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The Temperature and Mass Effects on Dust Grain Electrical Potential in Dusty Plasma

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Abstract. By orbit-limited motion (OLM) theory and the kinetic model, currents carried by electrons and ions on the dust grain are obtained and the effects of temperature and drift velocity of ions on the dust grain electrical potential are considered. It is shown that the dust grain electrical potential and thus the dust grain charge are affected by density of ions (electrons) and dust grain. Moreover, it is found that as the ratio of temperature electron-to-ion is raised, the plasma with heavier ions experiences the larger electric potential of the dust grain. Furthermore, it is indicated that the dust grain electrical potential for the potassium plasma is significantly higher than the oxygen plasma. Finally, it is shown that by decreasing the dust grain electrical potential and the electron temperature, the drift velocity of ions is increased.

Keywords: electron temperature, drifting ions, charging dust grains, orbit motion limited theory, dusty plasma

1 Introduction

The study of particle charging process is one of the most important parts of the new type of plasmas known as dusty plasma. In fact, the study of dusty plasmas concerns the collective behavior of dust grains in such plasmas [1, 2, 3].These particles may have sizes ranging from tens of nanometers to hundreds of microns. Although the particles are commonly solid, they might also be fluffy ice crystals or even liquid droplets [4, 5]. They are typically much more massive than the plasma electrons and ions [6]. Dusty plasmas exist in a number of active fields of plasma research. They are dealt with in a wide range of environments from experimental [6, 7] to theoretical plasmas [8, 9, 10, 11, 12]. There seems to be dusts everywhere in: cosmic plasmas, cometary plasmas, interplanetary plasmas, planetary plasmas, plasmas near the earth and plasmas in the laboratory [8, 9, 10, 13, 14]. In other words, most of plasmas are dusty plasmas which means that there may be some dust particles in them[15, 16]. It is known in astrophysics that dust grains are very prevalent throughout the universe [15, 16, 17]. It is thought that the evolution of the solar system into its present state to have proceeded from a stage of solid matter in the form of dust particles, and larger bodies are state has through the coagulation of dust particles. Since the components of the solar nebula gases were likely to have been at least partially ionized, the initial evolution of dust into larger objects is included the dusty plasma physics [14, 15]. What makes the study of the dusty plasma in astrophysical and space plasmas necessary and interesting is the fact that the dust particles are charged in the plasma. The processes of dust charging has been investigated using theoretical and empirical methods [15, 18]. For example, charging a spherical probe in connection with the charge of satellites and rockets in the Earth's ionosphere and the magnetosphere has been considered, assuming that in space conditions, the Debye radius of electron is bigger than the probing size. Due to the interaction of the solar wind with the Earth's magnetosphere and ionosphere, the drift velocity of the ions changes as well as the electron-to-ion temperature ratios, especially for oxygen ions. Therefore, it is important to investigate the process of dust charging under the influence of these factors, especially for satellites and rockets [19, 20]. A theory that considers the mechanism of dust charging is needed for evaluating the overall behavior of dust in the plasma. The orbit-limited motion (OLM) theory is the most prevalent dust charging theory. This theory was first proposed by Langmuir and Mott-Smith [21] and was completed further in 1960$ [20, 21, 22]. Although this theory uses simple concepts such as angular momentum and collision cross-section, it predicts the dust potential with an acceptable accuracy for a wide range of grain sizes [22, 23, 24, 25, 26, 27]. In the present work, we study the effects of the temperature and drift velocity of plasma particles on dust grain charging using a kinetic model and the OLM theory. We assume that dust potential is negative because the mass of ion is much larger than the mass of electron and the electron reaches the dust grain faster than ion. The Maxwellian distribution for dust grain charging currents in equilibrium plasma is considered. It has now been proved that the powerful standard Boltzmann-Gibbs (BG) and the thermodynamics of the BG statistical mechanics are valid when certain conditions are satisfied. This is a scenario that typically occurs for the short-range interactions in the many-body Hamilton systems [28, 29, 30]. These currents are carried by electrons and ions that the role of electrons and ions temperatures on the dust grain electrical potential is evaluated. Moreover, for studying a more general case, a drift velocity is considered for ions, and its role on the electrical potential of dust grain is calculated. The results are presented for oxygen and potassium plasmas to find the effect of ion mass on the results.

