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Reconstruction of Solar Coronal Linear Force-Free Magnetic Field

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Abstract. Solar magnetic fields has an important role in the construction of the corona. In the coronal condition, the gas pressure and gravity are neglected as compaired with the magnetic pressure. Here, the model of Linear Force-Free Field (LFFF) is introduced. Based on this model and appropriate boundary conditions, the structure of coronal magnetic field is reconstructed.

First, using semi-analytic Seehafer method, the LFFF is reconstructed using the Wiegelmann code for the line of sight (LOS) magnetogram. Then, the magnetic field lines and loops on a given magnetogram is plotted on the corresponding AIA image for estimating the compatibility. Finally, by calculating the variance between a given traced loop and corresponding reconstructed loops, we find a proper magnetic field parameter for reconstructing a loop before and after flaring.

Keywords: magnetic fields, solar corona, reconstruction, force-free

1 Introduction

As the struture and dynamics of the solar corona is dominated by the magnetic field, this quantity plays a key role in solar coronal activities. While, the photospheric magnetic field is measured regularly on vector magnetographs, it is not feasible to measure the coronal magnetic field directly. These photospheric measurments are extrapolated into the corona using a proper boundary conditions. The extrapolated coronal magnetic field relies on asumptions concerning the coronal plasma conditions, e.g, force-freeness. Force-free means that all non magnetic forces like pressure gradients and gravity are disregarded due to the low plasma beta ($\beta = \frac{2\mu_0 p}{B^2} \ll 1$). Force-free fields are charactrized by the equations

$$\vec{J} \times \vec{B} = 0, \tag{1}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J}, \tag{2}$$

$$\nabla \cdot \vec{B} = 0, \tag{3}$$

Where $\vec{B}$ is the magnetic field, $\vec{J}$ the electric current density and μ_0 the permeability of vacuum. Equation (1) implies that the magnetic field and current density are parallel, i.e.,

$$\nabla \times \vec{B} = \alpha \vec{B}, \tag{4}$$

where α, as force-free magnetic field parameter, is the function of position. Here, by taking the divergence of Eq. (4)and using Eq. (3), one gets

$$\vec{B} \cdot \nabla \alpha = 0. \tag{5}$$

This equation tells us that α is constant on every field line. Taking the curle of Eq.(4) and using the vector identity

$$\nabla \times (\nabla \times \vec{B}) = \nabla(\nabla \cdot \vec{B}) - \nabla^2 \vec{B}, \tag{6}$$

one gets the Helmholtz equation for constant α,

$$\nabla^2 \vec{B} + \alpha^2 \vec{B} = 0. \tag{7}$$

Equation(7) can be solved by diffrent methods such as a Fourier method (Alissandrakis, 1981)[1] or Green's functions method (Seehafer 1978)[2]. We will discuss about the Seehafer method in the next section.

2 The Seehafer solution for LFFF

For extrapolating the solar coronal magnetic field with LFFF model, one needs measurements of the z-component of magnetic field. The force-free parameter α is unknown and we will explain later how α could be estimated from the observed data. The solutions of LFFF equations by a Green function method is given by [2][3]:

$$B_x = \Sigma_{m,n=1}^{\infty} \frac{C_{mn}}{\lambda_{mn}} exp(-r_{mn}z).[\alpha \frac{\pi n}{L_y} sin(\frac{\pi mx}{L_x}) cos(\frac{\pi ny}{L_y}) - r_{mn} \frac{\pi m}{L_x} cos(\frac{\pi mx}{L_x}) sin(\frac{\pi ny}{L_y})], \tag{8}$$

$$B_y = -\Sigma_{m,n=1}^{\infty} \frac{C_{mn}}{\lambda_{mn}} exp(-r_{mn}z).[\alpha \frac{\pi m}{L_x} cos(\frac{\pi mx}{L_x}) sin(\frac{\pi ny}{L_y}) + r_{mn} \frac{\pi n}{L_y} sin(\frac{\pi mx}{L_x}) cos(\frac{\pi ny}{L_y})], \tag{9}$$

$$B_z = \Sigma_{m,n=1}^{\infty} C_{mn} exp(-r_{mn}z).sin(\frac{\pi mx}{L_x}) sin(\frac{\pi ny}{L_y}). \tag{10}$$

Here, $\lambda_{mn} = \pi^2(\frac{m^2}{L_x^2} + \frac{n^2}{L_y^2})$ and $r_{mn} = \sqrt{\lambda_{mn} - \alpha^2}$.

