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Calculating special relativistic viscous stress and heat flux tensors.

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Abstract. The study of the thermodynamics of relativistic fluid is important in astrophysics and modern physics. In this paper, we calculate two important parameters of the special relativistic fluids which are the viscous stress and heat flux tensors in the Cartesian coordinate system. We calculated the explicit relations of viscous stress tensor with velocity, spatial and time derivative of velocity. Also, the relation of heat flux tensor is derived with velocity, temperature and spatial and time derivatives of them. The components of viscous stress and heat flux tensors in the other coordinates can be derived by the transformation matrix.

We use the relativistic method for deriving the non-relativistic viscous stress and heat flux tensors in the limit of the relativistic method. So, to improve the accuracy of non-relativistic studies of shear rate, viscous stress tensor, heat flux vector, and heat flux tensor, especially in the fast fluids, the special relativistic corrections can be used.

Keywords: Special relativistic viscosity, Special relativistic heat flux, Special relativistic fluids.

1 Introduction

If we study the fluids around the black holes, neutron stars, etc., we must study the relativistic fluids. In the relativistic fluids, momentum and energy transportation and energy dissipations can be caused by some internal processes. Two effective mechanisms to this transportations are viscous stress and heat conduction.

The first studies of dissipative relativistic fluids were done by [2] and [3]. Also, in the wide range of papers viscosity and heat conduction of relativistic fluid were studied, for example [1] studied viscosity and magnetic field in the disks of neutron stars. The influence of viscous heating was studied in this paper. The non-ideal dynamics of the relativistic fluid has been studied by Muronga [8]. In this paper, the components of shear and bulk tensor and heat flux in some special cases were derived.The components of shear and bulk tensor in the Kerr metric in the equatorial plane were calculated in [6] and [7].

In the relativistic fluids, the energy-momentum tensor is important, because this tensor shows the energy and momentum of fluids. Especially, the energy density is given by T^{tt}. The energy-momentum tensor of non-magnetic fluid includes three types of energy-momentum, which are perfect fluid energy-momentum, viscous stress and heat flux energy-momentum tensors.

The heat flux energy-momentum tensor is caused by heat flux and velocity. In the non-relativistic fluids, the heat flux is created by divergence of temperature. In this paper, the relations of heat flux of relativistic fluids are obtained. So, in the relativistic fluids with the constant temperature, the heat flux is seen.

In most papers of the relativistic fluids, the ordinary scaling $\hbar = c = G = 1$ are used, we don't use these units because we use the relativistic relations to calculate the non-relativistic relations. Therefore, with the relativistic method, the non-relativistic viscous stress, and heat flux tensors can be calculated.

The outline of this paper is as follows: Metric and scaling are in section 2. In section 3, energy momentum tensor are introduced. In section 4, the relations of viscous stress tensor are derived. Relativistic heat flux tensor is obtained in section 5. Non-relativistic viscous stress tensor and heat flux tensor are in section 6. Summary and conclusion are in section 7.

2 Metric and scaling

In the relativistic fluids, if the influences of gravitation is negligible and the space-time is flat. In the astrophysics, the flat space-time is used around the proto stars, young stars, far from the black holes, etc. But in the fluids around the black holes, the flat space-time has occurred if the amounts of the diagonal components of the Kerr metric are equal to 1 and the non-diagonal components are zero. So, in this region the curvature of space-time is ignorable. This condition has happened if we study $r >> \frac{GM}{c^2}$ (M, G and c are the black hole mass, gravitational constant and the speed of light).

All calculations are done in the Cartesian coordinate systems. Minkowski metric (flat space-time) in the Cartesian coordinate systems is given by

$$ds^2 = -c^2 d\tau^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2, \tag{1}$$

where τ is the proper time; also, the variables of this coordinate are ct, x, y and z. $g_{\mu\nu}$ and $g^{\mu\nu}$ are the components of metric and the inverse components of metric which are obtained as

$$g_{tt} = g^{tt} = -1, \qquad g_{xx} = g^{xx} = 1, \qquad g_{yy} = g^{yy} = 1, \qquad g_{zz} = g^{zz} = 1. \tag{2}$$

3 Energy momentum tensor

In the relativistic fluids, the energy-momentum tensor is written as ([9])

$$T^{\mu\nu} = \rho u^\mu u^\nu + p g^{\mu\nu} + t^{\mu\nu} + q^\mu u^\nu + q^\nu u^\mu, \tag{3}$$

where ρ is density, p is the pressure, $t^{\mu\nu}$ is the viscous stress tensor and q^μ is the heat-flux four vector. In this paper, we derived the explicit and useful relation for two important factor to energy-momentum distribution which are viscous stress tensor and heat flux tensor.