This work is organized as follows: In section 2, the basic formulas in OLM theory are presented. Also, the electron and ion currents on dust grain are calculated and the formulation needed for the dust grain electrical potential and ion electron drift velocities are obtained. In section 3, the numerical simulations of nonlinear equations are discussed and the influence of some parameters such as: relative temperature of electron to ion, drift velocities and mass of plasma particles on the dust grain electrical potential in the dusty plasma are investigated and discussed. Finally, a summary and conclusions are given in Section 4.

2 Basic equations and charging processes

In this section, we find the dust charging currents in plasma under two conditions, i.e., ions distribution is either a standard Maxwellian or a shifted Maxwellian. Dust charging current I_j (carried by plasma species j) can be calculated using the OLM theory [25, 31]. Based on this theory,I_j results from plasma particles with the charge q_j and the velocity distribution $f_j(v_j)$ as

$$I_j = q_j \int_{v_j^{\min}}^{\infty} v_j \sigma_j^d f_j(v_j) dv_j, \tag{1}$$

where $v_{j\,\min}$ the minimum velocity for which the collision with the grain is possible. Assuming that v_j and v_{gj} are plasma particle velocity before and after grazing collision with the dust grain then the conservation of angular momentum is as the following

$$m_j v_j b_j = m_j v_{gj} r_d \tag{2}$$

that m_j is the mass of the incident charge particle, rd is the grain radius and b_j is the impact parameter between the dust grain and the plasma particle j. The energy conservation is as $E_1 = E_2$ that E_1 and E_2 are plasma particle energy before and after grazing collision with the dust grain, respectively. Thus, the conservation of energy is as

$$\frac{1}{2} m_j v_j^2 = \frac{1}{2} m_j v_{gj}^2 + \frac{q_j q_d}{r_d}, \tag{3}$$

Where q_d is the charge of dust grain and q is the charge of incident particle. Then, by considering conservation of momentum and energy, the collision cross-section of the dust grain with electron and ion $\sigma_j^d = \pi b_j^2$ is defined as

$$\sigma_j^d = \pi r_d^2 \Big(1 - \frac{2 q_j \phi_d}{m_j v_j^2}\Big) \tag{4}$$

where ϕ_d is the electrical potential between the plasma and the dust grain. For a standard Maxwellian distribution, Eq.1 can be calculated using either the Gaussian integrals or integration by parts [32] and the combination of both methods. The calculation for the case of a repulsive potential ($q_j \varphi_d > 0$) is as the following: [4, 33, 34, 35]

$$I_j = 4\pi r_d^2 q_j n_j \Big(\frac{k_B T_j}{2\pi m_j}\Big)^{\frac{1}{2}} \exp\Big(-\frac{q_j \phi_d}{k_B T_j}\Big), \tag{5}$$

where k_B is the Boltzmann constant and T_j is the temperature of species j. In an attractive potential ($q_j \varphi_d < 0$), the calculation is somewhat different because an initial velocity is not required for the incident particles; however, the procedure for integrating is similar to Eq.5 and the current is obtained as

$$I_j = 4\pi r_d^2 q_j n_j \Big(\frac{k_B T_j}{2\pi m_j}\Big)^{\frac{1}{2}} \Big(1 - \frac{q_j \phi_d}{k_B T_j}\Big). \tag{6}$$