This method uses the distribution of $B_z(x, y)$ on the photosphere $z = 0$ as the boundary condition. The coeffients C_{mn} can be obtained using Eq. (10) for $z = 0$ and the magnetogram data.

By this method, LFFF can be calculated for a given magnetogram (e.g., obtained by HMI on SDO) and a specific value for α. The obsereved data which has domain as $0 \le x \le L_x$ and $0 \le y \le L_y$ is artificially extended to a region like $-L_x \le x \le L_x$and $-L_y \le y \le L_y$ taking an antisymmetric mirror image of the original magnetogram in extended region., i.e.,

$$B_z(-x, y) = -B_z(x, y),$$
$$B_z(x, -y) = -B_z(x, y).$$

Becuase of this assumption, the total magnetic flux will be zero in the whole extended region. The real and positive values for r_{mn} indicates that α^2 should not pass the following maximum value for a given L_x and L_y

$$\alpha_{max}^2 = \pi^2(\frac{1}{L_x^2} + \frac{1}{L_y^2}).$$

It can be shown that the values of α fall into the range of $-\sqrt{2}\pi < \alpha < \sqrt{2}\pi$.

3 The Wiegelmann Code

In reconstruction of the magnetic field, we have used the code written by Wiegelmann in five versions[4]. This code contains two parts for the computing and reconstructing solar coronal magnetic fields. The IDL-program includes many diffrent tools to compute and display LFFF. All basic features are controlled with the help of mouse. The other part of this code is written in C language and is based on an optimization principle. This program is not a part of the IDL-Widget program paket, but the IDL-tools are used for the preprocessing of vector magnetograph data and for the analysis of reconstructed 3D magnetic field. The connection between IDL and C is done via ASCII-files[4].

4 Results

In this paper, we make use of semi-analytic Seehafer method in order to reconstruct LFFF with constant α on a given magnetogram data. These data are collected from SDO/HMI space telescope for C-class solar flare in 171 $\dot{A}$ on 27 Apr, 2013 between 05:44:22UT and 05:50:34UT[5]. The results of the reconstruction have been plotted on AIA data, i.e., Fig. (1) and Fig. (2). The reconstructed open and close magnetic field lines are roughly compatible with the magnetic field lines and loops of AIA image. This is done for the magnetogram and the corresponding AIA image obtained before and after this flare[7].

4.1 Reconstruction of LFFF

In Fig. 1, the magnetic field lines are plotted before flare occurrence. In Fig. 1a, a group of reconstructed 2D magnetic lines for $\alpha = 0.88$ are shown. The same lines are plotted in 3D case in Fig. 1b. A projection of the field lines tracing are shown on AIA image in Fig. 1c. As you see, this reconstruction is roughly compatible to the main AIA image in 171$\dot{A}$.

The magnetic field lines and loops are reconstructed by the same procedure and are shown for 2D and 3D cases of post flare occurrence in Fig. 2. Again, the same as Fig. 1, the compatibility of reconstructed magnetic field lines with its AIA image is roughly seen.

4.2 The comparison between traced and reconstructed loops

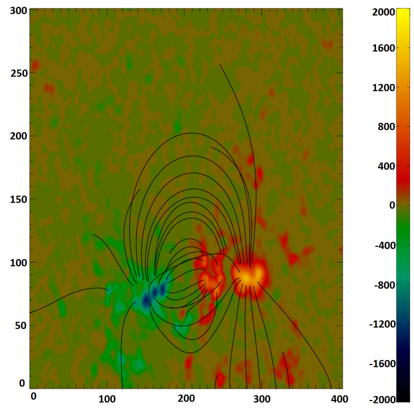
The compatibility of reconstructed and traced loops can be evaluated by

