual for the stellar atmosphere, can be also valid, as long as the disc is geometrically thin. Furthermore, gray approximation, where the opacity is assumed to be independent of frequency, and non-gray approximation, where the opacity does not depend on frequency, was developed under numerical treatments ([18, 8, 5]) and under analytical ones ([9, 2, 7]). The analytical solutions of the radiative-transfer equations beside the development of the numerical codes can be useful to clarify the properties of radiative transfer in such systems.

Fukue (2011) solved analytically the radiative transfer equations and studied scattering effects on the radiative quantities for the linear plank function and uniform-heating cases. The result of his work showed the scattering has a significant effect on the radiative quantities and the emergent intensity becomes a modified blackbody spectrum. In this paper, we reexamine radiative transfer equations for both optically thin and thick discs with a linear plank function and proper boundary conditions. Here, we assume that the optical depth of disc is finite which cause our work becomes different from that of Fukue (2011). In the next section, we will describe the basic equations. In section 3, we show analytical solutions. The final section is devoted to concluding remarks.

2 Basic Assumptions and Equations

In this section, we derive the radiative-transfer equations in the vertical direction (z). To begin with, we here consider the following standard assumptions:
(i) The disc is steady and axisymmetric.
(ii) It is also geometrically thin and plane parallel.
(iii) The viscous heating rate is concentrated at the equator or uniform in the vertical direction.
(iv) The non-gray approximation, where the opacity depend on frequency, is adopted.
(v) The disc atmosphere is under local thermodynamic equilibrium (LTE) i.e. $j_\nu = 4\pi\kappa_\nu B_\nu$ which j_ν, B_ν are the mass emissivity and Plank function, respectively.
(vi) As a closure relation, we use the Eddington approximation in the sense that $K_\nu = fJ_\nu$ where J_ν, K_ν and f are the mean intensity, the mean radiation stress and Eddington factor, respectively.

On the basis of these assumptions and simplifications, the radiative-transfer equations are involved in the frequency-integrated transfer equation, the zeroth moment equation, and the first moment equation that can be respectively written as follows:

$$\mu\frac{dI_\nu}{dz} = \rho\left[\frac{j_\nu}{4\pi} - (\kappa_\nu + \sigma_\nu)I_\nu + \sigma_\nu J_\nu\right], \tag{1}$$

$$\frac{dH_\nu}{dz} = \rho\left[\frac{j_\nu}{4\pi} - \kappa_\nu J_\nu\right], \tag{2}$$

$$\frac{dK_\nu}{dz} = -\rho\big(\kappa_\nu + \sigma_\nu\big)H_\nu, \tag{3}$$

where μ is the direction cosine ($= cos\theta$), I the specific intensity, H_ν the Eddington flux and ρ the gas density. The absorption opacity, κ_ν, and the scattering one, σ_ν, generally depend on the frequency. We define the source function as:

$$S_\nu = \frac{1}{\kappa_\nu + \sigma_\nu}\frac{j_\nu}{4\pi} + \frac{\sigma_\nu}{\kappa_\nu + \sigma_\nu}J_\nu = \epsilon_\nu B_\nu + (1-\epsilon_\nu)J_\nu, \tag{4}$$

where $\epsilon_\nu = \sigma_\nu/(\sigma_\nu + \kappa_\nu)$ is the photon destruction probability. Introducing the optical depth τ_ν as: $d\tau_\nu \equiv -\rho\big(\kappa_\nu + \sigma_\nu\big)dz$, we can rewrite the above equations as follow

$$\mu\frac{dI_\nu}{d\tau_\nu} = I_\nu - S_\nu, \tag{5}$$

$$\frac{dH_\nu}{d\tau_\nu} = J_\nu - S_\nu, \tag{6}$$

$$\frac{d(fJ_\nu)}{d\tau_\nu} = H_\nu, \tag{7}$$

Also, the total optical depth of the disc is:

$$\tau_{\nu 0} = -\int_h^0 \rho(\kappa_\nu + \sigma_\nu)dz, \tag{8}$$

where h is the disc half-thickness. The radiative-transfer equations governing atmosphere of such disc are represented by equations (5)-(7). It is still hard to solve this system of equations with several kinds (i.e. algebraic, differential and integral equations). Similar to the stellar atmosphere, we assume that the Planck function, $B(\nu)$, linearly be depend on the optical depth. In the next section, we will solve the basic equations of this system for a semi-infinite optical depth with proper boundary conditions.

