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# Obliquely Propagating Dust-Ion Acoustic Solitary Waves in Magnetized Plasmas with Superthermal Electrons

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**Abstract**. In this paper, the problem of small amplitude dust-ion acoustic (DIA) solitary waves is discussed using the reductive perturbation theory in magnetized plasmas consisting of superthermal electrons, inertial ions, and negatively charged stationary dust grains. Presented investigation shows that the effects of external magnetic field and superthermal electrons significantly modify the basic properties of DIA solitary waves, so that an enhancement in the electron superthermality causes a reducion in solition width. Moreover, when the magnetic field increases, the width of soliton decreases, and in a consequence, it makes the solitary structures more spiky. In the following, it is found out that the soliton compression and rarefaction are sensitive to the degree of electron superthermality and dust concentration. Also, the value of obliqueness index has bright effect on adjusting the amplitude of the positive and negative solitary waves. It is expected that the results of this study will give more insight into the DIA dynamics in dusty astrophysical and laboratory plasmas.

*Keywords*: Dust-ion-acoustic solitary waves; Magnetic field; Superthermal electrons.

# 1 Introduction

The behavior of nonlinear waves in plasmas has received considerable attention in the past few decades [1-6]. The study of dusty plasmas as a nonlinear medium is important to understand the space environments and astrophysical phenomena, such as the planetary rings, the comets, the interstellar medium, the earths ionosphere, and the magnetosphere as well as industrial plasma devices [7-12]. The low-frequency dust ion-acoustic (DIA) waves are one of the important electrostatic dust-associated waves. For the first time, Shukla and his co-worker [13] have theoretically shown that due to the conservation of equilibrium charge density $n_{e0} + Z_d n_{d0} = n_{i0}$, and the strong inequality $n_{e0} \ll n_{i0}$ (where $n_{e0}$, $n_{d0}$, and $n_{i0}$ are electron, dust, and ion number density at equilibrium, respectively, and $Z_d$ is the number of electrons residing onto the dust grain surface) a dusty plasma, with negatively charged static dust, supports low-frequency DIA waves with phase speed much smaller (larger) than the electron (ion) thermal speed. It should be noted that DIA waves [14] are more suitable than the dust-acoustic waves [15] to observe in laboratory dusty plasma conditions. For this reason, characteristics of the DIA waves have studied by many authors. For instance, Mamun and Shukla [16-17] have investigated DIA solitary waves in unmagnetized dusty plasmas consisting of cold ion fluid, isothermal electrons, and negatively charged static dust particles, theoretically. Mamun [18] discussed the propagation of nonlinear DIA solitary waves in an unmagnetized adiabatic dusty plasma containing adiabatic inertialess electrons, adiabatic inertial ions, and negatively charged static dust grains. Recently, Dutta et al. [19] have studied DIA solitons in the presence of superthermal electrons. In this study, they have concluded that presence of superthermal electrons accelerates the damping of the wave. Apart from these, numerous observations of space plasmas [20-23] clearly indicate the presence of superthermal electron and ion structures as ubiquitous in a variety of astrophysical plasma environments. Superthermal particles may arise due to the effect of external forces acting on the natural space environment plasmas, and the wave particle interaction. Plasmas with an excess of superthermal (non-Maxwellian) electrons are generally characterized by a long tail in the high energy region. To model such space plasmas, the generalized Lorentzian of $\kappa$-distribution has been found to be appropriate rather than the Maxwellian distribution [24-28]. The Kappa distribution has been used by several authors [29-35] in studying the effect of Landau damping on various plasma modes. Superthermal plasma behavior was observed in various experimental plasma contexts, such as laser matter interactions or plasma turbulence [36]. At very large values of the spectral index $\kappa$, the velocity distribution function approaches to the Maxwellian distribution, while for low values of $\kappa$, they represent a hard spectrum with a strong non-Maxwellian tail having a power-law form at high speeds. Recently, some studies have been done on electron, ion, dust, and dust-ion acoustic solitary waves using superthermal distribution for ions or electrons [32, 37-42]. Most of the studies on the DIA solitary waves are restricted to unmagnetized dusty plasmas in the cold plasma limit. However, for a bounded plasma, which is a common feature of laboratory and most space plasmas, it is inevitable to take into account the magnetic field, and one cant omit the role of magnetic field. Atteya et al. [25] have studied the role of electrons superthermality on DIA solitary waves in magnetized plasmas with positive and negative ions. The effect of the ion pressure on the obliquely propagated DIA solitary waves in external magnetic fields had been investigated in [43]. Obliquely propagating DIA solitary waves in hot adiabatic magnetized dusty plasmas with negatively/positively charged static dust grains were investigated by Shalaby et al. [44]. The combined effects of electron nonextensivity and external magnetic field (obliqueness) on the DIA waves have been investigated in [45]. As well as, obliquely propagations of DIA solitary waves and double layers in rotational magnetoplasma have been explored in [46]. But, as far as we know, no such study of obliquely propagation of DIA solitary waves in magnetized dusty plasmas with superthermal electrons has been carried out, so far. Therefore, we have considered superthermal electrons along with stationary dust particles in magnetized dusty plasmas.

