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Bulk Viscous Bianchi Type VI_0 Cosmological Model in the Self-creation Theory of Gravitation and in the General Theory of Relativity

E. A. Hegazy
Mathematics Department, Faculty of Science, Minia University, 61519 El-Minia, Egypt;
emails: sayed00ali@gmail.com; elsayed.mahmoud@mu.edu.eg

Abstract. In the second self-creation theory of gravitation and in the general theory of relativity, Bianchi type VI_0 cosmological model in the presence of viscous fluid is studied. An exact solution of the field equations is given by considering the cosmological model yields a constant decelerations parameter q=constant and the coefficients of the metric are taken as $A(t) = [c_1 t + c_2]^{\frac{3L}{(q+1)(L+1)}}$, $B(t) = [c_1 t + c_2]^{\frac{3}{2(q+1)(L+1)}}$, $C(t) = [c_1 t + c_2]^{\frac{3}{2(q+1)(L+1)}}$,where c_1, c_2 and L are constants . Effect of the viscosity on the entropy of the universe is given by a composition of the second law of thermodynamics with the the energy momentum tensor T^i_j with bulk viscous term in a conservative manner. We obtained a formula for calculating the entropy of the universe in term of the viscosity and used it to study and compare the Entropy, Enthalpy, Gibbs energy and Helmholtz energy of the universe in the presence of viscosity term in the self-creation theory of gravitation and in the general theory of relativity. The physical and geometrical properties of the obtained models are discussed.

Keywords: General theory of relativity ; Einstein field equations; Bianchi type VI_0 cosmological models; Viscosity ; Entropy

1 Introduction

In 1982, Barber [1] has proposed two self-creation theories of cosmology that allows the scalar field ϕ to cooperate with the particle and photon momentum four vectors. The first theory, which can be considered as a modification of the Brans-Dicke theory [2], is unsatisfactory since the equivalence principle is violated [3]. The second theory is an adjustment in the general theory of relativity [4] to a variable G-theory. In the second theory, the scalar field ϕ does not directly gravitate, but simply divides the matter tensor, acting as a reciprocal of the gravitational constant. Here, ϕ couples to the trace of the energy-momentum tensor. In (2010), a review for theory of self-creation was given by Barber [5].

Several authors dealt with cosmological models in the presence of bulk viscosity in the general theory of relativity and modified theories of it. In $f(R, T)$ theory of gravity, Ram and Kumari [6] studied Bianchi types I and V bulk viscous fluid cosmological models, bulk viscous cosmological model was studied by Debnath [7] and locally rotationally symmetric Bianchi type I cosmological model was studied by Mahanta [8]. Bianchi type-II bulk viscous cosmological model in the theory of relativity was studied by Sharma [9]. In the presence of variable bulk viscous coefficient, constant bulk viscous coefficient and in the absence of bulk viscosity five dimensional string cosmological models were studied by Mohanty [10]. Ali [11] studied spatially homogeneous and anisotropic axially symmetric Bianchi type-I space time in the presence of bulk viscous fluid with variable cosmological term Λ. With a new form of time varying deceleration parameter, Singh and Bishi [12] studied Friedmann Robertson Walker (FRW) cosmological model in the Brans-Dicke theory of gravitation. Mete et. al. [13] studied Bianchi type IX string space time in the general relativity theory in the presence of bulk viscosity and magnetic field. Mak and Harko [14] investigated Bianchi type I space times with causal bulk viscous cosmological fluid. A new class of LRS Bianchi type-I cosmological model with variable deceleration parameter with bulk viscous fluid in the general theory of relativity was investigated by Pradhan et. al. [15]. In the Brans-Dicke theory of gravitation, Singh and Bishi [12] studied FRW cosmological model in the presence of bulk viscosity and time varying deceleration parameter. Kremer and Sobreiro [16] dealt with bulk viscous cosmological model with interacting dark fluids. In the framework of Lyra geometry, spatially homogeneous and anisotropic Bianchi type-VI_0 cosmological models in the presence of bulk viscosity were studied by Asgar and Ansari [17]. In the presence of Chaplygin gas with time varying- Λ in Lyra geometry, Patra and Sethi [18] studied Bianchi type-III bulk viscous cosmological models. By considering time dependant deceleration parameter and a constant viscosity coefficient, FRW cosmological models in the presence of viscous fluid in Lyra's geometry were investigated by Singh et al.[19]. Singha and Srivastavab studied flat FRW cosmological model filled with dark matter and viscous new holographic dark energy [20]. Kotambkar et. al. [21] investigated Bianchi type I anisotropic cosmological models filled with a bulk viscous cosmic fluid in the presence of variable gravitational and cosmological constants.