The two obtained currents are valid for plasma with a symmetrical Maxwellian distribution, i.e., the drift velocity of plasma particles v_j is very smaller than their thermal velocity v_{Tj} and the ratio $v_j/v_{Tj} (= u.)$ can be neglected. However, in a more general case, a drift velocity may be attributed to the ions of the plasma. The calculations, in part similar to the case of nondrifting ions, yield as

$$I_j = 4\pi r_d^2 e n_i \Big(\frac{k_B T_i}{2\pi m_i}\Big)^{\frac{1}{2}} \Big[F_1(u_0) - F_2(u_0) \frac{e\phi_d}{k_B T_i}\Big], \tag{7}$$

where e is the electron charge and n_i is the ion density; F_1 and F_2 are functions of the ratio of drift velicity to thermal velocity of ions u_0 and are defined as follows:

$$F_1(u_0) = \frac{\sqrt{\pi}}{4u_0}(1 + 2u_0^2)\mathrm{erf}(u_0) + \frac{1}{2}\exp(-u_0^2), \tag{8}$$

and

$$F_2(u_0) = \frac{\sqrt{\pi}}{2u_0}\mathrm{erf}(u_0), \tag{9}$$

where erf denotes the error function and by its calculating we have

$$I_i \simeq \pi r_d^2 e n_i v_i . \Big[1 - \frac{2e\phi_d}{v_i^2 m_i}\Big], \tag{10}$$

Since ions are much heavier than electrons, at first, the ion current is significantly less than electron current, and the dust grain acquires negative charge. A little later, the ion current to the grain increases. Finally, both ions and electron currents become equal, which is the interesting situation for our study. The quasi-neutrality condition of the system implies tha

$$\frac{n_e}{n_i} = 1 - Z_d \frac{n_d}{n_i}, \tag{11}$$

where Z_d is the number of charges on the dust grain. It is cleared that the ratio $Z_d n_d / n_i$ is an important factor in determining the behavior of dusty plasma. When $Z_d n_d \ll 1$, the dust particles can be assumed to be isolated, and when this ratio is comparable to unity, the seed can be assumed to be non-isolated. In non-isolated cases, increasing the density of the number of dust grains means that the seeds have a great deal of appetite for electrons, but the number of electrons in each dust grain is small. As noted above, the ratio $Z_d n_d \ll 1$ is an important factor in determining the dusty plasma behavior, and it is common in the literature that a parameter is defined for taking account of this ratio; that is

$$P = \frac{r_d Z_d n_d k_B T_i}{e n_i}. \tag{12}$$

Now, as the final step for evaluating the role of electron temperature in dust grain charging process, we have to use Eq. 5 for electrons and Eq. 6 for ions and owing to the fact that $I_i + I_e = 0$, we can write

$$\Big(\frac{1}{\tau}\frac{m_e}{m_i}\Big)^{1/2} \exp\Big(-\frac{e\phi_d}{k_B T_i}\frac{1}{\tau}\Big)(1 - \frac{e\phi_d}{k_B T_i}\Big) = 1 + P\frac{e\phi_d}{k_B T_i}, \tag{13}$$

where τ is the ratio of electron to ion temperature. Another interesting case can be the studying of the effect of drift velocity on dust grain charging process and thus the Eq. 5 is as follow (note that here we deal with only the case $u_0 \ll 1$ because the case $u_0 \ll 1$ can be considered as the situation of non-drifting ions).

$$\Big(\frac{1}{\tau}\frac{m_e}{m_i}\Big)^{1/2} \exp\Big(-\frac{e}{k_B}\frac{\phi_d}{k_B T_i}\frac{1}{\tau}\Big)\frac{\sqrt{\pi}}{2}u.\Big(1 - \frac{e\phi_d}{u^2 . k_B T_i}\Big) = 1 + P\frac{e\phi_d}{k_B T_i}. \tag{14}$$

Now, we have to solve Eqs. 13 and 14. Obviously, these equations are very nonlinear and have no analytical solutions. Therefore, with boundary conditions, we can obtain the distribution profiles of the electric potential of the dust in the plasma.

