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$\alpha -^4 He$ Elastic Scattering with Halo Effective Field Theory at Low Eenerg

H. Khalili[1] · S. M. Moosavi khansari[2]

[1] Department of Physics, Faculty of Sciences, Arak University, Arak 38156-8-8349, Iran; E-mail: H-Khalili@araku.ac.ir

[2] Department of Physics, Faculty of Science, Ayatollah Boroujerdi University, Boroujerd, Iran; Email: M.Moosavikhansari@gmail.com

Abstract. We studied the alpha-alpha particle scattering ($\alpha\alpha$) with halo Effective Field Theory (hEFT) as well as halolike systems. In this paper, we considered $\alpha - \alpha$ elastic scattering in s-wave state at the very low energy. Because of the unnaturally large scattering length of $a_{\alpha\alpha}$, it is considered as a halo system. We defined the lagrangian of this system and then calculated the T-matrix for the scattering amplitude at leading order by using of the Feynman diagrams. For the $\alpha - \alpha$ elastic scattering at energy about 0.29 MeV, the total cross section that with this approach has been calculated will be $\sigma_t \simeq 16.0226$ mb.

Keywords: Elastic scattering; T-matrix; Cross section; Halo effective field theory

1 Introduction

The radiative capture at the very low-energy involves the many-body systems for the studies of the stellar structure and the big-bang nucleosynthesis. This fact that these reactions are too dicult to evaluate, makes them even more interesting from the standpoint of a nuclear physics theorist. The accurate cross-section predictions are the important input for the nucleosynthesis forming and in the solar-fusion procedures these reactions occur at the low energies. Consequently, the studies for determination of accurate cross sections for the radiative capture reactions have a mutual partnership between the nuclear experiment and nuclear theory. In this regard, experimental nuclear physics can only measure some hundred keV, while the theoretical nuclear physics can provide good models for the extrapolation of the data. When the captured nucleus is an Alpha, the cross section is exponentially repressed at the very low energies due to the propulsive coulomb barrier. For this reason, low-energy experimental investigations of these types of reactions are extremely difficult and it is required to be developed by numerical codes. Unfortunately, it has not been investigated enough in literature.

Reactions such as the scattering of alpha particles (^4He), the triple-alpha reaction, and alpha capture pretend a significant role in stellar nucleosynthesis. Especially, alpha capture on carbon affects the ratio of carbon to oxygen throughout helium burning, and influences succedent carbon, neon, oxygen, and silicon burning phases. It also considerably influences models of thermonuclear kind Ia supernovae, owing to carbon ignition in accreting carbon-oxygen white-dwarf stars.

In these processes, the accurate calculation of the elastic scattering of alpha particles and alpha-like nuclei-nuclei with equal numbers of protons and neutrons is important for comprehending background and resonant scattering contributions. Nucleons have characteristic momentums that are not large contrasted to the typical QCD scale of 1 GeV. At these low momentums, QCD can easily be depicted by a hadronic theory including all possible interactions steadfast with the QCD symmetries.

Effective field theory produces a controlled frame work for promoting the separation of scales in the nucleuses. Effective field theory is established to give a perfect description for interacting particles in terms of the quantum fields which can be stimulated below a characteristic energy scale Λ [23]. One more important idea of effective field theory is to identify the observables that are independent from any short-distance regulators. Such regulators have to be introduced to distribute with the singularities that a low-momentum expansion characteristically introduces.

One well-known example is that, the three-boson system (where unsuspecting dimensional analysis implies the threebody interaction) should enter at N^2LO. Another problem that can occur in few-body systems is the semblance of artificial bound states in the two-body subsystem at cutoffs larger than the breakdown scale.

For example, this occurs in the deuteron system when a chiral potential with a large momentum-space cutoff is occupied. The systems that interact resonantly in a relative P-wave are measured in halo effective field theory

The low-lying P-wave two-body resonance can fix the P-wave scattering length and effective range such that the two-body propagator reproduces this resonance. In any event, the denominator of such a propagator is then an order-three polynomial, which implys that additional two-body poles are present in the theory.

The halo effective field theory analysis of the Helium-6 system is one example for artificial bound states that become an instantaneous problem.