In the second self-creation theory in the general theory of relativity, Katore et. al. [22] studied spatially homogeneous and isotropic FRW space time in the presence of viscous fluid. A five dimensional Kaluza-Klein cosmological model in the presence of a bulk viscous fluid was investigated by Kumar and Reddy [23]. Borkar and Ashtankar [24] studied Bianchi type I cosmological model with bulk viscous barotropic fluid with varying Λ and functional relation on Hubble parameter H with deceleration parameter q and an average scale factor R. LRS Bianchi type-I cosmological model with constant deceleration parameter where the metric potentials are taken as function of x and t and the coefficient of bulk viscosity is assumed to be a power function of the mass density was given by Pradhan and Pandey [25]. Borkar and Ashtankar [26] studied LRS Bianchi type-II cosmological model with string viscous fluid and magnetic field. Raut et. al. [27] studied interacting two fluid viscous dark energy cosmological models in Bianchi type II cosmological model. Tade et. al. [28] investigated a Kaluza-Klein bulk viscous string cosmological model. Kantowski Sachs cosmological model in the presence of bulk viscous fluid with one dimensional cosmic strings was studied by Naidu et al [29]. When the energy momentum tensor is a bulk viscous fluid containing one-dimensional cosmic strings, Rao and Sireesha [30] studied axially symmetric string cosmological model. As a consequence for these studies we derive the Bianchi type VI_0 cosmological model in the presence of viscous fluid in the self-creation theory. In section 2, we derive the field equations for the model. Solution of the field equation is given in section 3. The effect of the viscosity term on the thermodynamic functions (Entropy, Gibbs free energy, Helmoheltz energy and Enthalpy) of the universe is given in section 4. Bulk viscous Bianchi type VI_0 cosmological model in the general theory of relativity is given in section 5. Conclusion is indicated in section 6.

2 The metric and field equations

The line element ds^2 for Bianchi type VI_0 cosmological model reads as:

$$ds^2 = [A(t)]^2 dx^2 + [B(t)]^2 e^{2x} dy^2 + [C(t)]^2 e^{-2x} dz^2 - dt^2 \tag{1}$$

$$(x^1 = x, x^2 = y, x^3 = z, x^4 = t),$$

In the self-creation theory in the general relativity theory, Einstein field equations take the form [1]:

$$G^i_j = -8\pi\phi^{-1}T^i_j, \tag{2}$$

and

$$\Box\phi = \frac{8\pi}{3}\eta T. \tag{3}$$

Here, T is the trace of the energy momentum tensor, $\Box\phi = \phi^{;k}_{;k}$ is d'Alembertian invariant and η is a coupling constant can be determined from experiments. The measurements of the deflection of light restrict the value of coupling to $\eta < 10^{-1}$. In the limit $\eta \to 0$, the Barber's second self creation theory approaches the standard general relativity in every respect.
In the presence of bulk viscous fluid, the energy momentum tensor T^i_j read as [31],[32]:

$$T^i_j = (\rho + p)u^i u_j + pg^i_j - \xi\Theta(g^i_j + u_i u^j). \tag{4}$$

ξ is the coefficient of bulk viscosity, Θ the expansion scalar of the cosmological model, ρ energy density and p isotropic pressure. In the co-moving coordinates ($u_4 = -1, u^4 = 1, u^i u_j = -1, u^1 = 0 = u^2 = u^3$), the components of T^i_j read as:

$$T^1_1 = (p - \xi\Theta) = T^2_2 = T^3_3, \qquad T^4_4 = -\rho. \tag{5}$$

For the line element (1), equations (2) gives:

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + A^{-2} = -8\pi\phi^{-1}[p - \xi\Theta], \tag{6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - A^{-2} = -8\pi\phi^{-1}[p - \xi\Theta], \tag{7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - A^{-2} = -8\pi\phi^{-1}[p - \xi\Theta] \tag{8}$$

$$\frac{\dot{C}}{C} = \frac{\dot{B}}{B} \tag{9}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - A^{-2} = 8\pi\phi^{-1}\rho. \tag{10}$$

$$\ddot{\phi} + \dot{\phi}(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}) = \frac{8\pi}{3}\eta(3(p - \xi\Theta) - \rho), \tag{11}$$

where dot means ordinary differentiation with respect to t.
From the conservation of the energy momentum tensor $T^i_{j;i} = 0$ with equivalent pressure and equivalent density as defined by Solong [33], we get:

$$(\frac{\rho}{\phi})^{\cdot} + (\frac{\rho + p - \xi\Theta}{\phi})(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}) = 0 \tag{12}$$

3 Solution of the field equations

For the universe, the volume element $V=\sqrt{-g}=ABC$, the average scale factor $R=\sqrt[3]{V}=\sqrt[3]{(ABC)}$. If we consider the model yields a constant deceleration parameter $q\,(constant)=-\frac{R\ddot{R}}{\dot{R}^2}$, then we have .

$$R=[c_1t+c_2]^{\frac{1}{q+1}}, \tag{13}$$

where c_1 and c_2 are constants of integrations.

The motivations behind assuming a constant deceleration parameter that solution of the equations (6) - (11)(Einstein field equations) is not possible in the case of time dependent deceleration parameter. Also, the familiar cosmological models of the general relativity theory and Brans-Dike theory with curvature parameter equal zero are cosmological models which have a constant deceleration parameter. Several authors (Berman [34], Berman and Gomide [35], Pradhan and Aotemshi [36], Pradhan et al. [37], Singh and Desikan [38] and Maharaj and Naidoo [39]) dealt with cosmological models which have a constant deceleration parameter.

From (13) with the definition of the scale factor R, the coefficients of the metric A, B and C can be take the form:

$$A(t)=[c_1t+c_2]^{\frac{3L}{(q+1)(L+1)}},\ B(t)=[c_1t+c_2]^{\frac{3}{2(q+1)(L+1)}},$$

$$C(t)=[c_1t+c_2]^{\frac{3}{2(q+1)(L+1)}}, \tag{14}$$

where L is a constant will be determined from the achievement of the field equations (6)-(10). Now from (7) and (8), we must have:

$$\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{\ddot{C}}{C}-\frac{\dot{A}\dot{C}}{AC}=0 \quad (\ automatically\ satisfied),$$

and from (6) and (7) we get:

$$\frac{\ddot{A}}{A}+\frac{\dot{A}\dot{C}}{AC}-\frac{\ddot{B}}{B}-\frac{\dot{B}\dot{C}}{BC}-\frac{2}{A^2}=0,$$

hence,

$$-\frac{3(2L-1)(q-2)c_1^2}{2(1+L)(1+q)^2[c_1t+c_2]^2}-2[c_1t+c_2]^{\frac{-6L}{(1+L)(1+q)}}=0. \tag{15}$$

To find zeros of (15), we put $q=\frac{2L-1}{L+1}$, then equation (15) reduces to:

$$\frac{4L^2+(1-2L)c_1^2}{2L^2[c_1t+c_2]^2}=0 \tag{16}$$

which has a solution in the form $L=\frac{1}{4}(c_1^2\pm\sqrt{-4c_1^2+c_1^4}\ )$. Here, we deal with negative sign, that is $L=\frac{1}{4}(c_1^2-\sqrt{-4c_1^2+c_1^4}\ )$ (positive sign gives the same result).