To this end, we prepare the present paper as follows: The basic equations, governing considered electronegative dusty plasma system, are given in Sec. 2. The features of DIA waves are investigated in Sec. 3, and the considerable characteristics of DIA solitary waves are evaluated in Sec. 4. Finally, the discussion is presented in Sec. 5.

## 2 Basic equations

Let us consider a dusty plasma system, which consists of negatively charged stationary dust, inertial ions, and superthermal electrons in the presence of a uniform external magnetic field $\vec{B} = B_0\hat{z}$ along the $z$-axis. Due to the very large inertia, the dust grains do not participate in the motion in the case of studying DIA solitary waves. Accordingly, the dust grains are assumed to be stationary. The nonlinear dynamics of DIA solitary waves is described by [47-48]

$$\frac{\partial n}{\partial t} + \vec{\nabla}.(n\vec{u}) = 0 \tag{1}$$

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u}.\vec{\nabla})\vec{u} = -\vec{\nabla}\phi - \omega_{ci}(\vec{u} \times \hat{z}) \tag{2}$$

$$\nabla^2 \phi = -n + (1-\mu)n_e + \mu \tag{3}$$

Where $n$ is the ion number density normalized by its equilibrium value $n_{i0}$, $u$ is the ion fluid speed normalized by $C_i = (k_B T_e/m_i)^{\frac{1}{2}}$, and $\phi$ is the electrostatic wave potential normalized by $k_B T_e/m_i$ with $k_B$ being the Boltzmanns constant and $m_i$ being the mass of positively charged ions. The time $t$ and the distance $x$ are normalized by the ion plasma frequency $\omega_{pi} = \sqrt{\frac{4\pi n_{i0} e^2}{m_i}}$ and the Debye length $\lambda_{Di} = \sqrt{\frac{k_B T_i}{4\pi n_{i0} e^2}}$, respectively. $T_i(T_e)$ being the ion (electron) temperature and $\mu = \frac{Z_d n_{d0}}{n_{i0}}$.

To model the effects of superthermal electrons, we refer to an appropriate Bernstein-GreeneKruskal solution that solves the collisionless Vlasov equation. Thus, we choose [26]

$$f_k(v_e) = C_e\left(1 - \frac{e\varphi}{km\theta_{the}^2} + \frac{v_e^2}{2k\theta_{the}^2}\right)^{-(k+1)} \tag{4}$$

where the normalization is provided for any value of the spectral index $k > 1/2$ by

$$C_e = n_{e0}(2\pi k \theta_{the}^2)^{-\frac{1}{2}} \frac{\Gamma(k+1)}{\Gamma(k+\frac{1}{2})} \tag{5}$$

Here, the parameter $k$ shapes predominantly the superthermal tail of the distribution, the quantity $C$ stands for the standard gamma function, and

$$\theta_{the} = \left[\frac{k-\frac{1}{2}}{k}.\frac{T_e}{m_e}\right]^{\frac{1}{2}} \tag{6}$$

In the limit $k \to \infty$, distribution $f(v_e)$ reduces to the well known MaxwellBoltzmann

$$f(v_e) = n_{e0}(2\pi v_{the}^2)^{-\frac{1}{2}}\left(-\frac{v_e^2 - 2e\varphi/m_e}{2v_{the}^2}\right) \tag{7}$$

where $v_{the} = \sqrt{\frac{T_e}{m_e}}$. Integrating $f(v_e)$ over all velocity space, we get for the normalized number density of electron

$$n_e = \left(1 - \frac{\phi}{k-\frac{1}{2}}\right)^{-k-\frac{1}{2}} \tag{8}$$

where $\phi = \frac{e\varphi}{T_e}$.