$$\sigma = \sqrt{\frac{1}{N}\Sigma_{i=1}^{N}(\vec{r}_{Obs} - \vec{r}_{Rec})^2}$$

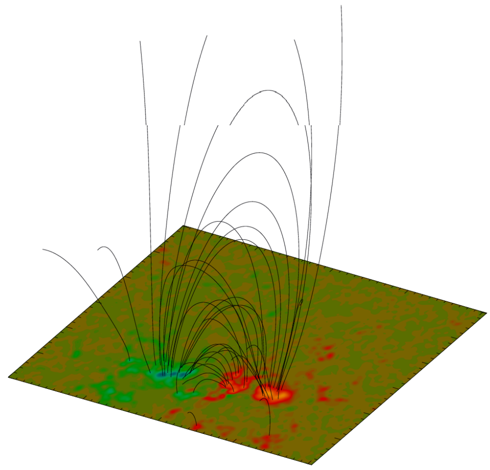
Here, $\vec{r}_{Obs}$ is the traced loop coordinations on AIA image and $\vec{r}_{Rec}$ is the reconstructed loop coordinations on HMI magnetogram data[8].

In Fig. 3, field lines tracing are shown on AIA image before and after flare using Oriented Coronal Curved Loop Tracing (OCCULT) method[9]. These lines and loops are chosen for comparing with reconstructed LFFF on HMI data.

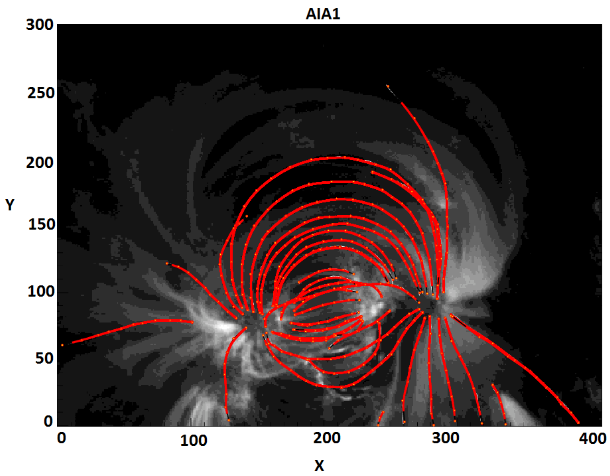
The values of σ are calculated and plotted versus α in Figs. 4 and 5. Here, according to the different values of α , the variation of reconstructed and traced loop are clearly visible. In these figures, the minimum variance versus α for a given loop is shown.



(a) Reconstructed lines and loops in 2D

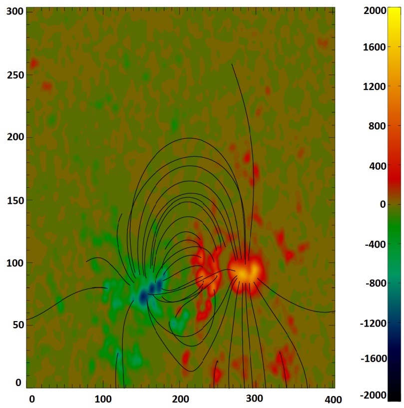


(b) Reconstructed lines and loops in 3D

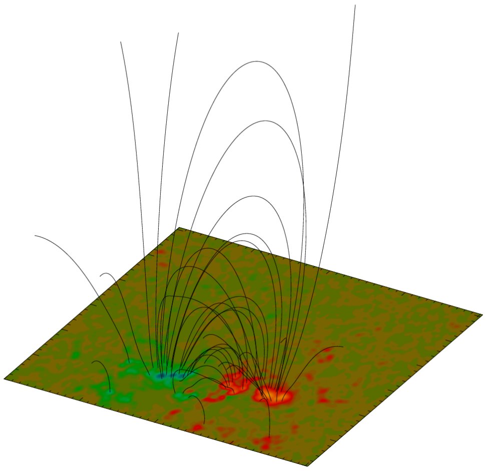


(c) Some reconstructed field lines on AIA image

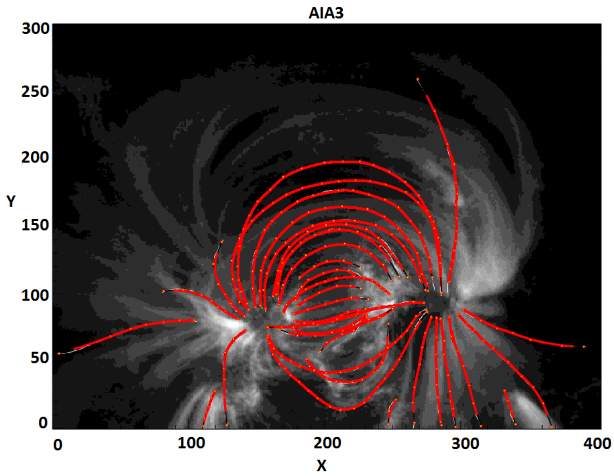
Figure 1: (a) shows some extrapolations from photosphere in 2D with the help of a LFFF model for $\alpha = 0.88$. (b) the same as Fig. (a), 3D Fig. (c) shows a projection of some reconstructed field lines on AIA image registered by AIA telescope in 171 $\dot{A}$. These data are obtained before flaring in 27 Apr, 2013.