From (10) the equivalent density $\frac{\rho}{\phi}$ takes the form:

$$\frac{\rho}{\phi}=\frac{-4+3c_1^2+3\sqrt{c_1^2(c_1^2-4)}}{16\pi[c_1t+c_2]^2} \tag{17}$$

From (6) - (8), $\frac{p-\xi\Theta}{\phi}$ reads as:

$$\frac{p-\xi\Theta}{\phi}=\frac{4-c_1^2-\sqrt{c_1^2(c_1^2-4)}}{16\pi[c_1t+c_2]^2} \tag{18}$$

Equation (18) can be split to gives:

$$\frac{p}{\phi} = \frac{4 - \sqrt{c_1^2(c_1^2-4)}}{16\pi[c_1 t + c_2]^2}, \qquad \frac{\xi\Theta}{\phi} = \frac{c_1^2}{16\pi[c_1 t + c_2]^2}. \tag{19}$$

The metric coefficient $A(t)$, $B(t)$ and $C(t)$ read as:

$$A(t) = c_1 t + c_2, \quad B(t) = (c_1 t + c_2)^{\frac{2}{c_1^2 - \sqrt{c_1^2(c_1^2-4)}}} = C(t). \tag{20}$$

Line element (1) reads as:

$$ds^2 = T^2 dx^2 + T^{\frac{4}{c_1^2 + \sqrt{c_1^2(c_1^2-4)}}}[e^{2x}dy^2 + e^{-2x}dz^2] - dt^2, \tag{21}$$

where $T = [c_1 t + c_2]$.

Equation (11) reduces to:

$$T^2 \frac{d\phi}{dT^2} + MT\frac{d\phi}{dT} - N\phi = 0, \tag{22}$$

where M and N are constants.

From (22), the scalar field ϕ reads as:

$$\phi(T) = K_1 T^{M_1} + K_2 T^{M_2}, \tag{23}$$

where K_1, K_2, M_1 and M_2 are constants.

For the model (1), we have the volume element $V = T^{2+\frac{\sqrt{c_1^2-4}}{c_1}}$, the scalar expansion $\Theta = \frac{2c_1+\sqrt{c_1^2-4}}{T}$, the mean Hubble parameter $H = \frac{2c_1+\sqrt{c_1^2-4}}{3T}$ and the shear $\sigma = \sqrt{\frac{-2+c_1^2-c_1\sqrt{c_1^2-4}}{6T^2}}$.

From (17), (19) and (23) we get:

$$\rho = \frac{-4 + 3c_1^2 + 3\sqrt{c_1^2(c_1^2-4)}}{16\pi}(K_1 T^{M_1-2} + K_2 T^{M_2-2}), \tag{24}$$

$$p = \frac{4 - \sqrt{c_1^2(c_1^2-4)}}{16\pi}(K_1 T^{M_1-2} + K_2 T^{M_2-2}), \tag{25}$$

and

$$\xi = \frac{c_1^2}{16\pi[2c_1 + \sqrt{c_1^2-4}]}(K_1 T^{M_1-1} + K_2 T^{M_2-1}). \tag{26}$$

4 The Effect of the viscosity on the Thermodynamic functions (Entropy, Gibbs free energy, Helmoheltz energy and Enthalpy) of the universe

Introducing the scalar field ϕ in the trace of the energy momentum tensor has a significant role; it helps us in studing the entropy of the universe for stages different about adiabatic case. Hegazy [40] showed that the scalar field ϕ has effect on the entropy of the universe and gives a formula for calculating the entropy with help of the scalar field ϕ. Hegazy and Farook [41] proved that the displacement vector field introduced in the Sen field equations used in the framework of Lyra geometry has no effect on the entropy of the universe as it arises from geometry and not as a part of the energy momentum tensor. Hegazy and Farook [42] studied the effect of the electromagnetic field on the entropy of the universe. Hegazy [43] formulated a new class of Bianchi type I cosmological model with source of gravitation as a perfect fluid and gave a good explanation for the thermodynamic functions (Gibbs free energy, Helmoheltz energy and Enthalpy) of the universe. As consequences for these results, we study the effect of the viscosity on the entropy of the universe.