# 3 DIA solitary waves

In order to study the DIA solitary waves in the plasma model under consideration, we construct a weakly nonlinear theory of the electrostatic waves with small but finite amplitude which leads to a scaling of the independent variables through the stretched coordinates $\xi = \varepsilon^{\frac{1}{2}}(l_x x + l_y y + l_z z - \lambda t)$, $\tau = \varepsilon^{\frac{3}{2}} t$; where $\varepsilon$ is a small dimensionless parameter measuring the weakness of the dispersion and nonlinearity, $\lambda$ is the phase velocity (to be determined later) normalized by ion-acoustic speed $(C_i)$. Parameters $l_x$, $l_y$, and $l_z$ are the directional cosines of the wave vector $\vec{k}$ along the $x$, $y$, and $z$ axes, respectively, so that $l_x^2 + l_y^2 + l_z^2 = 1$.

We also expand $n_c$, $u_{cx,y,z}$, and $\phi$ in a power series of $\varepsilon$ as

$$\begin{aligned} n_c &= 1+\varepsilon n_1+\varepsilon^2 n_2+\dots \\ u_{cx} &= \varepsilon^{\frac{3}{2}} u_{1x}+\varepsilon^2 u_{2x}+\dots \\ u_{cy} &= \varepsilon^{\frac{3}{2}} u_{1y}+\varepsilon^2 u_{2y}+\dots \\ u_{cz} &= \varepsilon u_{1z}+\varepsilon^2 u_{2z}+\dots \\ \phi &= \varepsilon\phi_1+\varepsilon^2\phi_2+\dots \end{aligned} \tag{9}$$

and write equations in various powers of $\varepsilon$. From the lowest order in $\varepsilon$ of the continuity equation, the $z$ component of the momentum equation and Poissons equation we have $n_1 = \frac{l_z^2}{\lambda^2}\phi_1$, $u_{1z} = \frac{l_z}{\lambda}\phi_1$, and $\lambda = l_z\sqrt{\frac{2k-1}{(2k+1)(1-\mu)}}$. One can write the lowest order of the $x$ and $y$ components of the momentum equation as

$$u_{1x} = \frac{l_y}{\omega_{ci}}\frac{\partial\phi_1}{\partial\xi}, \quad u_{1y} = \frac{l_x}{\omega_{ci}}\frac{\partial\phi_1}{\partial\xi} \tag{10}$$

which represent the $y$ and $x$ components of the electric-field drift arising due to the balance between the electric and Lorentz forces respectively. We can also obtain the next higher order $x$ and $y$ components of the momentum equation

$$u_{2x} = \frac{l_x\lambda}{\omega_{ci}^2}\frac{\partial^2\phi_1}{\partial\xi^2}, \quad u_{2y} = \frac{l_y\lambda}{\omega_{ci}^2}\frac{\partial^2\phi_1}{\partial\xi^2} \tag{11}$$

To next higher order in $\varepsilon$, from the continuity equation, the $z$-component of the momentum equation and Poissons equation, we obtain

$$\begin{cases} \frac{\partial n_1}{\partial\tau} - \lambda\frac{\partial n_2}{\partial\xi} + l_x\frac{\partial u_{2x}}{\partial\xi} + l_y\frac{\partial u_{2y}}{\partial\xi} + l_z\frac{\partial u_{2z}}{\partial\xi} = 0 \\ \frac{\partial u_{1z}}{\partial\tau} - \lambda\frac{\partial u_{2z}}{\partial\xi} + u_{1z}l_z\frac{\partial u_{1z}}{\partial\xi} - l_z\frac{\partial\phi_2}{\partial\xi} = 0 \\ \frac{\partial^2\phi_1}{\partial\xi^2} = -n_2 + \frac{l_z^2}{\lambda^2}\frac{2k+1}{2k-1}\phi_2 + \frac{l_z^2}{\lambda^2}\frac{(2k+1)(2k+3)}{2(2k-1)^2}\phi_1^2 \end{cases} \tag{12}$$