3 Linear Plank function

In order to explain the scattering effect and the effect of the finite optical depth, we find radiative quantities for a simple case proposed for the stellar atmosphere. In this case, we assume that:

$$B_\nu = B_{\nu 0} + b_\nu \tau_\nu, \tag{9}$$

Now referring to the basic equations (4), (6) and (7), we have:

$$\frac{d^2}{d\tau_\nu^2}(fJ_\nu) - \epsilon(J_\nu - B_\nu) = 0, \tag{10}$$

The Eddington factor, f is given by

$$f(\tau) = \frac{1+\tau_\nu}{1+3\tau_\nu}, \tag{11}$$

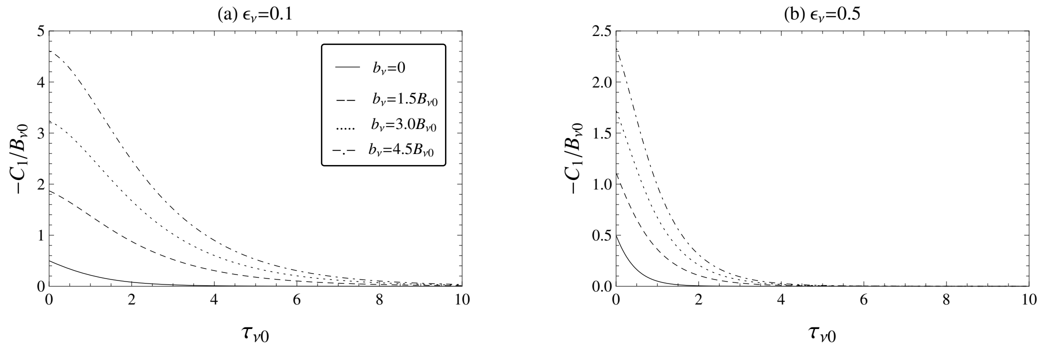

Figure 1: Solutions for simple case: Variation of the first coefficient of J_ν normalized by $B_{\nu 0} = B_\nu(\tau_\nu = 0)$ (which is omitted in a disc with infinite optical depth) with respect to the disc optical depth, $\tau_{\nu 0}$.

However, in the plan-parallel case, we can employ the approximation of $f = \frac{1}{3}$ (Kato et al. 2008). With this selection (i. e $f = 1/3$), we obtain a simple analytical solution for the ordinary differential equation (10)

$$J_\nu = B_\nu + C_1 e^{\sqrt{\epsilon_\nu/f}\tau_\nu} + C_2 e^{-\sqrt{\epsilon_\nu/f}\tau_\nu} \tag{12}$$

If we consider an optically thick disc with $\tau_\nu \to \infty$, we find $C_1 = 0$. However, for a disc with finite optical depth τ_b, we use a different boundary condition as:

$$H_\nu(\tau_b) = 0, \quad \rightarrow \quad \left.\frac{dJ_\nu}{d\tau_\nu}\right|_{\tau_\nu=\tau_b} = 0 \tag{13}$$

The other boundary conditions are defined at $\tau_\nu = 0$:

$$H_\nu(0) = H_{\nu 0}, \tag{14}$$

$$J_\nu(0) = c_\nu H_{\nu 0}, \tag{15}$$

So we have:

$$f\left.\frac{dJ_\nu}{d\tau_\nu}\right|_{\tau_\nu=0} = \frac{J_\nu(0)}{c_\nu} \tag{16}$$