4 Viscous stress tensor

In this paper, we want to calculate the viscous stress tensor($t^{\mu\nu}$) which is ([5])

$$t^{\mu\nu} = -2\lambda\sigma^{\mu\nu} - \zeta\Theta h^{\mu\nu} = S^{\mu\nu} + B^{\mu\nu}, \tag{4}$$

where λ is the coefficient of the dynamical viscosity, $\sigma^{\mu\nu}$ is the shear tensor, ζ is the coefficient of the bulk viscosity, $b^{\mu\nu} = \Theta h^{\mu\nu}$ is the bulk tensor, $S^{\mu\nu} = -2\lambda\sigma^{\mu\nu}$ is the relativistic shear viscosity and $B^{\mu\nu} = -\zeta b^{\mu\nu}$ is the relativistic bulk viscosity. In this paper the bulk viscosity are ignored. The relativistic shear tensor is

$$\sigma^{\alpha\beta} = g^{\alpha\mu} g^{\beta\nu} \sigma_{\mu\nu}, \tag{5}$$

where the shear rate, $\sigma_{\mu\nu}$ is ([5])

$$\sigma_{\mu\nu} = \frac{1}{2}(u_{\mu;\nu} + u_{\nu;\mu} + \frac{1}{c^2}(a_\mu u_\nu + a_\nu u_\mu)) - \frac{1}{3}\Theta h_{\mu\nu}. \quad (6)$$

Also, the covariant derivative of u^ν and u_ν, the projection tensor ($h^{\mu\nu}$), the expansion of fluid world line (Θ), four acceleration(a_μ) are obtained as

$$\begin{aligned} u^\nu_{;\gamma} &= u^\nu_{,\gamma} + \Gamma^\nu_{\gamma\lambda}u^\lambda, \qquad u_{\nu;\gamma} = u_{\nu,\gamma} - \Gamma^\lambda_{\gamma\nu}u_\lambda, \\ h^{\mu\nu} &= g^{\mu\nu} + \frac{u^\mu u^\nu}{c^2}, \qquad a_\mu = u_{\mu;\gamma}u^\gamma, \\ \Theta &= u^\gamma_{;\gamma} = u^\gamma_{,\gamma} + \Gamma^\gamma_{\gamma\nu}u^\nu, \end{aligned} \quad (7)$$

where ; is used for covariant derivative. Also, the components of the Christoffel symbols are

$$\Gamma^\alpha_{\mu\nu} = \frac{1}{2}g^{\alpha\rho}(\partial_\mu g_{\nu\rho} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu}). \quad (8)$$

4.1 four velocity

In the Cartesian coordinate system velocity is

$$\mathbf{v} = v_x(t,x,y,z)\hat{i} + v_y(t,x,y,z)\hat{j} + v_z(t,x,y,z)\hat{k}. \quad (9)$$

Also, the four velocity in the Cartesian coordinate system is

$$u^\mu = (u^t, u^x, u^y, u^z). \quad (10)$$

Therefore, the four velocity in the Minkowski metric is shown as

$$\begin{aligned} u^t &= c\frac{dt}{d\tau} = \frac{c}{\sqrt{1-\frac{v^2}{c^2}}} = c\gamma, \\ u^i &= \frac{dt}{d\tau}\frac{dx^i}{dt} = \gamma v^i, \qquad i = 1,2,3, \\ u^\mu &= \gamma(c, v^x, v^y, v^z) = \gamma(c, \mathbf{v}), \end{aligned} \quad (11)$$

$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ is the Lorentz factor. In this paper Greek indices are used for spacetime components (0-3), but the Latin indices are for spatial components which are 1-3.