The neutron-α interaction is resonant in the P-wave and the resulting two-body T-matrix has three poles in the complex plane with one of them being unphysical. Halo nuclei have become another field for the implementation of effective field theories. These are systems of firmly bound cores with faintly bound valence nucleons that can be found close to the neutron and proton driplines.

The halo effective field theory uses the ratio of the valence-nucleon separation energy and the binding of the core as the expansion parameter for the low-energy effective field theory expansion. The core and valence nucleons are the effective degrees of freedom used within this approximation and the intricacy of the problem is meaningfully reduced.

An advantage of this approximation is that the uncertainties of the model can be methodically reduced by including more terms in the low-energy expansion.

Degrees of freedom of higher energies are described by an expansion of the Lagrangian in terms of local operators of growing dimensions. For the nucleon system at energies under the pion mass m_π the effective theory will thus include only the nucleon field and derivatives there of [23]. It is judgmental to formulate a power counting that justifies a orderly truncation of the Lagrangian leading to observables with the requested severity.

Effective field theory is one of the most important tools for understanding of the light nuclei formation in Big-Bang Nucleosynthesis (BBN). At very low energies ($\frac{Q}{\Lambda_\pi}$), where pions can be removed, nuclei structure can be described by pionless effective field theory, which nucleons are degrees of freedom and be inserted as well as external current [2].

Nucleuses present a non-trivial challenge because one wants such a perturbative expansion in addition to the non-perturbative treatment of particular leading operators, which is needed by the existence of shallow bound states. By now, principally few-body systems have been investigated within effective field theory. The effective field theories are established to provide a perfect picture of interaction between particles in designates of only the quantum fields theory which can be aroused under a distinctive energy scale Λ [23]. Degrees of freedom of higher energies are depicted by an expansion of the Lagrangian in designates of local operators of growing dimensions [23].

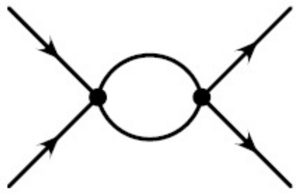

Figure 1: One loop correction to the scattering amplitude of the nuclear interaction. The solid black lines depict the alpha particles and the dark circles depict the nuclear interactions regardless of the Coulomb interaction in the center of mass frame.

Here, we evaluate the observables for a system of nucleons at energies under the first excitation energy of α particle, -7.1294 MeV [3, 4, 5, 6].

In an attempt regarding understanding alpha procedures from first principles, we describe the first order computation of Coulomb effects of alpha-alpha scattering.

The paper is organized as follows: In the section 2, we describe briefly the formalism of halo effective field theory, interactions two bodies system, T- matrix for the coulomb and the coulomb-short interactions. We have calculated the total cross section of the $\alpha - \alpha$ elastic scattering in section 3. Summary and conclusions follow in section 4.

2 Formalism

In the latest years, there has been much promote in nuclear scattering and reactions including light nucleuses. However, the calculative scaling for most numerical methods raises significantly when the projectile nucleus has more than a few nucleons.

Hence, it abides a challenge to address important alpha procedures relevant for stellar astrophysics such as ${}^4He +^4 He$ scattering and ${}^{12}C +^4 He$ scattering and radiative capture, as well as carbon and oxygen burning in enormous star evolution and in thermonuclear supernovae. The relative coordinates for a two body system can wrote using bare Green function that is as below

$$\widehat{G}_0^{(+)}(E) = \frac{1}{E - \widehat{H}_0 + i\varepsilon}. \tag{1}$$

where $M/2$ is the reduced mass of the system and M is the mass of α particle also $\widehat{H}_0 = \widehat{\mathbf{p}}^2/M$ and $E = p^2/M$ is the kinetic energy of system in the CM frame [11, 12, 13]. This bare Green function is expressed in the momentum space using completeness relation as

$$\widehat{G}_0^{(+)}(E) = M \int \frac{d^3q}{(2\pi)^3} \frac{\mid \mathbf{q}\rangle\langle\mathbf{q} \mid}{\mathbf{p}^2 - \mathbf{q}^2 + i\varepsilon}. \tag{2}$$

