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Omori's Law in the Occurrence of Solar Flare : Approach of Hybrid Model

Amin Najafi
Department of Basic Science, Faculty of Shahid Mofateh, Hamedan Branch, Technical & Vocational University (TVU), Hamedan, Iran;
email: najafi.amin@znu.ac.ir

Abstract. Solar flares are large-scale phenomena that have a significant effect on the Earth's climate. Using the solar flare time series from January 1, 2006 to July 21, 2016, we develop a complex network to predict solar flares. In the work, hybrid model is employed to construct complex network. In addition to the position of the flares and their occurrence times, the energy of events according to the Telesca-lovallo model is also used to develop solar flare network. Using the complex network, we retrieve the Omori's law for fore flare and after flares related to a main flare. The frequency of occurrence of solar flares decreases with time as a power law. Statistical analysis of flaring events catalogues indicates that a power-law dependence characterizes the occurrence of both fore flares and after flares corresponding to a main flare. The hybrid model retreive the Omori's law for fore flare–after flare process related to a main event.

Keywords: Solar Flare, Complex Network, Hybrid Model, Omori's Law

1 Introduction

Solar flares are accompanied by high-energy, rapid explosions on the surface of the sun. Solar flares are determined by complex spectral and temporal events in active solar regions. Magnetic reconnection in the solar active regions is responsible for the production of solar flare [1]. Since the precise nature of solar phenomena is not fully understood, so probabilistic methods are used to predict of the occurrence probability of them [2, 3, 4]. Because the flaring events occur in the certain position of the solar surface, they have spatio-temporal features with complex dynamics [5].

Solar flares and earthquakes phenomena are characterized by complex temporal occurrence. They release huge and rapid energy and momentum. The issue is related to their time series. There is evidence in measurements the size and time occurrence of solar flares and seismicity phenomena which proves that they follow the same empirical law [6].

By using the application of complex network theory, a number of properties of seismic phenomena have been revealed [7, 8]. A number of empirical laws of earthquake have also been investigated in seismic networks [9].

In this work, we introduce the hybrid model to develop solar flares network. Using the position of the flares on the surface of the sun and their temporal occurrence, the Abe-Suzuki approach is implemented to build a network of solar flares. To complete the construction of the flare network, we use the visibility graph condition for the occurence time and the magnitude of the flare energy flux [10, 11].

In this view, fore events and after events are part of the same process of flaring events. This is supported by an observed relationship between the rate of fore events and the rate of after events related to a main flare. The complexity of the solar flare network is further illustrated by the classical law known as the Omori's law [12].

2 Omori's Method

Omori's Law was first introduced in seismic research by Japanese scientist Fusakichi Omori in 1894 [12]. It stated that the frequency of aftershocks decreases roughly with the reciprocal of time after the mainshock.

This law describes the frequency of events before or after the mainshock. Therefore, we will have the decay law of foreshock or the aftershock events activity according to:

$$n(t) = K(t + c)^{-1} \tag{1}$$

where K and c are constants. Parameter n(t) is the foreshock or the aftershock events frequency measured over a certain interval of time. This law defines the distribution of the time interval of seismic events. According to Omori's law, the number of aftershock or foreshock decreases with time as power law. Otsu also obtained a modified version of the Omori's Law, which was expressed by the following relation [13] :

$$n(t) = \frac{K}{(t + c)^p}. \tag{2}$$

where t is the time after the large event, p is a rather universal exponent and takes a value between 0.8 and 1.5 , c is a case-dependent time scale and K is a productivity that depends on the mainshock size.

The Utsu-Omori law has also been obtained theoretically, as the solution of a differential equation describing the evolution of the aftershock activity [14].

Use foreshocks to help predict upcoming earthquakes have been obtained by some seismologists. One of their successes is related to the 1975 Haicheng earthquake in China. On the East Pacific Rise however, transform faults show quite predictable foreshock behavior before the main seismic event [15].

In this study, Omori's law is defined as the frequency of events before or after the main flare. There are the power law decay of after events or fore events associated with the main event according to equation (1).

3 Flare Data Sets

We analyze the solar flare data available in the Lockheed Martin Solar and Astrophysics Laboratory (LMSAL) between 2006 January 1 and 2016 July 21, Fig. 1. Our data source is taken from the following site: (http://www.lmsal.com/solarsoft/latest-events-archive.html). The time series of solar flares has long range correlation. The data contains the following information: start time, end time, peak time, GOES classification system , NOAA number (4-digit digital number for tracking a group of sunspots), longitude solar, latitude solar and peak fluxes . We use the letters A, B, C, M or X, according to the peak flux in watts per square meter of X-rays with wavelengths 100 to 800 available in the GOES system for the classification of solar flare[16, 17, 18, 19, 20, 21].