3 Results and discussion

In the previous section, the nonlinear equations the electrical potential was obtained. These equations show that the dust grain electrical potential is dependent to the characteristics of the dusty plasma such as the density of ions, electrons and of grains dust, the particles size of the dust, electrons and ions temperature and mass of ions. In this section, we obtain numerical solution of nonlinear Eqs.13 and 14. The effect of electron-to-ion temperature ratio on the dust grain electrical potential is presented in Fig 1. Fig 1 shows the variation of the dust grain electrical potential in terms of log P. In these figures, plasmas are assumed as O^+ plasma for Figs. 1_a and 1_b and K^+ plasma for Figs. 1_c and 1_d. Also, the changing in ratio (electron-to-ion) temperatures (τ) is considered which in Figs. 1_a and 1_c are less than unity (i.e., the temperature of electrons is less than the temperature of ions) and Figs. 1b and 1d are larger than unity. As the ion density, ion temperature and the grain radius are assumed to be constant, in fact; the fig 1 illustrates the effect of dust density on the electrical potential φ_d. From Fig. 1, it is obvious that as the density of dust grains increases (i.e., as the mean distance between dust grains decreases), the value of $e\varphi_d/k_BT_e$ decreases, or equivalently, the mean charge of dust grains ($Z_d = -(r_d/e)\varphi_d)$) decreases. Fig 1 can be analyzed from another point of view. If it is assumed that the ion density, ion temperature and the grain radius are constant, then the effects of ion or electron density on the electrical potential φ or dust charge Z_d can be concluded from the Fig 1. This shows that as the plasma density (ne and ni) increases, the value of $-e\varphi_d/kBTe$ inceases, i.e., the charge density of dust increases. For example, the dust grain electrical potential is larger in weakly ionized plasma. Furthermore, from Fig 1 , it is obvious that a smaller electron temperature leads to a smaller potential value. It is indicated that a smaller τ leads to a smaller electrical potential of dust grain. Moreover, from Figs. 1_b and 1_d , it can be seen easily that heavier ions may lead to a larger dust grain electrical potential (compare with Figs. 1_a and 1_c which illustrates the behavior of an O^+ plasma). The values of τ unity (and the solid curve corresponding to $\tau = 1$) is also plotted for comparison purposes. Scales on these two plots are considered the same for more clarity. The absolute value of electrical potential for plasma with potassium ions, which are about two times heavier than oxygen ions, is larger. The physical reason for that is the principle of energy conservation. According to Eq. 3, as the mass of the ions increases, the amount of kinetic energy increases and therefore the amount of potential energy and therefore the electric potential of dust grain must also be increased.

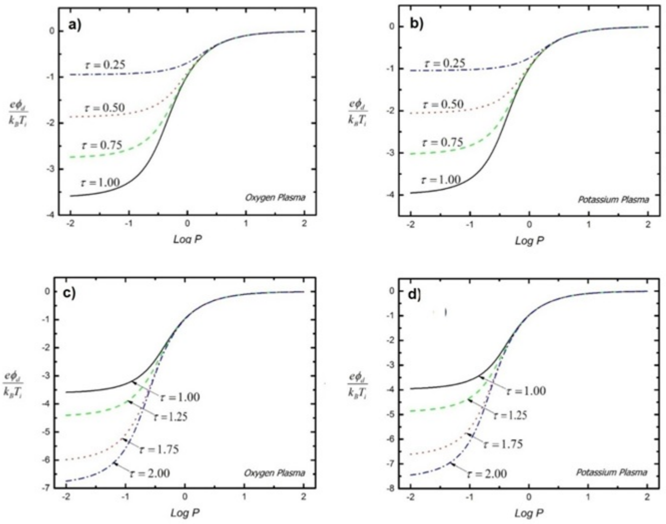

Figure 1: The variation of the dust grain electrical potential in terms of log P with different ratio (electron to ion) temperatures (τ) (a) O^+ plasma and is less than unity (b) K^+ plasma and τ is less than unity (c) O^+ plasma and τ is larger than unity (d) K^+ plasma and τ is larger than unity