4.2 Shear rate

In the Cartesian coordinate all components of metric is constant; therefore, all components of Christoffel symbols are zero, so the covariant derivative and the expansion of fluid world line are

$$\Theta = \gamma_{,t} + u^i_{,i}. \quad (12)$$

The components of projection tensor in the Cartesian coordinate system are calculated as $(i, j = 1, 2, 3)$.

$$\begin{aligned} h^{tt} &= -1 + \gamma^2, \qquad h^{ti} = h^{it} = \frac{u^i u^t}{c^2} = \frac{\gamma^2 v^i}{c}, \\ h^{ii} &= 1 + \frac{(u^i)^2}{c^2} = 1 + \frac{(\gamma v^i)^2}{c^2}, \qquad h^{ij} = h^{ji} = \frac{u^i u^j}{c^2} = \frac{\gamma^2 v^i v^j}{c^2}. \end{aligned} \quad (13)$$

In the cartesian coordinate system, the components of four acceleration are calculated as

$$\begin{aligned}
a_t &= u_{t;\gamma}u^\gamma = -c\gamma\gamma_{,i}v^i - \gamma c\gamma_{,t}\\
a_x &= u_{x;\gamma}u^\gamma = \gamma(\gamma\dot{x})_{,i}v^i + \gamma(\gamma\dot{x})_{,t},\\
a_y &= u_{y;\gamma}u^\gamma = \gamma(\gamma\dot{y})_{,i}v^i + \gamma(\gamma\dot{y})_{,t},\\
a_z &= u_{z;\gamma}u^\gamma = \gamma(\gamma\dot{z})_{,i}v^i + \gamma(\gamma\dot{z})_{,t}.
\end{aligned} \tag{14}$$

Also, the components of relativistic shear rate in the Cartesian coordinate system are calculated as

$$\begin{aligned}
\sigma_{tt} &= \frac{2(-1+\gamma^2)}{3}\gamma_{,t} + \gamma^2\gamma_{,i}v^i - \frac{-1+\gamma^2}{3}(\gamma v^i)_{,i}\ ,\\
\sigma_{tx} &= -\frac{1}{2}[c\gamma_{,x} + \frac{(-1+\gamma^2)}{c}(\gamma v^x)_{,t} + \frac{\gamma^2 v^x}{3c}\gamma_{,t} + \frac{\gamma^2 v^x}{c}\gamma_{,i}v^i + \gamma^2(\gamma v^x)_{,i}v^i - \frac{2\gamma^2 v^x}{3c^2}(\gamma v^i)_{,i}]\ ,\\
\sigma_{ty} &= -\frac{1}{2}[c\gamma_{,y} + \frac{(-1+\gamma^2)}{c}(\gamma v^y)_{,t} + \frac{\gamma^2 v^y}{3c}\gamma_{,t} + \frac{\gamma^2 v^y}{c}\gamma_{,i}v^i + \gamma^2(\gamma v^y)_{,i}v^i - \frac{2\gamma^2 v^y}{3c^2}(\gamma v^i)_{,i}]\ ,\\
\sigma_{tz} &= -\frac{1}{2}[c\gamma_{,z} + \frac{(-1+\gamma^2)}{c}(\gamma v^z)_{,t} + \frac{\gamma^2 v^z}{3c}\gamma_{,t} + \frac{\gamma^2 v^z}{c}\gamma_{,i}v^i + \gamma^2(\gamma v^z)_{,i}v^i - \frac{2\gamma^2 v^z}{3c^2}(\gamma v^i)_{,i}]\ ,\\
\sigma_{xx} &= (\gamma v^x)_{,x} + \frac{\gamma^2 v^x}{c^2}((\gamma v^x)_{,i}v^i + (\gamma v^x)_{,t}) - \frac{1}{3}[(1+\frac{(\gamma v^x)^2}{c^2})((\gamma v^i)_{,i} + \gamma_{,t})]\ ,\\
\sigma_{yy} &= (\gamma v^y)_{,y} + \frac{\gamma^2 v_y}{c^2}((\gamma v_y)_{,i}v^i + (\gamma v^y)_{,t}) - \frac{1}{3}[(1+\frac{(\gamma v^y)^2}{c^2})((\gamma v^i)_{,i} + \gamma_{,t})]\ ,\\
\sigma_{zz} &= (\gamma v^z)_{,z} + \frac{\gamma^2 v^z}{c^2}((\gamma v^z)_{,i}v^i + (\gamma v^z)_{,t}) - \frac{1}{3}[(1+\frac{(\gamma v^z)^2}{c^2})((\gamma v^i)_{,i} + \gamma_{,t})]\ ,\\
\sigma_{xy} &= \frac{1}{2}[(\gamma v^x)_{,y} + (\gamma v^y)_{,x} + \frac{\gamma^2}{c^2}((\gamma v^x v^y)_{,i}v^i + (\gamma v^x v^y)_{,t} + \frac{4v^x v^y}{3}\gamma_{,t} - \frac{2v^x v^y}{3}(\gamma v^i)_{,i}]\ ,\\
\sigma_{xz} &= \frac{1}{2}[(\gamma v^x)_{,z} + (\gamma v^z)_{,x} + \frac{\gamma^2}{c^2}((\gamma v^x v^z)_{,i}v^i + (\gamma v^x v^z)_{,t} + \frac{4v^x v^z}{3}\gamma_{,t} - \frac{2v^x v^z}{3}(\gamma v^i)_{,i}]\ ,\\
\sigma_{yz} &= \frac{1}{2}[(\gamma v^y)_{,z} + (\gamma v^z)_{,y} + \frac{\gamma^2}{c^2}((\gamma v^y v^z)_{,i}v^i + (\gamma v^y v^z)_{,t} + \frac{4v^y v^z}{3}\gamma_{,t} - \frac{2v^y v^z}{3}(\gamma v^i)_{,i}].
\end{aligned} \tag{15}$$