Then, we find the two constants of J_ν as:

$$C_1 = -\frac{B_{\nu 0}\sqrt{\epsilon_\nu} + b_\nu\sqrt{f}[e^{\tau_b\sqrt{\epsilon_\nu/f}}(1 + c_\nu\sqrt{f\epsilon_\nu}) - c_\nu\sqrt{f\epsilon_\nu}]}{\sqrt{\epsilon_\nu}[e^{2\tau_b\sqrt{\epsilon_\nu/f}}(1 + c_\nu\sqrt{f\epsilon_\nu}) - c_\nu\sqrt{f\epsilon_\nu} + 1]} \tag{17}$$

$$C_2 = \frac{-B_{\nu 0} + b_\nu c_\nu f + C_1(c_\nu\sqrt{f\epsilon_\nu} - 1)}{c_\nu\sqrt{f\epsilon_\nu} + 1} \tag{18}$$

Fukue (2011) studied the linear Plank function for a disc with infinite optical depth. As seen in figure (1), the first coefficient of J_ν tends to zero when a disc becomes optically thick and this is in agreement with the result of Fukue (2011). According to Fig.(1), C_1 decreases considerably as ϵ_ν or $\tau_{\nu 0}$ increases. Moreover, this coefficient is less effective in the absence

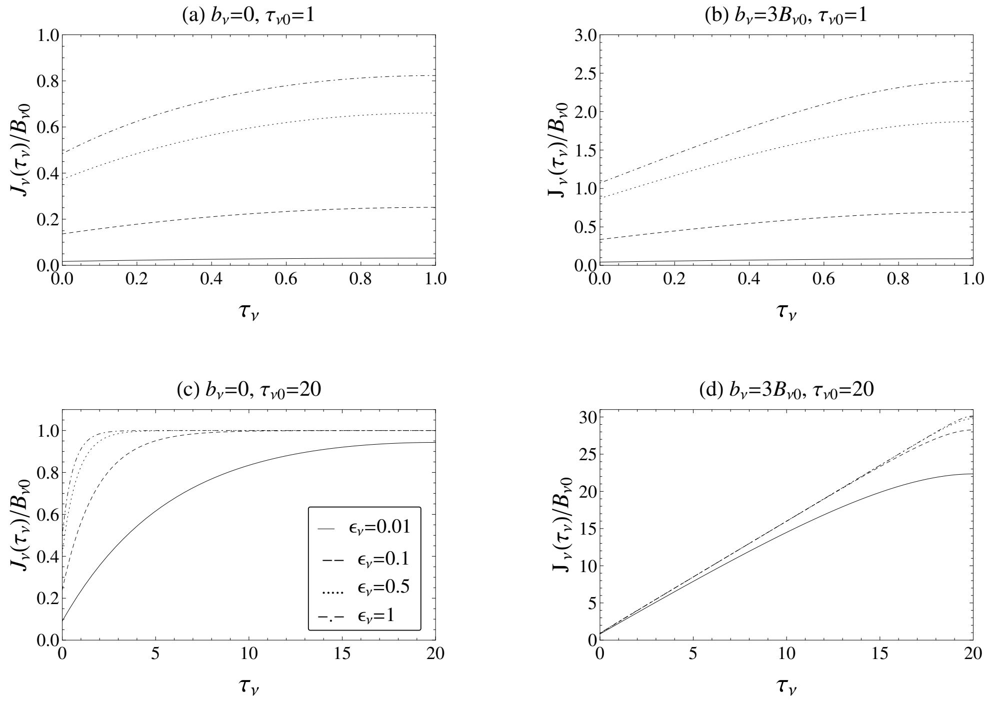

Figure 2: Variation of mean intensity, J_ν normalized by $B_{\nu 0} = B_\nu(\tau_\nu = 0)$ with respect to the optical depth, τ_ν.