The propagator of one separation distance, $\mathbf{r}$ to the other separation distance, $\mathbf{r}'$ is

$$\langle \mathbf{r}' \mid \widehat{G}_0 \mid \mathbf{r}\rangle = G_0(E; \mathbf{r}', \mathbf{r}). \tag{3}$$

Corresponding to the FIG. 1 both $\mathbf{r}$ and $\mathbf{r}'$ are equal to zero for a loop diagram and we can write

$$\widehat{G}_0(E; 0, 0) = \int \frac{d^3q}{(2\pi)^3} \frac{1}{E - \mathbf{q}^2/M + i\varepsilon}. \tag{4}$$

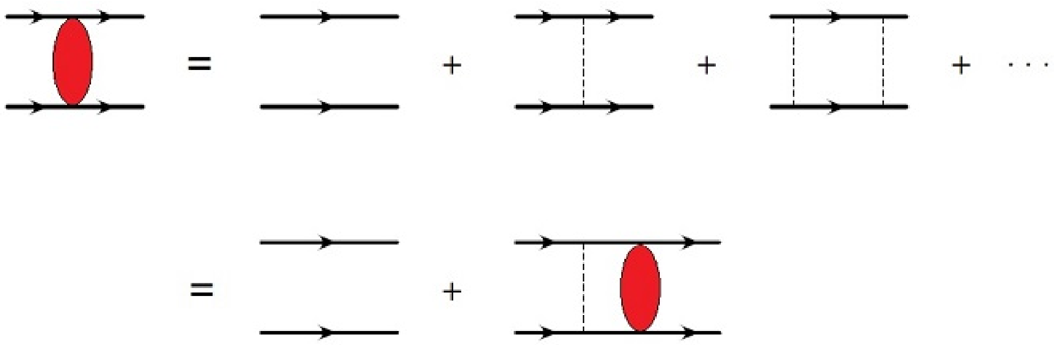

Figure 2: The integral equation of the Feynman diagrams of this interaction that demonstrate Coulomb propagator as an infinite sum. The solid black lines depict the alpha particles and the dashed lines demonstrate the photon exchange between the two alpha particles and the shaded bubble demonstrate the Coulomb interaction between the two alpha particles.

But, the coulomb repulsive force in this problem as $V_C = z_1 z_2 e^2/4\pi r$;therefore, we define the full Green function for the Coulomb interaction as

$$\widehat{G}_C^{(\pm)}(E) = \frac{1}{E - \widehat{H}_0 - \widehat{V}_C \pm i\varepsilon}. \tag{5}$$

corresponding to the FIG. 2, the integral equation of photons exchange using Feynman diagrams is as

$$\widehat{G}_C^{(\pm)} = \widehat{G}_0^{(\pm)} + \widehat{G}_0^{(\pm)} \widehat{V}_C \widehat{G}_C^{(\pm)}. \tag{6}$$

Now, the Schrodinger equation is $(\widehat{H} - E) \mid \psi\rangle = 0$ where $\widehat{H} = \widehat{H}_0 + \widehat{V}_C$ is the full Hamiltonian and $\mid \psi_{\mathbf{p}}^{(-)}\rangle$ and $\mid \psi_{\mathbf{p}}^{(+)}\rangle$ are the incoming and outgoing relative wave functions of system, respectively. These are functions using the eigenstates of the free Hamiltonian are defined as

$$\mid \psi_{\mathbf{p}}^{(\pm)}\rangle = [1 + \widehat{G}_C^{(\pm)} \widehat{V}_C] \mid \mathbf{p}\rangle \tag{7}$$

The normalization relation in the momentum space is

$$\langle \psi_{\mathbf{p}}^{(\pm)} \mid \psi_{\mathbf{p}'}^{(\pm)}\rangle = (2\pi)^3 \delta(\mathbf{p} - \mathbf{p}') \tag{8}$$