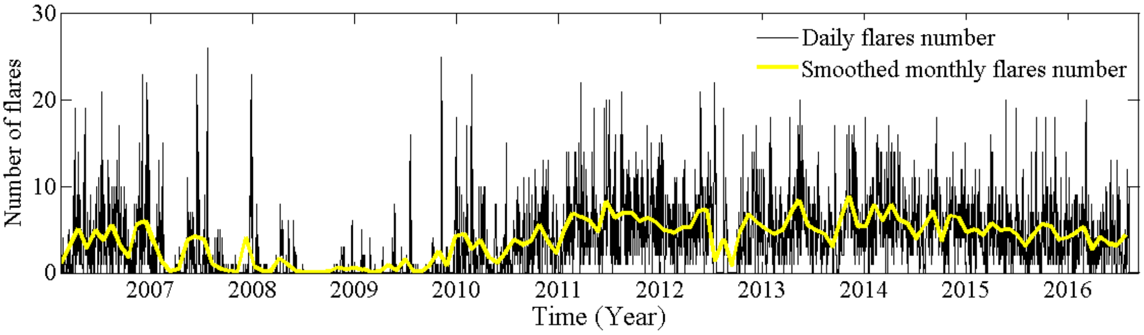

Figure 1: The time series of daily solar flares (black lines) and monthly flares (yellow line) from January 1, 2006 to July 21, 2016 [16].

4 Hybrid Model to Construction of Solar Flare Network

4.1 The Abe-Suzuki Model (AS Method)

According to the Abe-Suzuki model, the location (latitude and longitude of flare on the sun) and occurrence time of them are used to construct flare complex network. The first, the surface of the sun is divided into cells of equal size without overlapping . Then, solar network is simulated by nodes and links. We remove the empty cells in which the flare events did not occur. The construction method is similar to earthquakes network made by Abe and Suzuki [22, 23, 24].

$$\begin{aligned}
&\phi_{i+1} = \phi_i + \frac{2\pi}{n}, \phi \in [-180^\circ, 180^\circ],\\
&\sin(\theta_{j+1}) = \sin(\theta_j) - \frac{2\pi}{n}, \theta \in [-90^\circ, 0^\circ],\\
&\sin(\theta_{j+1}) = \sin(\theta_j) + \frac{2\pi}{n}, \theta \in [0^\circ, 90^\circ],
\end{aligned} \tag{3}$$

where n is the resolution of network.The number of cells varies between 1936 to 7744. The flares are spanned from 90N to -90S in solar latitude and 180E to -180W in solar longitude.

4.2 The Telesca-Lovallo Model (TL Method)

In the Telesca-Lovallo model (hearafter the TL method), the visibility graph condition is applied to connect nodes to each other . Visibility graph is a condition in which a time series can be converted to a graph. The time series of solar events is simulated by vertical bars. Events (nodes) are connected by visibility condition, Fig. 2. According to the occurrence time of the flares and their energy flux, we develop solar flares complex network by the TL model [9, 10, 11, 25].

The visibility graph condition is given by following inequality:

$$m_c \prec m_b + (m_a - m_b)\frac{t_b - t_c}{t_b - t_a}. \tag{4}$$

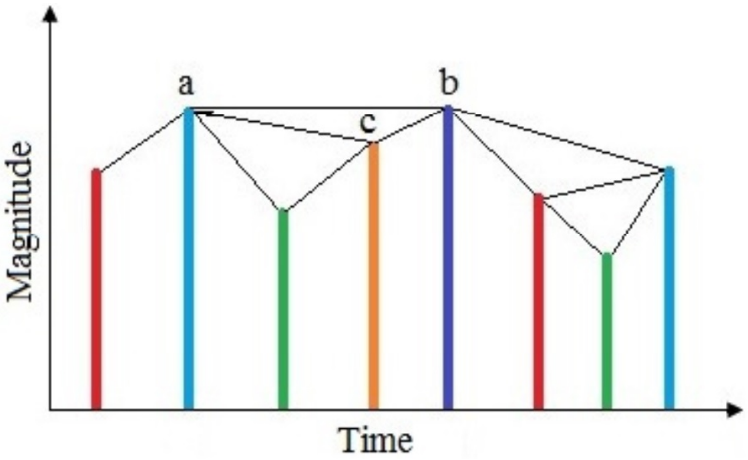

Figure 2: Graph determines which data are connected to each other.