For the problem of entropy ($\mathbf{S}$) and according to the second law of thermodynamics, we must have at least $d\mathbf{S} \geq 0$ for a part of evolution of the universe [44] ,[45]. By using ρV as a definition for the internal energy U, the entropy equation reads as:

$$\mathbf{T}d\mathbf{S} = d(\rho V) + pdV. \tag{27}$$

Here, $\mathbf{T}$ represents the temperature. Since $d\mathbf{S} \geq 0$, then:

$$\dot{\rho} + (\rho + p)(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}) \geq 0. \tag{28}$$

Dividing (28) by ϕ, then $\frac{\dot{\rho}}{\phi} + \frac{(\rho+p)}{\phi}(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}) \geq 0$. But $(\frac{\rho}{\phi})^{\cdot} = \frac{\dot{\rho}}{\phi} - \frac{\rho\dot{\phi}}{\phi^2}$, and $\Theta = (\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C})$, then we have $(\frac{\rho}{\phi})^{\cdot} + (\frac{\rho+p}{\phi})(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}) + \frac{\rho}{\phi^2}\dot{\phi} \geq 0$. By adding and subtracting the term $(\frac{\xi\Theta^2}{\phi}) = \frac{\xi\Theta}{\phi}(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C})$, we get:

$$(\frac{\rho}{\phi})^{\cdot} + (\frac{\rho + p - \xi\Theta}{\phi})(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}) + \frac{\rho}{\phi^2}\dot{\phi} + (\frac{\xi\Theta^2}{\phi}) \geq 0. \tag{29}$$

The two equations (12) and (29) give:

$$\frac{\rho}{\phi^2}\dot{\phi} + \frac{\xi\Theta^2}{\phi} \geq 0. \tag{30}$$

Hence, the viscosity term ξ affects entropy on the universe, since $d\mathbf{S} \geq 0 \Rightarrow \frac{\rho}{\phi^2}\dot{\phi} + \frac{\xi\Theta^2}{\phi} \geq 0$

From (27) one can write:

$$\frac{d\mathbf{S}}{dt} = \frac{V}{\mathbf{T}}[\dot{\rho} + (\rho + p)(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C})]. \tag{31}$$

By rearranging (31) as (29) and using (12) with defining the temperature $\mathbf{T}$ as $\mathbf{T} \simeq \frac{H}{2\pi}$ [46],[47] then:

$$\frac{d\mathbf{S}}{dt} = \frac{2\pi V}{H}[\xi\Theta^2 + \rho(\frac{\dot{\phi}}{\phi})], \tag{32}$$

then:

$$\begin{aligned}\mathbf{S} = &\frac{1}{8(c_1^2(M_1+1) + \sqrt{c_1^2(c_1^2-4)})(c_1^2(M_2+1) + \sqrt{c_1^2(c_1^2-4)})}[3c_1^3(c_1t+c_2)^{\frac{\sqrt{c_1^2(c_1^2-4)}}{c_1^2}+1}(K_1(M_1(\sqrt{c_1^2(c_1^2-4)}M_2 \\ &+ 2\sqrt{c_1^2(c_1^2-4)} - 4) + \sqrt{c_1^2(c_1^2-4)})(c_1t+c_2)^{M_1} + K_2(M_2(\sqrt{c_1^2(c_1^2-4)}M_1 + 2\sqrt{c_1^2(c_1^2-4)} - 4) \\ &+ \sqrt{c_1^2(c_1^2-4)})(c_1t+c_2)^{M_2} + c_1^2(K_1(M_2 + M_1(M_2+2)+1)(c_1t+c_2)^{M_1} \\ &+ K_2(M_1 + (M_1+2)M_2+1)(c_1t+c_2)^{M_2}))].\end{aligned} \tag{33}$$

The Enthalpy (**H**), Helmholtz free energy **F**, and Gibbs free energy (**G**) read as:

$$\mathbf{H} = U{+}pV = \frac{\left(3c_1^2+2\sqrt{c_1^2\left(c_1^2-4\right)}\right)\left(c_1t+c_2\right)^{\frac{\sqrt{c_1^2\left(c_1^2-4\right)}}{c_1^2}}\left(K_1\left(c_1t+c_2\right)^{M_1}+K_2\left(c_1t+c_2\right)^{M_2}\right)}{16\pi}, \tag{34}$$