After solving the Schrodinger equation, we have these relations for incoming and outgoing wave functions respectively as below

$$\psi_{\mathbf{p}}^{(-)}(\mathbf{r}) = e^{-\pi\eta/2}\Gamma(1 - i\eta) M(i\eta, 1; -ipr - i\mathbf{p}.\mathbf{r}) e^{i\mathbf{p}.\mathbf{r}} \tag{9}$$

$$\psi_{\mathbf{p}}^{(+)}(\mathbf{r}) = e^{-\pi\eta/2}\Gamma(1 + i\eta) M(-i\eta, 1; ipr - i\mathbf{p}.\mathbf{r}) e^{i\mathbf{p}.\mathbf{r}} \tag{10}$$

where $\eta = \alpha M/2p$ is the Sommerfield parameter and $M(a, b; z)$ is the Kummer function. Using Sommerfeld parameter only as a consequence of this relation, we can calculate the probability of being in the zero separation distance, $\mathbf{r} = 0$ or Sommerfield factor, as

$$C_\eta^2 = \mid \psi_{\mathbf{p}}^{(\pm)}(0) \mid^2 = e^{-\pi\eta}\Gamma(1 + i\eta)\Gamma(1 - i\eta) = \frac{2\pi\eta}{e^{2\pi\eta} - 1} \tag{11}$$

Now we go back to the Coulomb Green function and the relation $\langle \mathbf{r}' \mid \widehat{G}_C^{(+)} \mid \mathbf{r}\rangle$ redefine as below

$$\langle \mathbf{r}' \mid \widehat{G}_C^{(+)} \mid \mathbf{r}\rangle = M \int \frac{d^3q}{(2\pi)^3} \frac{\psi_{\mathbf{q}}^{(+)}(\mathbf{r}')\psi_{\mathbf{q}}^{(+)*}(\mathbf{r})}{\mathbf{p}^2 - \mathbf{q}^2 + i\varepsilon} \tag{12}$$

the strong force insert in the problem; thus, we have a new hamiltonian as

$$\widehat{H} = \widehat{H}_0 + \widehat{V}_C + \widehat{V}_S \tag{13}$$

where $\widehat{V}_S$ is the short range potential (Strong interaction); thus, we have a new Green function (the Coulomb-Strong Green function) as

$$\widehat{G}_{SC}^{(\pm)} = \frac{1}{E - \widehat{H} \pm i\varepsilon} \tag{14}$$

Also, the new incoming and outgoing wave functions are [14, 15, 16, 17]

$$\mid \Psi_{\mathbf{p}}^{(\pm)}\rangle = [1 + \widehat{G}_{SC}^{(\pm)}(\widehat{V}_S + \widehat{V}_C)] \mid \mathbf{p}\rangle \tag{15}$$

Using the mathematical union as $A^{-1} - B^{-1} = B^{-1}(B - A)A^{-1}$ we have

$$\widehat{G}_{SC}^{(\pm)} - \widehat{G}_C^{(\pm)} = \widehat{G}_C^{(\pm)}\widehat{V}_S\widehat{G}_{SC}^{(\pm)} \tag{16}$$

and eventually,

$$\mid \Psi_{\mathbf{p}}^{(\pm)}\rangle = [1 + \sum_{n=1}^{\infty}(\widehat{G}_C^{(\pm)}\widehat{V}_S)^n] \mid \psi_{\mathbf{p}}^{(\pm)}\rangle \tag{17}$$

here, we only consider the $n = 1$ case. But, in the two body system for the relative incoming and outgoing momenta, the S-matrix elements are as

$$S(\mathbf{p}', \mathbf{p}) = \langle \Psi_{\mathbf{p}'}^{(-)} \mid \Psi_{\mathbf{p}}^{(+)}\rangle = (2\pi)^3\delta(\mathbf{p}' - \mathbf{p}) - 2\pi i\delta(E' - E)T(\mathbf{p}', \mathbf{p}) \tag{18}$$

the T-matrix itself is as $T(\mathbf{p}', \mathbf{p}) = T_C(\mathbf{p}', \mathbf{p}) + T_{SC}(\mathbf{p}', \mathbf{p})$ where

$$T_C(\mathbf{p}', \mathbf{p}) = \langle \mathbf{p}' \mid \widehat{V}_C \mid \psi_{\mathbf{p}}^{(+)}\rangle \tag{19}$$

is the Coulomb T matrix and

$$T_{SC}(\mathbf{p}', \mathbf{p}) = \langle \psi_{\mathbf{p}'}^{(-)} \mid \widehat{V}_S \mid \Psi_{\mathbf{p}}^{(+)}\rangle \tag{20}$$