Finally, using equations (10-12) the KdV equation yields

$$\frac{\partial\phi_1}{\partial\tau} + A\phi_1\frac{\partial\phi_1}{\partial\xi} + B\frac{\partial^3\phi_1}{\partial\xi^3} = 0 \tag{13}$$

where the nonlinear and dispersive coefficients $A$ and $B$, respectively, are defined as

$$A = \frac{1}{2}\{\frac{3l_z^2}{\lambda} - \lambda(\frac{2k+3}{2k-1})\}, \quad B = \frac{\lambda^3}{2l_z^2}\{1 + \frac{1-l_z^2}{\omega_{ci}^2}\} \tag{14}$$

Equation (13) is well known KdV equation describing the nonlinear propagation of the DIA waves in a magnetized superthermal dusty plasma.

# 4 Discussion

In order to study the dynamics of small amplitude DIA solitary waves in a superthermal magnetized dusty plasma, we derived a KdV equation, by employing the reductive perturbation method. The nonlinear dynamical equation (13) describes obliquely propagating DIA solitary waves in a magnetized dusty plasma with superthermal electrons. In order to study a stationary solitary wave solution of equation (13), we assume that the stationary

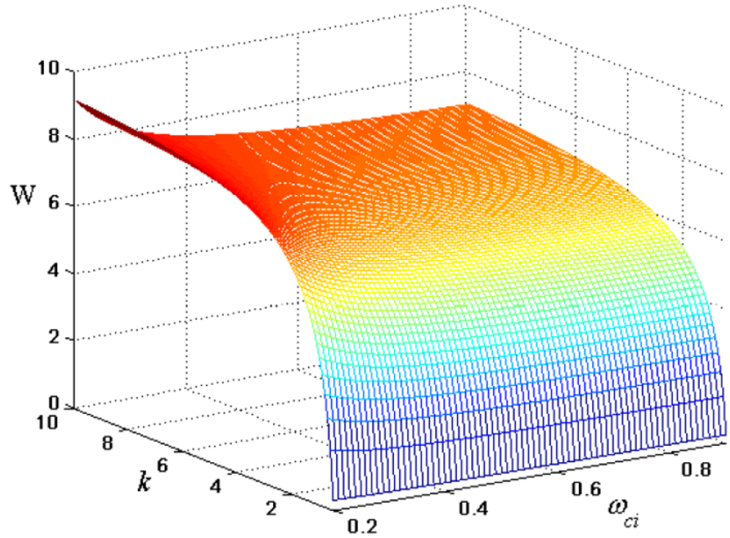


Figure 1: Soliton width as functions of $k$ and $\omega_{ci}$ with $\mu = 0.5$, $\delta = 10°$, and $u = 0.1$.

solution can be expressed as $\phi_1 = \phi_1(\chi)$, where $\chi = \xi - u\tau$. Substituting this expression into equation (13), we can obtain the stationary solitary wave solution

$$\phi_1 = \phi_m sech^2(\frac{\chi}{W}) \tag{15}$$

where $\phi_m = 3u/A$ is the soliton amplitude and $W = 2\sqrt{B/u}$ is its width.