Figs. 2*a* and 2*b* show the variation of the dust grain electrical potential in terms of log P for values of τ smaller and larger than unity, respectively. Fig. 2 is dedicated to both O^+ and K^+ plasmas, and the scales are the same for easier comparison. An important point that is concluded from Fig. 2 is the fact that not only the values of the dust grain electrical potential increases with τ , but also the difference between the values of the electrical potential of dust grain corresponding to oxygen and potassium plasmas becomes more and more significant as the value of τ increases.

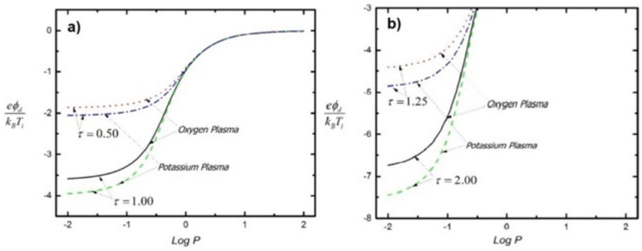

Figure 2: The variation of the dust grain electrical potential in terms of log P for different values τ that in (a) is less than unity and (b) is larger than unity. For comparison purposes, curves corresponding to K^+ and O^+ plasmas are plotted together.

Fig. 3 shows the variation of the dust grain electrical potential in terms of log P for different values of relative (drift to thermal) velocities (u_0). For comparison purposes, curves corresponding to K^+ and O^+ plasmas are plotted together. By comparing curves, it is obvious that as u_0 increases, the dust grain electrical potential decreases. In other words, an increase in the drift velocity of ions acts as a reduction in the electron temperature. It can be seen in 3 that an increase in ions drift velocity decreases the electrical potential of dust grain; however, the dust grain electrical potential depends on the mass of plasma ions, that is, the electrical potential is larger for heavier plasma ions.

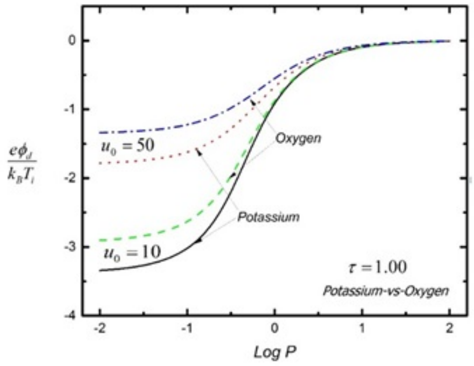

Figure 3: The variation of the dust grain electrical potential in terms of log P for different values of relative (drift to thermal) velocities (u_0). For comparison purposes, curves corresponding to K^+ and O^+ plasmas are plotted together.

4 Conclusion

In this work, the dust grain charging currents carried by electrons and ions were calculated using a kinetic model, Maxwellian distribution function and OLM theory. The calculations were performed for finding the role of densities of dust grains and ions on the dust grain electrical potential which is the main factor in determining the tendency of the grain for acquiring more charge. Also, the effects of the ratio temperature of electron-to-ion and ion drift velocity were studied. It was found that increasing dust grain charge (that is, reduction in distance between dust grains) is caused by decreasing the dust grain electrical potential. The dust grain electrical potential and the dust grain charge were affected by density of ions (electrons), too. It was shown that as the density of ambient plasma increases, the dust grain electrical potential increases and thus the charge on the dust grain increases. It was found that as the ratio of temperature electron-to-ion raised, the plasma with heavier ions experiences the larger electric potential of the dust grain. These results were concluded through involving oxygen and potassium ions and it indicated that the dust grain electrical potential for the potassium plasma was significantly higher than the oxygen plasma. Finally, it was found that the dust grain electrical potential decreases by increasing drift velocity of ions and an increase in the drift velocity acts as a reduction in electron temperature.

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