$$\begin{aligned}\mathbf{F} = U-\mathbf{T}\ \mathbf{S} = {} & \frac{1}{16\pi(c_1^2(M_1+1)+\sqrt{c_1^2(c_1^2-4)})(c_1^2(M_2+1)+\sqrt{c_1^2(c_1^2-4)})}[c_1^2(c_1t+c_2)^{\frac{\sqrt{c_1^2(c_1^2-4)}}{c_1^2}}(K_1(c_1^4(4M_2+9)\\ & +c_1^2((5\sqrt{c_1^2(c_1^2-4)}-16)M_2+9\sqrt{c_1^2(c_1^2-4)}-40)-4(\sqrt{c_1^2(c_1^2-4)}M_2\\ & +5\sqrt{c_1^2(c_1^2-4)}-4))(c_1t+c_2)^{M_1}+K_2(c_1^4(4M_1+9)+c_1^2((5\sqrt{c_1^2(c_1^2-4)}-16)M_1\\ & +9\sqrt{c_1^2(c_1^2-4)}-40)-4(\sqrt{c_1^2(c_1^2-4)}M_1+5\sqrt{c_1^2(c_1^2-4)}-4))(c_1t+c_2)^{M_2})]\end{aligned} \tag{35}$$

and

$$\begin{aligned}\mathbf{G} = \mathbf{H}-\mathbf{TS} = {} & \frac{(c_1t+c_2)^{\frac{\sqrt{c_1^2(c_1^2-4)}}{c_1^2}}}{16\pi}[(3c_1^2+2\sqrt{c_1^2(c_1^2-4)})(K_1(c_1t+c_2)^{M_1}+K_2(c_1t+c_2)^{M_2})\\ & -\frac{1}{(c_1^2(M_1+1)+\sqrt{c_1^2(c_1^2-4)})(c_1^2(M_2+1)+\sqrt{c_1^2(c_1^2-4)})}[c_1^2(2c_1^2\\ & +\sqrt{c_1^2(c_1^2-4)})(K_1(M_1(\sqrt{c_1^2(c_1^2-4)}M_2+2\sqrt{c_1^2(c_1^2-4)}-4)+\sqrt{c_1^2(c_1^2-4)})(c_1t+c_2)^{M_1}+\\ & K_2(M_2(\sqrt{c_1^2(c_1^2-4)}M_1+2\sqrt{c_1^2(c_1^2-4)}-4)+\sqrt{c_1^2(c_1^2-4)})(c_1t+c_2)^{M_2}+c_1^2(K_1(M_2+\\ & M_1(M_2+2)+1)(c_1t+c_2)^{M_1}+K_2(M_1+(M_1+2)M_2+1)(c_1t+c_2)^{M_2}))].\end{aligned} \tag{36}$$

5 Bulk Viscous Bianchi Type VI_0 Cosmological Model in the General Theory of Relativity

In the general theory of relativity ($\phi = 1$) and ($\eta = 0$), the metric coefficients will not be changed since it does not depend on the scalar field ϕ. So, we have the same line element (21) and the same kinematics quantities. Physical quantities (the pressure p_{GR} and the density ρ_{GR}) and the coefficient of the bulk viscosity ξ_{GR} will be changed and take the form:

$$p_{GR} = \frac{4-\sqrt{c_1^2\left(c_1^2-4\right)}}{16\pi\left(c_1t+c_2\right)^2}, \tag{37}$$

$$\rho_{GR} = \frac{3c_1^2+3\sqrt{c_1^2\left(c_1^2-4\right)}-4}{16\pi\left(c_1t+c_2\right)^2}, \tag{38}$$

$$\xi_{GR} = \frac{c_1^3}{16\pi\left(2c_1^2+\sqrt{c_1^2\left(c_1^2-4\right)}\right)\left(c_1t+c_2\right)}. \tag{39}$$

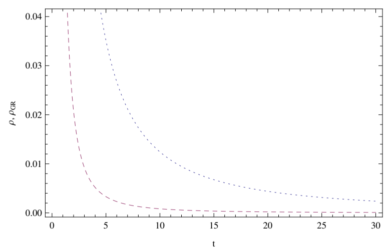

Figure 1: The density ρ in the self-creation theory (dot line) and the density ρ_{GR} in the general relativity theory (dash line) vs time t.