(a) Reconstructed lines and loops in 2D

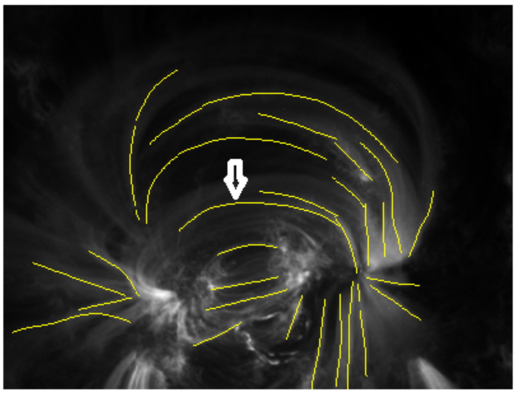


(b) Reconstructed lines and loops in 3D

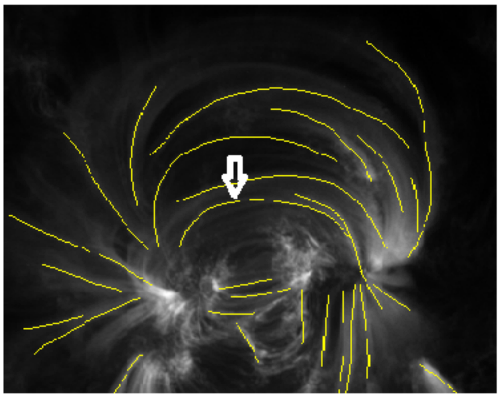


(c) Some reconstructed field lines on AIA image

Figure 2: (a) shows some extrapolations from photosphere in 2D with the help of a LFFF model for $\alpha = 0.88$. (b) the same as Fig. (a), 3D Fig. (c) shows a projection of some reconstructed field lines on AIA image registered by AIA telescope in 171 $\dot{A}$. These data are obtained after flaring in 27 Apr, 2013.

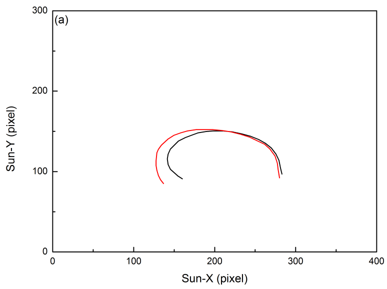


(a) Before Flare

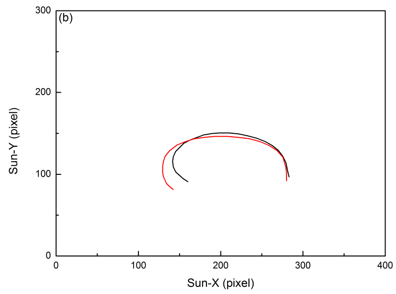


(b) After Flare

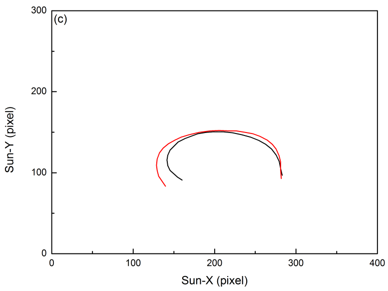
Figure 3: A projection of traced lines and loops on solar coronal AIA image for before and after flaring using OCCULT method.