Also, the components of shear tensor are derived with equations (5) and (6) as

$$\begin{aligned}
&\sigma^{tt} = g^{tt}g^{tt}\sigma_{tt} = \sigma^{tt}, && \sigma^{xx} = g^{xx}g^{xx}\sigma_{xx} = \sigma_{xx},\\
&\sigma^{yy} = g^{yy}g^{yy}\sigma_{yy} = \sigma_{yy}, && \sigma^{zz} = g^{zz}g^{zz}\sigma_{zz} = \sigma_{zz},\\
&\sigma^{tx} = \sigma^{xt} = g^{tt}g^{xx}\sigma_{tx} = -\sigma_{tx}, && \sigma^{ty} = \sigma^{yt} = g^{tt}g^{yy}\sigma_{ty} = -\sigma_{ty},\\
&\sigma^{tz} = \sigma^{zt} = g^{tt}g^{zz}\sigma_{tz} = -\sigma_{tz}, && \sigma^{xy} = \sigma^{yx} = g^{xx}g^{yy}\sigma_{xy} = \sigma_{xy},\\
&\sigma^{xz} = \sigma^{zx} = g^{xx}g^{zz}\sigma_{xz} = \sigma_{xz}, && \sigma^{yz} = \sigma^{zy} = g^{yy}g^{zz}\sigma_{yz} = \sigma_{yz}.
\end{aligned} \tag{16}$$

4.3 Components of viscous stress tensor

The relations of components of viscous stress tensor in the Cartesian coordinate are derived from equations (4), (15) and (16) as

$$t^{tt} = -2\lambda[\frac{2(-1+\gamma^2)}{3}\gamma_{,t} + \gamma^2\gamma_{,i}v^i - \frac{-1+\gamma^2}{3}(\gamma v^i)_{,i}]$$

$$t^{ti} = [c\gamma_{,i} + \frac{(-1+\gamma^2)}{c}(\gamma v^i)_{,t} + \frac{\gamma^2 v^i}{3c}\gamma_{,t} + \frac{\gamma^2 v^i}{c}\gamma_{,k}v^k + \gamma^2(\gamma v_i)_{,k}v^k - \frac{2\gamma^2 v^i}{3c^2}(\gamma v^k)_{,k}],$$
$$t^{ii} = -2\lambda[(\gamma v^i)_{,i} + \frac{\gamma^2 v^i}{c^2}((\gamma v^i)_{,k}v^k + (\gamma v^i)_{,t}) - \frac{1}{3}[(1+\frac{(\gamma v^i)^2}{c^2})((\gamma v^k)_{,k} + \gamma_{,t})]\,,$$
$$t^{ij} = -\lambda[(\gamma v^i)_{,j} + (\gamma v^j)_{,i} + \frac{\gamma^2}{c^2}((\gamma v^i v^j)_{,k}v^k + (\gamma v^i v^j)_{,t} + \frac{4v^i v^j}{3}\gamma_{,t} - \frac{2v^i v^j}{3}(\gamma v^k)_{,k}]$$
$$. \qquad (17)$$

where $i, j = 1, 2, 3$.