The event a with size m_a and occurrence time t_a is connected to event b with size m_b and occurrence time t_b. The above inequality applies to any event like c which is located between them.

5 Literature Review of Omori's Law in Complex Networks

Here, we review the results of Omori's law in different complex networks. Dynamical scaling and generalized Omori law was obtained for seismic phenomenain in [26]. The results of Omori's law for foreshocks and aftershocks in earthquake model were studied in [27, 28]. Generalized Omori–Utsu law for aftershock sequences in different region of earth was investigated in [29, 30].

Rezaei et al., 2017 examined Omori's law for seismic events [9]. They used the same hybrid model to derive the Omori's law. The magnitude, occurrence time and location of earthquakes were employed to construct their seismic complex network. They obtaind the decay rate in the weight of links for the earthquakes with different cell sizes. According to the Abe-Suzuki method (AS Model), the seismic events are network nodes. A link will be connected between two successive nodes. They also employed the visibility graph method in Telesca-Lovalo model (TL model) to connect between two nodes that were linked. Using the occurrence time of earthquakes and their magnitude, they developed their complex network. Using the network approach, omori's law was extracted for different main shocks.

In Ref [25], a comprehensive study of solar flare complex network was obtained. The Omori's law for successive after main flare was investigated. The AS and TL models were combined to retrieve the omori's law for solar events. The results well confirmed the universality of the solar flare complex network according to the mechanism of seismic phenomena.

After flare sequences with power law temporal correlations as the Omori law for seismic sequences had been studied in Ref [6]. There was universality in solar flare and earthquake which describes both phenomena in terms of the same driving physical mechanism.

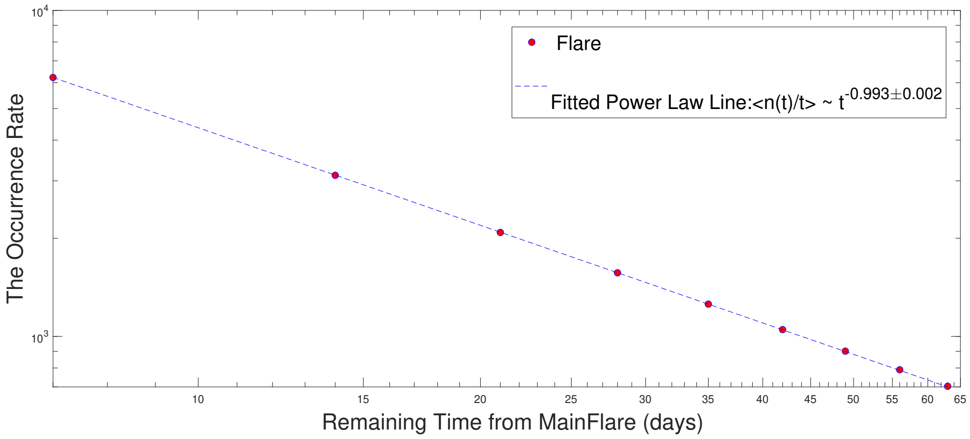

Figure 3: The occurrence rate of fore flare events in terms of the remaining time of large main flare of X=2.7. Occurrence time of main flare is 2015/05/05. The rate of changes in the cumulative number of links with time is considered as the occurrence rate of fore flare before a main flare. The occurrence rate of fore main flare has power-law time dependence. The power law index is approximately 0.993±0.002. The network resolution is equal to n=88.

6 Results

Establishing the solar flare hybrid network by combining the AS and TL models, we show that the universal scaling features of the solar flare complex system are pretty much similar to the Earth's seismic events [6, 9, 10, 11, 16, 25].

We obtain the omori's law by considering of the reduction rate in the cumulative number of links relative to main flare time. Flaring events are considered as nodes of the TL network. The main flares are more intense than the precede events and subsequent events. By applying the visibility graph condition, fore flares and after flares connect to their corresponding main event. Therefore, each link represents an after flare or fore flare connected to a main flare.

As exponent degree (hearafter power law index) is approximately close to one, it can be concluded that the Omori's law exhibit a universal feature of main flare.

By examining the statistical features, we find out more facts of the complexity of solar flare network, see Fig. 3 and Fig. 4. Because of the size of main events is larger than the fore flare and after flare sequences, therefore, links are made between them and their corresponding main flare. This is due to the application of the visibility graph condition of the TL model.