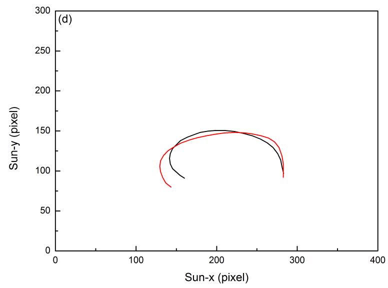
(a) $\alpha = -0.88$



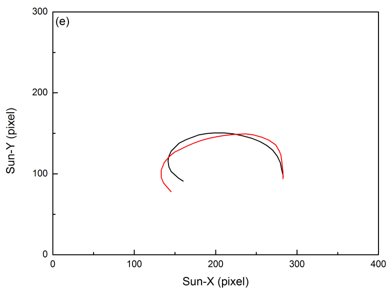
(b) $\alpha = -0.44$



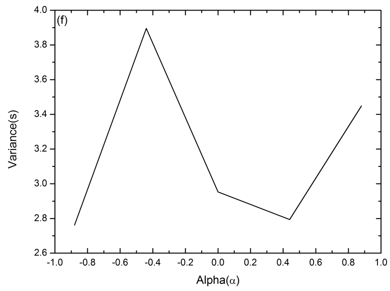
(c) $\alpha = 0$



(d) $\alpha = 0.44$

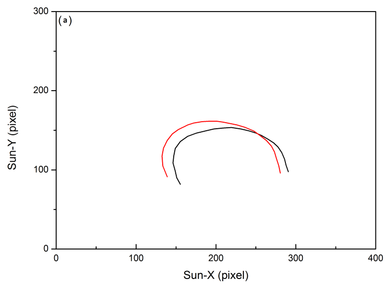


(e) $\alpha = 0.88$

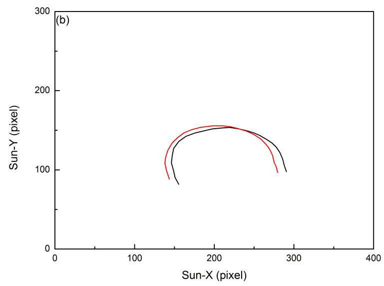


(f) Variance based on the alpha.

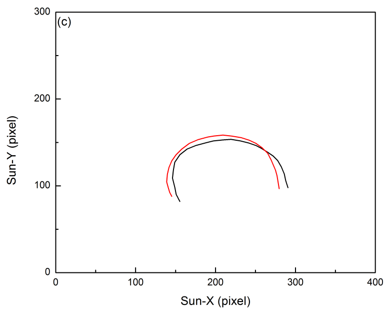
Figure 4: The comparison of traced loop on AIA image (the top one in Fig. 3, shown by pointer) and reconstructed loops with diffrent α for before flaring.



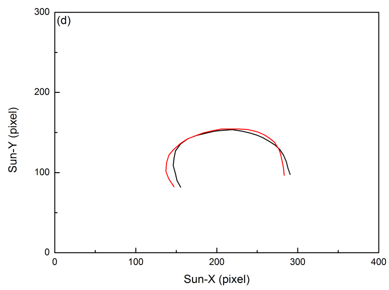
(a) $\alpha = -0.88$



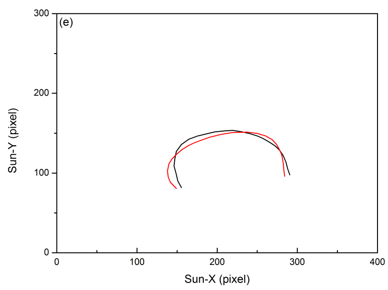
(b) $\alpha = -0.44$



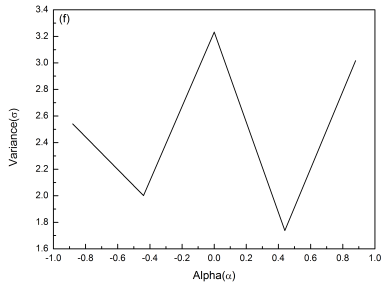
(c) $\alpha = 0$



(d) $\alpha = 0.44$



(e) $\alpha = 0.88$



(f) Variance based on the alpha.

Figure 5: The comparison of traced loop on AIA image (the down one in Fig. 3, shown by pointer) and reconstructed loops for various α for after flaring.

5 Conclusion

Here, we have reported the results obtained in an attempt to reconstruct the coronal magnetic field using LFFF model (constant α) for LOS magnetogram. These results are: a) The reconstructed LFFF in 2 and 3 dimentions on HMI magnetogram data. The reconstructed magnetic field lines are plotted on AIA image which are approximately compatible with the magnetic field lines of AIA data. b) We have compared direct observation of a magnetic loop in the upper chromosphere with the magnetic fields extrapolated from the photosphere with LFFF model. We have shown that how one is able to find an appropriate α for reconstructing the magnetic field lines with the LFFF model.

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