of the temperature gradient ($b_\nu = 0$ shows an isothermal acretion flow). Using Eq.(7), (9) and (12) yields the Eddington flux:

$$H_\nu = b_\nu f + \sqrt{f\epsilon_\nu}(C_1 e^{\sqrt{\epsilon_\nu/f}\tau_\nu} - C_2 e^{-\sqrt{\epsilon_\nu/f}\tau_\nu}) \tag{19}$$

And the source function is found by substituting B_ν and J_ν (Eq.(9), (12)) in equation (4):

$$S_\nu = B_{\nu 0} + b_\nu \tau_\nu + (1-\epsilon_\nu)(C_1 e^{\sqrt{\epsilon_\nu/f_0}\tau_\nu} + C_2 e^{-\sqrt{\epsilon_\nu/f_0}\tau_\nu}) \tag{20}$$

In figures (2) and (3), we have plotted J_ν and H_ν with respect to the optical depth, τ_ν. These quantities have been obtained with constant Eddington factor. The effect of scattering on the radiative quantities have been considered at these two figures. All plots in Fig.(2) show that a rise in the scattering effect (or a decrease in ϵ_ν) causes the mean intensity, J_ν falles considerably or slightly (depending on the disc optical depth) in a given optical depth τ_ν. In panels (a)-(c), we notice that the scattering effect is more remarkable in regions with thinner optical depth ($\tau_\nu < 2$), whereas in panel (d) differences apear in parts with large optical depth. Moreover, as seen in top panels of this figure, when the disc is optically thin, J_ν for both isothermal and temperature gradient cases has similar trend with respect to τ_ν; i.e. this quantity gradually grows as optical depth, τ_ν, increases. In the bottom panels of Fig.(2), the disc is assumed to be ratherly optically thick and then we see the behavior of J_ν becomes much more different for the isothermal and temperature gradient cases. In the isothermal case (Fig.2c), the value of function J_ν increases with respect to optical depth for smaller depths ($\tau_\nu \lesssim 5$) and tends to the constant, $B_{\nu 0}$, as optical depth

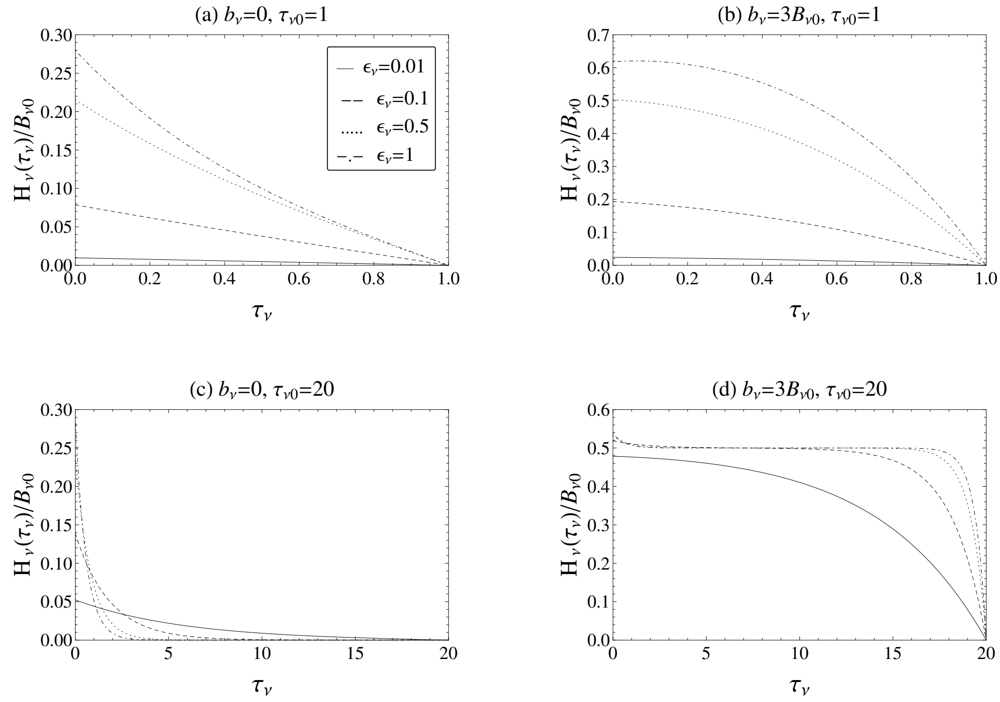

Figure 3: Variation of Eddington flux, H_ν normalized by $B_{\nu 0} = B_\nu(\tau_\nu = 0)]$ with respect to the optical depth, τ_ν.