In equation (14), derived coefficients are good agreement with the results of [45] in magnetized nonextensively dusty plasmas. The mentioned coefficients also are exactly equal to results in [49] for planar geometry in unmagnetized (with $l_z = 1$ or $\vec{B} = 0$) plasma with negative dusts. It is obvious from equation (15) that depending on whether $A$ is positive or negative, the solitary waves will be either compressive $\phi_m > 0$ or rarefactive $\phi_m < 0$. Thus, there exist solitary waves with positive potential (which are compressive solitary waves) when $A > 0$, i.e. $\mu < 4k/3(2k+1)$, and solitary waves with negative potential (which are rarefactive solitary waves) when $A < 0$, i.e. $\mu > 4k/3(2k+1)$. It is important to note that the amplitude of DIA solitary waves in equation (15) is independent of the magnitude of the external magnetic field. In addition, the amplitude is inversely proportional to $l_z$ ($l_z = \cos\delta$, where $\delta$ is the angle between the directions of the wave propagation vector $k$ and the external magnetic field $B_0$). It is observed that as $l_z$ increases, the amplitude of DIA solitary wave decreases. It can be easily find that phase velocity "$\lambda$" is correspond to $1/\sqrt{1-\mu}$ (and so $W$ correspond to $1/(1-\mu)^{3/2}$), thus the soliton width increases with an increasing $\mu$. To examine the effects of superthermal electrons and external obliquely magnetic field on the features of solitary waves, we have numerically analyzed both of the width and amplitude of DIA solitary waves. Figs. 13 show how the superthermality of electrons, external obliquely magnetic and dust concentration affect on the width of DIA solitary waves.

Variation of the soliton width respect to $\omega_{ci}$ and superthermal index ($k$) has been plotted in the Fig. 1 with $\mu = 0.5$, $\delta = 10°$ and $u = 0.1$. It is clear that increasing the superthermal index $k$ causes the width of soliton to increase. This implies that the soliton width decreases as the electrons evolve toward their thermodynamic equilibrium. We recall that a decrease

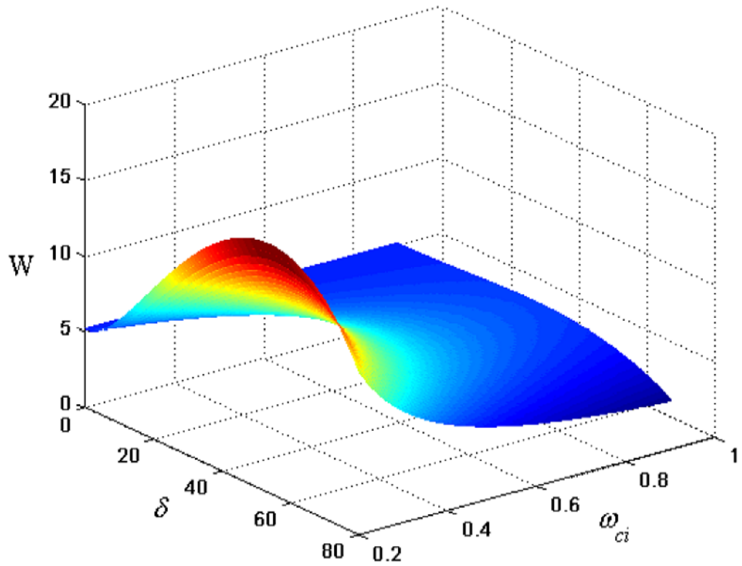


Figure 2: Soliton width as functions of $\delta$ and $\omega_{ci}$ with $\mu = 0.5$, $k = 2$, and $u = 0.1$.

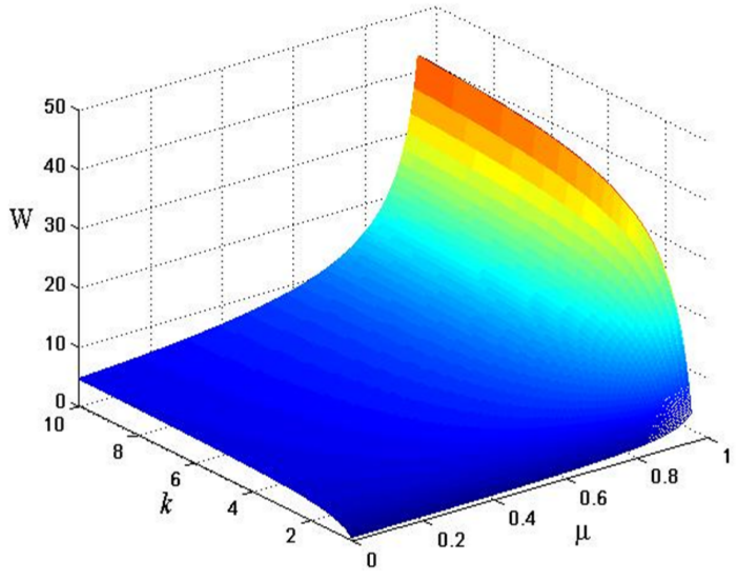


Figure 3: Soliton width as functions of $k$ and $\mu$ with $\omega_{ci} = 0.4$, $\delta = 10°$, and $u = 0.1$.