5 Heat flux tensor

In this section, we calculate the heat flux energy momentum tensor, as we see in equation (3) the heat flux energy momentum tensor is given by

$$T^{\mu\nu}_{heat} = q^\mu u^\nu + q^\nu u^\mu. \qquad (18)$$

The relativistic heat flux is shown as ([2])

$$q^\mu = -\kappa h^{\mu\nu}(T_{,\nu} + \frac{Ta_\nu}{c^2}), \qquad (19)$$

where κ is thermal conductivity and T is temperature. The components of the special relativistic heat flux in the Cartesian coordinate system are obtained as

$$\begin{aligned}
q^t &= -\frac{\kappa}{c}[(-1+\gamma^2)T_{,t} + \gamma^2(+v^x T_{,x} + v^y T_{,y} + v^z T_{,z}) + \frac{\gamma^4 T}{2c^2}(v^2_{,i}v^i + v^2_{,t})] \\
&= -\frac{\kappa}{c}[(-1+\gamma^2)T_{,t} + \gamma^2 v^i T_{,i} + \frac{\gamma^4 T}{2c^2}(v^2_{,i}v^i + v^2_{,t})], \\
q^x &= -\kappa[T_{,x} + \frac{\gamma^2 T}{c^2}(v^x_{,i}v^i + v^x_{,t}) + \frac{\gamma^2 v^x}{c^2}(T_{,i}v^i + T_{,t}) + \frac{\gamma^4 v^x T}{2c^2}(v^2_{,i}v^i + v^2_{,t})], \\
q^y &= -\kappa[T_{,y} + \frac{\gamma^2 T}{c^2}(v^y_{,i}v^i + v^y_{,t}) + \frac{\gamma^2 v^y}{c^2}(T_{,i}v^i + T_{,t}) + \frac{\gamma^4 v^y T}{2c^2}(v^2_{,i}v^i + v^2_{,t})], \\
q^z &= -\kappa[T_{,z} + \frac{\gamma^2 T}{c^2}(v^z_{,i}v^i + v^z_{,t}) + \frac{\gamma^2 v^z}{c^2}(T_{,i}v^i + T_{,t}) + \frac{\gamma^4 v^z T}{2c^2}(v^2_{,i}v^i + v^2_{,t})].
\end{aligned} \qquad (20)$$

So, the components of heat flux energy momentum tensor in the Cartesian coordinate system are given by

$$\begin{aligned}
T^{tt}_{heat} &= 2q^t u^t = -2\kappa\gamma[(-1+\gamma^2)T_{,t} + \gamma^2 v^i T_{,i} + \frac{\gamma^4 T}{2c^2}(v^2_{,i}v^i + v^2_{,t})], \\
T^{ti}_{heat} &= q^t u^i + q^i u^t = -\kappa[\gamma c T_{,i} + \frac{(-1+\gamma^2)\gamma v^i}{c}T_{,t} + \frac{\gamma^3 T}{c}(v^i_{,k}v^k + v^i_{,t}) + \frac{\gamma^5 v^i T}{c^3}(v^2_{,k}v^k + v^2_{,t}) \\
&\quad + \frac{\gamma^3 v_i}{c}(T_{,k}v^k + T_{,t})], \\
T^{ii}_{heat} &= 2q^i u^i = -2\kappa\gamma v^i[T_{,i} + \frac{\gamma^3 T v^i}{c^2}(v^i_{,k}v^k + v^i_{,t}) + \frac{\gamma^3 (v^i)^2}{c^2}(T_{,k}v^k + T_{,t}) + \frac{\gamma^5 (v^i)^2 T}{2c^4}(v^2_{,k}v^k + v^2_{,t})], \\
T^{ij}_{heat} &= q^i u^j + q^j u^i = -\kappa[\gamma(T_{,i}v^j + T_{,j}v^i) + \frac{\gamma^2 T}{c^2}((v^i v^j)_{,k}v^k + (v^i v^j)_{,t}) + 2\frac{\gamma^3 v^i v^j}{c}(T_{,k}v^k + T_{,t}) \\
&\quad + \frac{\gamma^5 v^i v^j T}{c^4}(v^2_{,k}v^k + v^2_{,t})].
\end{aligned} \qquad (21)$$

where $i, j = 1, 2, 3$.