From (32) with $\phi = 1$, the entropy equation takes the form:

$$\frac{d\mathbf{S}_{GR}}{dt} = \frac{2\pi V}{H}[\xi_{GR}\Theta^2], \tag{40}$$

by integration, we have:

$$\mathbf{S}_{GR} = \frac{3c_1^3 (c_1 t + c_2)^{\frac{\sqrt{c_1^2(c_1^2-4)}}{c_1^2}+1}}{8\left(c_1^2 + \sqrt{c_1^2 (c_1^2 - 4)}\right)}. \tag{41}$$

The thermodynamics functions (Enthalby, Gibbs energy and Helmoheltz energy) read as:

$$\mathbf{H}_{GR} = \frac{\left(3c_1^2 + 2\sqrt{c_1^2 (c_1^2 - 4)}\right)(c_1 t + c_2)^{\frac{\sqrt{c_1^2(c_1^2-4)}}{c_1^2}}}{16\pi}, \tag{42}$$

$$\mathbf{G}_{GR} = \frac{\left(-c_1^4 + \left(\sqrt{c_1^2 (c_1^2 - 4)} + 8\right)c_1^2 + 8\sqrt{c_1^2 (c_1^2 - 4)}\right)(c_1 t + c_2)^{\frac{\sqrt{c_1^2(c_1^2-4)}}{c_1^2}}}{64\pi}, \tag{43}$$

$$\mathbf{F}_{GR} = \frac{\left(-c_1^4 + \left(\sqrt{c_1^2 (c_1^2 - 4)} + 8\right)c_1^2 + 12\sqrt{c_1^2 (c_1^2 - 4)} - 16\right)(c_1 t + c_2)^{\frac{\sqrt{c_1^2(c_1^2-4)}}{c_1^2}}}{64\pi}. \tag{44}$$

In the following, we make a comparative study for the physical quantities (the pressure and the density) and the thermodynamics functions (Entropy, Gibbs free energy, Helmoheltz energy and Enthalpy) of the universe in the self-creation theory and in the general theory of relativity. In all figures, we take ($c_1 = 2.5,\ K_2 = 2,\ K_1 = 1,\ M_1 = 0.5$, and $M_2 = 0.5$). Also, without loss of generality we consider $c_2 = 0$. Here, we deal with absolute values of the functions $\mathbf{G}$, $\mathbf{G}_{GR}$, $\mathbf{F}$, $\mathbf{F}_{GR}$ and $\mathbf{H}$, $\mathbf{H}_{GR}$ to get positive values for them. The density ρ and ρ_{GR} begin with infinity at the beginning of evolution $t = 0$, and decreasing to reach zero at the end of evolution $t \to \infty$ (Fig. 1)

The pressure p and p_{GR} begin with infinity at $t = 0$, decreasing toward a constant value as $t = t_0$ (constant value) and reaches to zero at the end of evolution (Fig. 2). In the general theory of relativity, the entropy $\mathbf{S}_{GR}$ has a small change about zero as compared to its value in the self-creation theory. That means, approximately we deal with adiabatic process ($\frac{d\mathbf{S}}{dt} \simeq 0$). But, in the self-creation theory, the entropy $\mathbf{S}$ is an increasing function with the time t. The behavior of the entropy in the self-creation theory is consistent with the

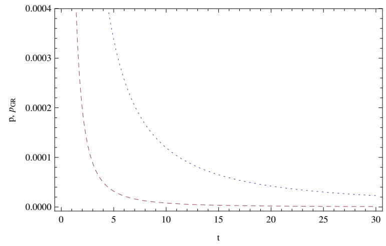

Figure 2: The pressure p in the self-creation theory (dot line) and p_{GR} (dashed line) vs time t.