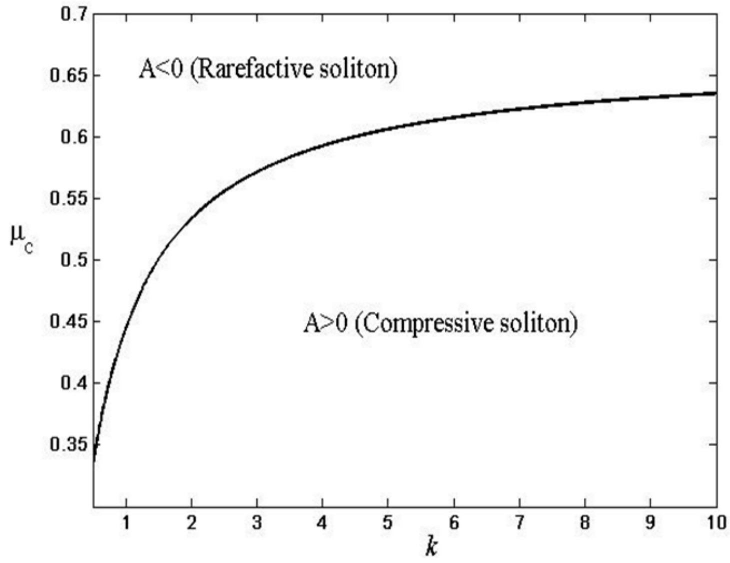


Figure 4: Plot of Critical density ($\mu_c$) with superthermal index ($k$) (when $A$ becomes zero).

in the superthermal index $k$ measures the deviation from the Maxwellian behavior through an increase in the superthermal electrons component and a concomitant decrease in the thermal part of the electron velocity distribution. It is also observed that an increase of the external magnetic field leads to a decrease of the potential width. Therefore, we find that the soliton is narrowed in the presence of magnetic field. It is not surprising; because greater magnetic field reduces the radius of particle circular motion. This is similar to that obtained earlier for ion-acoustic solitary waves in a magnetized superthermal plasma [50]. It should be noted that the width of solitary waves decreases with $\omega_{ci}$ for higher values of superthermal index, and it is almost constant (with respect to $\omega_{ci}$) for lower value of $k$. This means that the effect of magnetic field on the soliton width is noticeable for greater values of $k$. The above results also in general are in agreement with the results reported earlier in [45] for magnetized nonextensive dusty plasma, and also in [49] for unmagnetized superthermal dusty plasmas.

In Fig. 2, we observe the effects of the external magnetic field ($\omega_{ci}$) and the obliqueness of the wave propagation ($\delta$) on the formation of DIA solitary structure with $k = 2$, $\mu = 0.5$ and $u = 0.1$. It is obvious that the soliton width has a maximum value in the range of variation of $\delta$. This figure also shows that the maximum value of the soliton width decreases as $\omega_{ci}$ increases. The later is consistent with the result of [45] obtained for DIA waves with nonextensive electrons. Similar effect has been also observed in the study of the ion-acoustic [50] and DIA [51] solitary waves. Fig. 2 indicates that for $\delta = 0$ the soliton width is independent of $\omega_{ci}$. It is seen that external magnetic field ($\omega_{ci}$) doesn't affect on the angle in which the maximum width occurs.