becomes larger ($\tau_\nu \gtrsim 5$). On the other hand, in temperature gradient case (Fig.2d), J_ν increases approximately linearly with respect to optical depth τ_ν. Unlike the plots of panel (c), here differences between adjacent plots (with several values of ϵ_ν) are very small or even ignorable with $\tau_\nu \lesssim 2$. Generally, we can conclude that the scattering is less effective in optically thick regime.

Now in Fig.3, the Eddington flux H_ν is studied to examine the influences of τ_ν, ϵ_ν and $\tau_{\nu 0}$. As this figure represents the value of the Eddington flux at the surface of disc ($\tau_\nu = 0$) declines noticeably by increasing the scattering effect (smaller ϵ_ν's). Moreover, H_ν reduces from the surface towards the equatorial plane ($\tau_\nu = \tau_{\nu 0}$), and the slope of this reduction becomes steeper by growing the scattering effect. Like J_ν, the function of H_ν is very sensitive to the temperature distribution in the disc and also its total optical depth. The effect of parameter ϵ_ν is more visible for the optically thin discs. Focusing on the bottom panels of Fig.(3), we notice that H_ν becomes roughly constant with respect to τ_ν and also under the influence of various ϵ_ν's. Nevertheless, this uniform part is located near the surface of the non-isothermal disc (with $b_\nu = 3B_{\nu 0}$), whereas H_ν seems unchanged close to the darkest region of the isothermal (optically thick) disc.

Using the above solutions, we can also solve the transfer equation (5) to obtain the intensity, I_ν:

$$\pm\mu \frac{dI_\nu^\pm}{d\tau_\nu} = I_\nu^\pm - J_\nu \tag{21}$$

where the sign of $\pm$ is positive for outward intensity, I_ν^+, and it is negative for inward one,

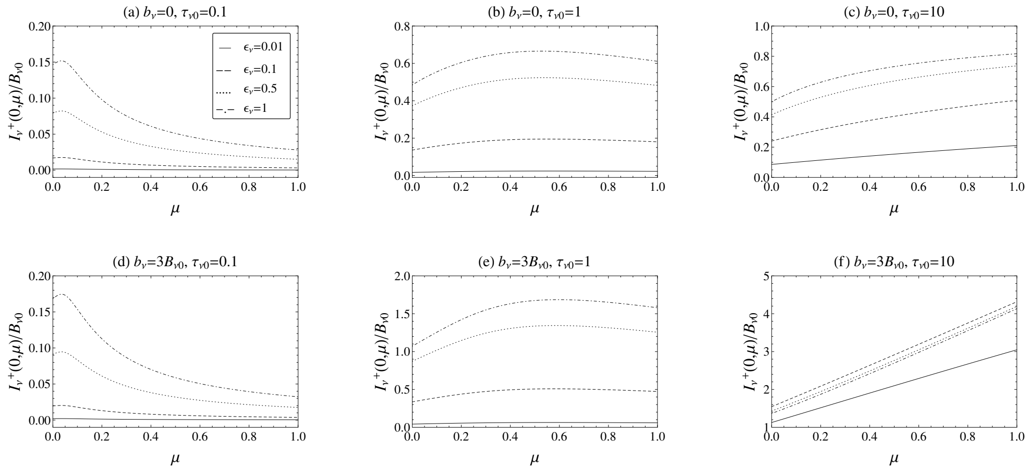

Figure 4: Emergent intensity, $I_\nu^+(\tau_\nu = 0, \mu)$ normalized by $B_{\nu0} = B_\nu(\tau_\nu = 0)]$ as a function of the direction cosine, μ.