6 Non-relativistic limits

In Mihalas & Mihalas [4], the components of shear and bulk tensor and heat flux in the non-relativistic fluids were calculated. In this section, the non-relativistic components of these two important parameters are computed. We use the relativistic methods and the limits of these calculation to deriving non-relativistic viscous stress tensor and heat flux tensor. For non-relativistic limits, we use $v << c$ or $\gamma = 1$, so we have $u^{\mu} = (c, v^x, v^y, v^z) = (c, \mathbf{v})$

6.1 Non-relativistic viscous stress tensor

If we use the non-relativistic limits in equations (17), the components of non-relativistic viscous stress tensor in the Cartesian coordinate system are derived as

$$\begin{aligned} t^{tt} &= t^{ti} = 0, \\ t^{ii} &= -2\lambda \frac{\partial v^i}{\partial x^i} + \frac{2\lambda}{3} \nabla \cdot \mathbf{v}, \\ t^{ij} &= -\lambda [\frac{\partial v^i}{\partial x^j} + \frac{\partial v^j}{\partial x^i}], i, j = 1, 2, 3. \end{aligned} \tag{22}$$

6.2 Non-relativistic heat flux tensor

The non-relativistic limits are used in equations (21) to calculate the components of non-relativistic heat flux energy momentum tensor in the Cartesian coordinate system, so we have

$$\begin{aligned} T^{tt}_{heat} &= -\kappa \nabla T \cdot \mathbf{v}, \\ T^{ti}_{heat} &= -\kappa c \frac{\partial T}{\partial x^i}, \\ T^{ij}_{heat} &= -\kappa [\frac{\partial T}{\partial x^i} v^j + \frac{\partial T}{\partial x^j} v^i], \\ T^{ii}_{heat} &= -2\kappa v^i \frac{\partial T}{\partial x^i}, i, j = 1, 2, 3. \end{aligned} \tag{23}$$

7 Summery and Conclusion

In this paper, components of viscous stress tensor and heat flux tensor are derived in the special relativistic cases. We study the relativistic fluids with zero bulk viscosity. The components of viscous stress tensor are given by the components of velocity and spatial and time derivative of velocity. So, we see that t^{tt} and t^{ti} $(i = 1, 2, 3)$ are time independent. But the components of t^{ii} and t^{ij} $(i, j = 1, 2, 3)$ are time-dependent. In the non-relativistic cases, shear stress tensor has 9 components which are creating with the spatial differential of velocity. In the special relativistic cases, viscous stress tensor has 16 non-zero components which are given by time and spatial variations of velocity. The t^{tt} is time-independent, so if the coefficients of shear viscosity are time independent, the local viscous energy of the special relativistic fluid is constant.

In this paper, the heat flux is calculated by velocity, temperature, and derivatives of them. Also, all components of heat flux are time-dependent, so the explicit time-dependent of temperature and velocity creates the heat flux in the relativistic fluids.

In the non-relativistic fluids, the heat flux is caused by the divergence of temperature, so if the fluid is isothermal, the heat flux is zero. But in the relativistic fluids with the constant temperature, the heat flux is seen. In these fluids, the time and spatial derivatives of velocity create the heat flux. Especially, in the high temperature fluids, this heat flux is more important.

In the special relativistic fluids, if the temperature has explicit time-dependent, the q^t and the heat flux energy-momentum tensor (T^{tt}_{heat}) are time-dependent, so the heat flux energy of fluids is not constant. But in the non-relativistic fluids, the heat flux energy-momentum tensor is time independent.

We use the flat metric without the ordinary scaling to calculate the relativistic and non-relativistic viscous stress tensor and heat flux tensor. So, we see that the non-relativistic expressions are the same as the classical studies, this method will be used for the other variables. Also, the components of special relativistic viscous stress and heat flux tensors can be used to improve the results of non-relativistic fast fluids.

The components of relativistic and non-relativistic viscous stress and heat flux tensors are calculated in the Cartesian coordinate, so, with the transformation matrix these tensors can be derived in the other coordinates.

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