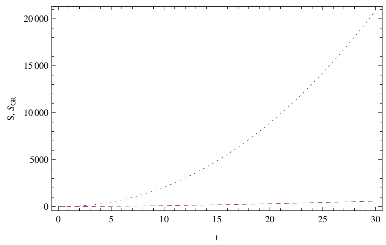

Figure 3: The entropy in the self-creation theory **S** (dot line) and the entropy in the general relativity theory $\mathbf{S}_{GR}$ vs time t.

second law of thermodynamics. So, introducing the scalar field ϕ in the trace of the energy momentum tensor makes the change in the entropy obvious comparing with its value in the general theory of relativity (Fig. 3). In the general relativity theory, the absolute value of $\mathbf{G}_{GR}$ has a small change about zero. In the self-creation theory, the absolute value of **G** is an increasing function with t due to the existing of the scalar field ϕ (Fig. 4). In the general relativity theory, the absolute values of $\mathbf{F}_{GR}$ and $\mathbf{H}_{GR}$ have a small changes about zero. In the self-creation theory, $\mathbf{F}_{GR}$ and $\mathbf{H}_{GR}$ are an increasing functions with t due to the existing of the scalar field ϕ (Fig. 5 and Fig. 6). It is obvious from figure 1 and figure 2 that the scalar field ϕ affected on the behaviors of the pressure and the density in the self-creation theory and makes them decreasing slower than their values in the general relativity theory. From figure 3 - figure 6, we see that the entropy, the Enthalby, the Helmoheltz energy

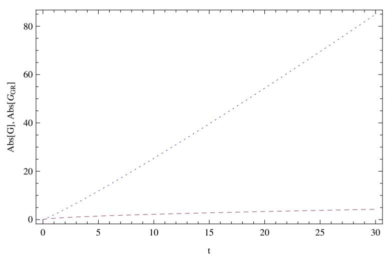

Figure 4: The absolute value of the Gibbs energy function in the self-creation theory **G** (dot line) and the absolute value of Gibbs energy function in the general relativity theory $\mathbf{G}_{GR}$ vs the time t.

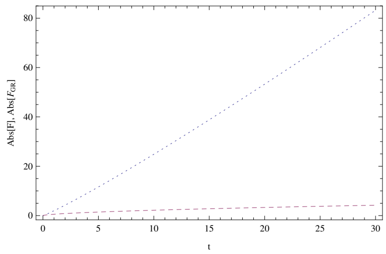

Figure 5: The absolute value of the Helmoheltz energy function in the self-creation theory **F** (dot line) and the absolute value of the Helmoheltz energy function in the general relativity theory $\mathbf{F}_{GR}$ vs the time t.

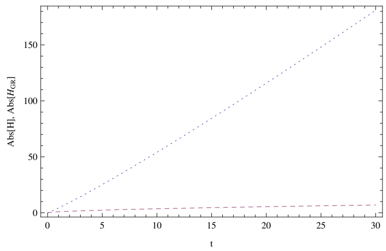

Figure 6: The absolute value of the enthalpy in the self-creation theory **H** and the absolute value of the enthalpy in the general relativity theory $\mathbf{H}_{GR}$ vs the time t.

function and the Gibbs energy function in the self-creation theory are increasing functions with t which represent stages begin after adiabatic process. In the general relativity theory, we deal also with the entropy, the Enthalby, the Helmoheltz energy function and the Gibbs energy function as an increasing functions of t, but, comparing with the self-creation theory, their values are too small that can be neglected.

6 Conclusions

In this paper, we had studied the effect of the bulk viscous term ξ on the entropy of the universe in the framework of the self-creation theory and in the general relativity theory. In the self-creation theory, we found that the scalar field ϕ and the bulk viscous term ξ have effects on the entropy of the universe. The entropy, the Helmoheltz and Gibbs energy are increasing functions with time (stages begin after adiabatic process). In the general theory of relativity we deal approximately with adiabatic process comparing with the results obtained in the self-creation theory. Introducing the scalar field ϕ in the trace of the energy momentum tensor leads to that the decreasing of the pressure p and the density ρ is slower than their behavior in the general relativity theory. Also, the entropy, the Helmohetz, Gibbs energy are increasing functions with time which agree with the second law of thermodynamics unlike the general relativity theory in which we deal with small changes on their values about zero. The kinematic quantities of the universe begin with infinity at the beginning of evolution, decreasing toward a finite quantities at a specific value of $t = t_0$ and have zero values at the end of evolution. The volume element V begins with zero value at $t = 0$ and increases to reach infinity at the end of evolution $t \to \infty$.

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