I_ν^-. We use the following boundary conditions:

$$I_\nu^-(0, \mu) = 0, \tag{22}$$

$$I_\nu^+(\tau_{\nu 0}, \mu) = I_{\nu 0} + I_\nu^-(\tau_{\nu 0}, \mu) \tag{23}$$

where $I_{\nu 0}$ is the uniform incident intensity, showing the equatorial heating rate and the second term is the inward intensity from the back side of the flow beyond the midplane (Fukue & Akizuki 2006, Fukue 2012). After some manipulations, the outward intensity I_ν^+ and inward intensity I_ν^- are obtained as:

$$I_\nu^\pm(\tau_\nu, \mu) = B_{\nu 0} + b_\nu(\tau \pm \mu) + e^{-\sqrt{\epsilon_\nu/f}\tau_\nu}\sqrt{f}\left(\frac{C_1 e^{2\sqrt{\epsilon_\nu/f}\tau_\nu}}{\sqrt{f} \mp \mu\sqrt{\epsilon_\nu}} + \frac{C_2}{\sqrt{f} \pm \mu\sqrt{\epsilon_\nu}}\right) + C_\pm e^{\mp\tau_\nu/\mu} \tag{24}$$

$$C_- = -B_{\nu 0} + b_\nu\mu - \sqrt{f}\left(\frac{C_1}{\sqrt{f} + \mu\sqrt{\epsilon_\nu}} + \frac{C_2}{\sqrt{f} - \mu\sqrt{\epsilon_\nu}}\right) \tag{25}$$

$$C_+ = C_- e^{-2\tau_{\nu 0}/\mu} + 2\mu b_\nu e^{-\tau_{\nu 0}/\mu} - \frac{2\mu\sqrt{f\epsilon_\nu}}{f - \mu^2\epsilon_\nu} e^{-\tau_{\nu 0}(1/\mu + \sqrt{\epsilon_\nu/f})}\left(C_1 e^{2\tau_{\nu 0}\sqrt{\epsilon_\nu/f}} - C_2\right) \tag{26}$$

Finally, the emergent intensity, $I_\nu^+(0, \mu)$, emitted from the disc surface becomes:

$$I_\nu^+(0, \mu) = B_{\nu 0} + \mu b_\nu + \sqrt{f}\left(\frac{C_1}{\sqrt{f} \mp \mu\sqrt{\epsilon_\nu}} + \frac{C_2}{\sqrt{f} \pm \mu\sqrt{\epsilon_\nu}}\right) + C_+. \tag{27}$$

Panels of Fig.4 display the emergent intensity normalized by the surface value $B_{\nu 0}$ with respect to direction cosine μ for isothermal case (top ones) and for the case with a temperature gradient (bottom ones). The effect of scattering and finite optical depth are shown at these plots. Their common feature is that the emergent intensity diminishes in all of them as the photon destruction probability, ϵ_ν, decreases, hence we can say the scattering effect has this role to reduce intensity significantly. In the case of discs with very thin optical depth (Fig.4a, 4d) , I_ν^+ with smaller values of μ is greater than that with larger μ's. Figure 4 reveals that

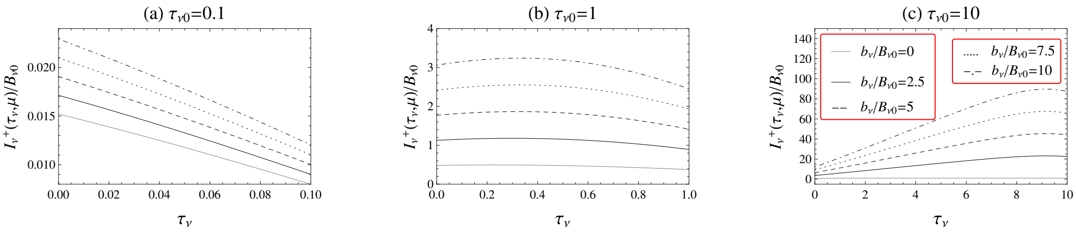

Figure 5: Variation of normalized outward specific intensity, I_ν^+, with respect to the optical depth, τ_ν. The other parameters are $\mu = 1$ and $\epsilon_\nu = 0.5$.