Fig. 3 presents soliton width as a function of $\mu$ and $k$ with $\omega_{ci} = 0.4$, $\delta = 10°$, and $u = 0.1$. This figure shows that increasing the electron superthermality leads to a decrease in the soliton width. This is consistent with the result obtained in Fig. 1. It is important to note that as the value of $k$ increases, the value of DIA soliton width increases (is almost constant) for higher (lower) value of dust concentration ($\mu$). This clearly indicates that the effect of superthermality on the soliton width in higher values of $\mu$ is more. It can be seen

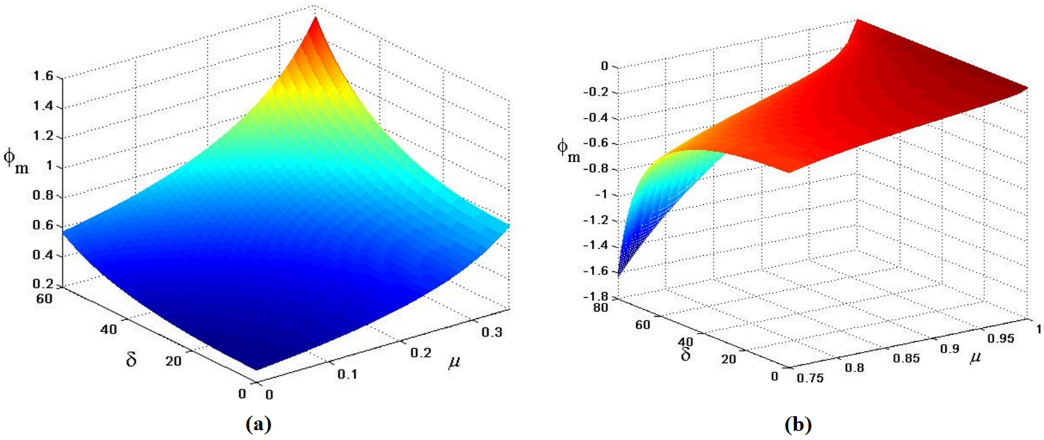


Figure 5: **a)** Compressive soliton amplitude $\phi_m$ respect to $\delta$ and $\mu$, and **b)** Rarefactive soliton amplitude $\phi_m$ respect to $\delta$ and $\mu$ for $k = 1.5$ and $u = 0.1$.

that the soliton width increases as $\mu$ increases. Fig. 3 also shows that the effect of relative density on the width increases as the electrons evolve toward thermal distribution. Figs. 1-3 indicate that width of the DIA solitary waves is highly sensitive to the superthermal electrons ($k$), dust concentration ($\mu$) and the magnetic field ($\omega_{ci}$).

In Fig. 4, we have shown how the critical value of relative density ($\mu_c$) varies with superthermal index ($k$). $\mu_c = 4k/3(2k+1)$ is the special value of density for which polarity of the solitary waves is transmitted from the negative kind ($\mu > \mu_c$) to positive ($\mu < \mu_c$). Fig. 4 clearly demonstrates that $\mu_c$ decreases with superthermality strength. Note that for $\mu < 1/3$, only positive polarity (compressive solitons) of DIA can be created and in this situation, superthermality will not be able to change the polarity of soliton. It is obvious that for other values of $\mu$ ($\mu > 1/3$), polarity of soliton drastically depends to superthermality. It is observed that the transition from a positive polarity to a negative polarity is sensitively depends on the variation of electrons superthermality [25, 42] and relative density.

Figs. 5 and 6 show how the compressive and rarefactive DIA solitary waves behave when the superthermality of electrons, external obliquely magnetic and dust concentration changes. Figs. 5-a and 5-b show the behavior of amplitude of compressive and rarefactive solitons as a function of superthermal parameter $k$ and angle $\delta$ with $\mu = 0.1$ and $u = 0.1$, respectively. It turns out that the amplitude of both compressive and rarefactive DIA solitons increases with increasing the values of $\delta$. It is also observed that the amplitude of compressive (rarefactive) solitons increases (decreases) with increasing the values of $\mu$. The interesting finding from Fig. 5-a is that, the higher values of angle ($\delta$) and density ($\mu$) (compared with lower values) have greater effects on the amplitude of positive potential. Fig. 5-b shows that the amplitude of negative soliton diminishes as $\mu$ increases and it becomes zero for $\mu = 1$ (and all values of $\delta$). Similar results were also obtained in [45] for DIA solitary waves in magnetized nonextensive dusty plasma. It should be noted that when the dust number density is increased, the number density of ions reduces and an increase in $\mu$ raises the phase speed as well as the amplitude of the solitary waves [45].