We show the frequency of occurrence of flaring events relative to main flare time of X-type in Fig. 3 and Fig. 4. The occurrence time of the desired main flare is May 05, 2015.

We display the occurrence rate of fore events related to main flare of X2.7 before the occurrence time of the main flare, Fig. 3. The rate of changes in the cumulative number of links with time is considered as the occurrence rate of fore flare events. The degree exponent confirms the Omori's law. The error of power law index is absolutely acceptable. The network resolution is equal to n=88 .

We obtain the Omori's law for after event sequence of main flare of X2.7 in Fig. 4. We display the ratio of the cumulative number of links to the number of elapsed days of the main flare of X2.7. The reduction rate of the cumulative number of links with respect to the

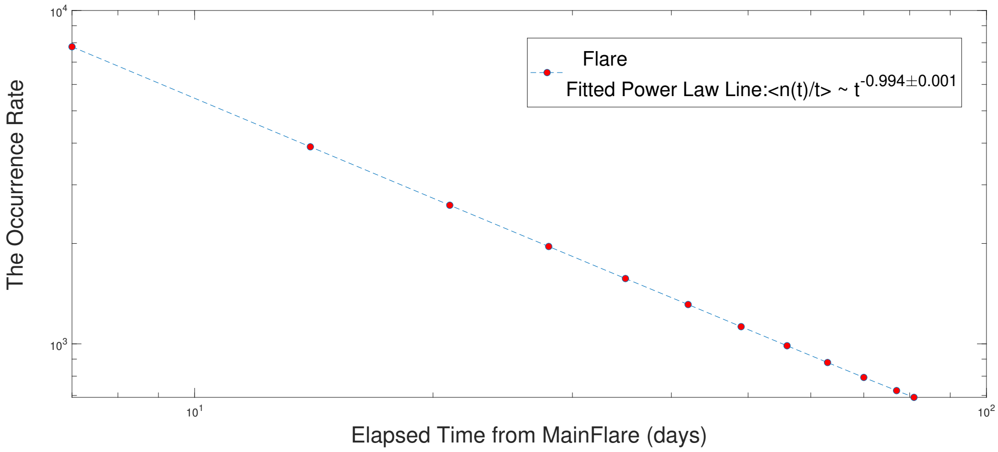

Figure 4: The occurrence rate of after flare events in terms of the elapsed time of large main flare of X=2.7. The network resolution is equal to n=88. Event time of main flare is May 05, 2015. The rate of changes in the cumulative number of links with time is considered as the occurrence rate of after main flare. We observe a power-law dependence with an index of approximately equal to 0.994±0.001.

elapsed days of main flare clearly indicates the Omori's law. The occurrence rate for both the fore flare and after flare events is decreased with time as the power law.

7 Conclusion

In the paper, we prepared a complete description of typical feature in the fore flare–after flare process related to a main flare. We demonstrated after flare and fore flare sequences with power law temporal correlations as the Omori's law for successive flaring event. Our results clearly confirmed the power law distribution in fore events sequence before a main flare. After events process also obey the power law. We developed a hybrid model to construct the solar flare complex network and investigated the utility of the model. In the new approach, the Abe-Suzuki model (AS Model) as well as the Telesca-Lovallo model (TL Model) were combined to construct solar flare hybrid network. We investigated how the empirial laws such as the Omori's law were retrieved.

In the study, by using the location of the flares and their occurrence times, the energy of events based on the TL model was also employed to construct solar flare complex network.

The alternative mapping method provides a huge improvement since it furnishes a control parameter over the number of cells (nodes of the network).

By comparing the results, we found that, the empirical laws in solar flare complex networks were dependent to construction method of complex network [25].

In the hybrid model, nodes were connected, if the visibility graph conditions were established between them.

The application of the TL Model increased the number of links in solar flare complex network. Therefore, solar flare hybrid network compared to previous network was more suitable to describe the empirical laws [25]. This was due to the application of the visibility graph condition for the time series related to flare energies.

The solar flare network presented by [16] was inspired by the Abe-Suzuki method (AS

Model) and was constructed by using only the start times of solar flares. However, in the hybrid model, the flare positions and energies are considered in construction of the network further to the start times.

Because time series of flare energy were not obtained in the previous solar flare network, it was not possible to study the Omori's law accurately. The hybrid model reconstructed the empirical laws such as the Omori's law for fore flare–after flare process associated with a main event.

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