in discs with optical depth less than unity, the limb brightening happens regardless of the scattering's influence or any initial assumptions (which we have considered in this paper). However, when the disc optical depth tends to unity, the outgoing radiation along the polar axis becomes less than one along the equatorial plane. Consequently, the limb-brightening effect disappears in both panels (b) and (e). The third column of Fig.4 shows the intensity coming from an optically (ratherly) thick disc. Although, for both top and bottom panels the emergent intensity increases with adding μ and we have limb-darkening, more differences between plots in panel (c) than panel (f) are seen. This can be referred to the more effective role of scattering in panel (c) than panel (f). Generally, as it is easily seen in bottom panels of Fig.4, the effect of finite optical depth on the emergent intensity is similar for both isothermal and non isothermal discs. Nevertheless, the presence of scattering on the emitted radiation intensity becomes less and less important as the disc optical depth increases. In figure (5), the outward intensity I_ν^+ is shown as a function of optical depth τ_ν. Here, we have focused on the effect of temperature gradient on the outward intensity. As can be seen in three panels of this figure, the larger temperature gradient makes the outward intensity increase approximately linearly (which implies that the second term of I_ν^+ in Eq.(24) is dominant comparing with the other terms including b_ν). Comparing this figure with the previous one, we find some similarities in the behavior of intensity under the influence of μ and τ. This might (again) indicate that the second term of Eq.(24) emergent intensity is more effective than other terms to change intensity with these two parameters (μ and τ). Like Fig.(5), the trend of plots varies signifacantly with different values of $\tau_{\nu 0}$. Depending on the disc optical depth, the emergent intensity can decrease or increase towards deeper optical depths.

4 Summary and Conclusions

In this paper, we solved the radiative transfer equations of a geometrically thin accretion disc while focusing attention on the scattering effect. We applied the approximation of LTE ($j_\nu = 4\pi\kappa_\nu B_\nu$) and assumed that the Plank function B_ν is a linear function of τ_ν. By using constant Eddington factor $f = \frac{1}{3}$ and under a consistent manner, we could obtain analytically expressions for the emergent intensity I_ν as well as other radiative quantities (i.e, J_ν and H_ν).

The effect of scattering on the radiative quantities is considered for isothermal and temperature gradient cases. We also explore the behavior of the radiative quantities for an optically thin disc and an optically thick disc, separately. The plots revealed the mean intensity and Eddington flux are more sensitive to the scattering factor in optically thin discs. Therefore, we can conclude that the scattering is less effective in optically thick regime.

Also, the plots of the mean intensity and Eddington flux with respect to the optical depth illustrated they grows in the opposite directions. Regarding radiation from the disc's surface, we found out that the parameter of ϵ_ν has a positive effect on growing the emergent intensity $I_\nu^+(\tau_\nu = 0, \mu)$ especially in discs with smaller optical depth. However, for the second case, the temperature gradient can influence its impact as there were seen much more coincidences in curves with different ϵ_ν related to a non-isothermal optically thick disc rather than that in an isothermal disc with the same $\tau_{\nu 0}$. We also showed the emergent intensity with respect to the direction cosine μ. The usual limb darkening effect is seen for large optical depths.

Here we employed the some assumptions to solve the radiative transfer equations. We neglect wind and the relativistic effects. However, in a realistic model, these effects should be be taken into account which can improve our theoritical studies. Also, we can substitute the assumption of plane-parallel with spherical atmosphere and use a the variable Eddington factor; other researchers can take this into account.

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