The amplitude of DIA solitary waves as depicted in Figures 6-a and 6-b reveal the possibility for the formation of compressive and rarefactive in a superthermal dusty plasma with $u = 0.1$ in cases (a) $\mu = 0.05$ and (b) $\mu = 0.85$. It can be seen from these figures that amplitude of both positive and negative potential increases when superthermal index ($k$)

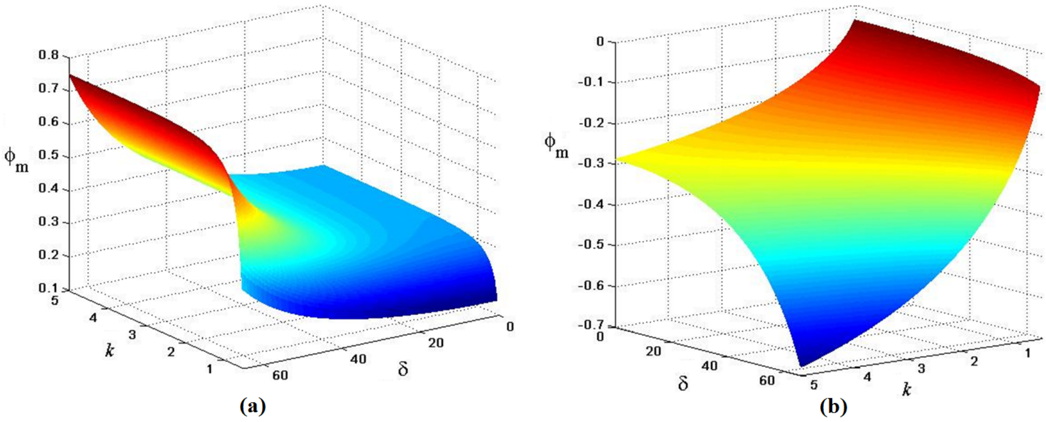


Figure 6: **a)** Compressive soliton amplitude $\phi_m$ respect to $\delta$ and $k$ with $\mu = 0.05$ and $u = 0.1$, and **b)** Rarefactive soliton amplitude $\phi_m$ respect to $\delta$ and $k$ with $\mu = 0.85$ and $u = 0.1$.

increases. Thus, the amplitude of DIA solitons decreases in the presence of superthermal electrons. This contrasts with the result obtained in [52] for dust acoustic waves in the presence of superthermal electrons, meaning that the behavior of dust acoustic and dust-ion acoustic waves is different in the superthemality plasma. It is seen that due to superthermality in the magnetized dusty plasma, the solitary wave overspreads specifically in large values of angle ($\delta$). It is observed that the magnitude of the external magnetic field has a significant effect on the width, but not the amplitude of the DIA solitary waves.

# 5 Conclusion

We have carried out a nonlinear analysis of dust ion acoustic waves by deriving the KdV equation in a magnetized dusty plasma system containing cold fluid ions, superthermal electrons and stationary negative dust. Behavior of obliquely propagating DIA solitary waves has been investigated in this model of plasma. It has been observed that an enhancement in the electron superthermality causes a decrease in the soliton width. At first, the width of solitary waves increases and then decreases with increasing the obliqueness index ($\delta$). Also, it has been found that the magnitude of the external magnetic field only has a significant effect on the width, and it has no effect on the amplitude of the DIA solitary waves. It is clear that as the magnetic field increases, the width of soliton decreases, and as a result the solitary structures seems to be more spiky. Moreover, the results have indicated that both compressive and rarefactive solitons can be propagated in this plasma. The solitary waves exist with positive (negative) potential when $A > 0$ ($A < 0$). It should be noted that the soliton compression and rarefaction are sensitive to the degree of electron superthermality ($k$) and dust concentration ($\mu$). The amplitude of both compressive and rarefactive solitary waves increases by increasing the superthermal index $k$. Therefore, the wave amplitude is reduced in the superthermal dusty plasma. Our results also have depicted that the amplitude of the positive (negative) solitary waves increases (decreases) with increasing the values of $\mu$. We have also found that due to the increase of obliqueness ($\delta$), the amplitude of the positive and negative solitary waves increases.

Eventually, it should be pointed out that the most of presented results are in agreement with the results of similar investigations, and one can find a better perspective about the previous analogous medium through combination of our results with them.

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