is the Coulomb-Strong T matrix. Eventually, the full coulomb part of T-matrix is

$$T_C(\mathbf{p}', \mathbf{p}) = -\frac{4\pi}{M}\sum_{\ell=0}^{\infty}(2\ell + 1)[\frac{e^{2i\sigma_\ell} - 1}{2ip}]P_\ell(\cos\theta) \tag{21}$$

where P_ℓ is the Legendre function of order ℓ. The Coulomb-Strong part of T matrix is as [18, 19, 20, 21]

$$T_{SC}(\mathbf{p}', \mathbf{p}) = -\frac{4\pi}{M}\sum_{\ell=0}^{\infty}(2\ell + 1)e^{2i\sigma_\ell}[\frac{e^{2i\delta_\ell} - 1}{2ip}]P_\ell(\cos\theta) \tag{22}$$

where θ is the scattering angle in the center of mass frame and the coulomb phaseshift σ_ℓ is equal to

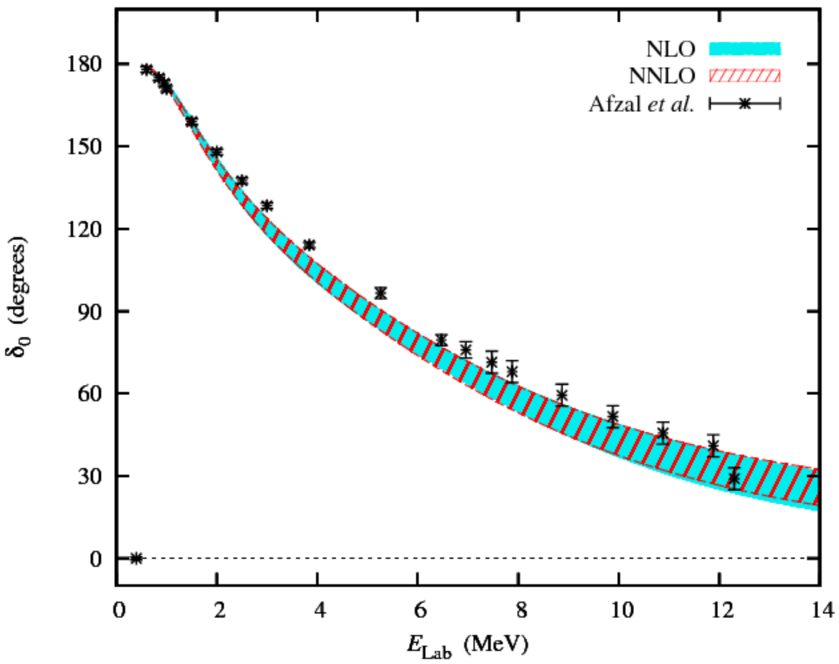

Figure 3: s-wave phase shifts at NLO and NNLO and comparison with experimental data [28, 29, 30, 31].

$$\sigma_\ell = arg\Gamma(1+\ell+i\eta) = \frac{1}{2i}\ln[\frac{\Gamma(\ell+1+i\eta)}{\Gamma(\ell+1-i\eta)}] \tag{23}$$

It should be stressed that the phaseshift δ_ℓ is not the same as one would have in the failure to appear of the Coulomb interaction. But, it can be immediately measured from the experimental differential cross sections (FIG. 3) [28, 29, 30, 31].

In our situation, the strong interaction potential $V_S = C_0\delta(r)$ and will solely affect s-wave amplitude which we symbolize by $T_{SC}(p)$ here C_0 is the first order nuclear coupling constant. The approximation is straightforward and can also be implemented in other non-relativistic field theories [23].

The phenomenological consequential quantities in these systems are the scattering lengths and effective ranges. This approximation should be put on a firmer basis or replaced by a more direct method, possibly in coordinate space [23].

3 Results

We have demonstrated that Coulomb effects in $\alpha - \alpha$ scattering and other hadronic systems at not high energies can be calculated methodically in a non-perturbative method based immediately on the full Coulomb propagator within the effective field theory of Kaplan, Savage and Wise for nucleons. We used the mathematical code and numerical values of physical parameters and below the relations are done numerical solutions of the T-matrix for coulomb and strong interaction and the cross section of alpha-alpha elastic scattering. These

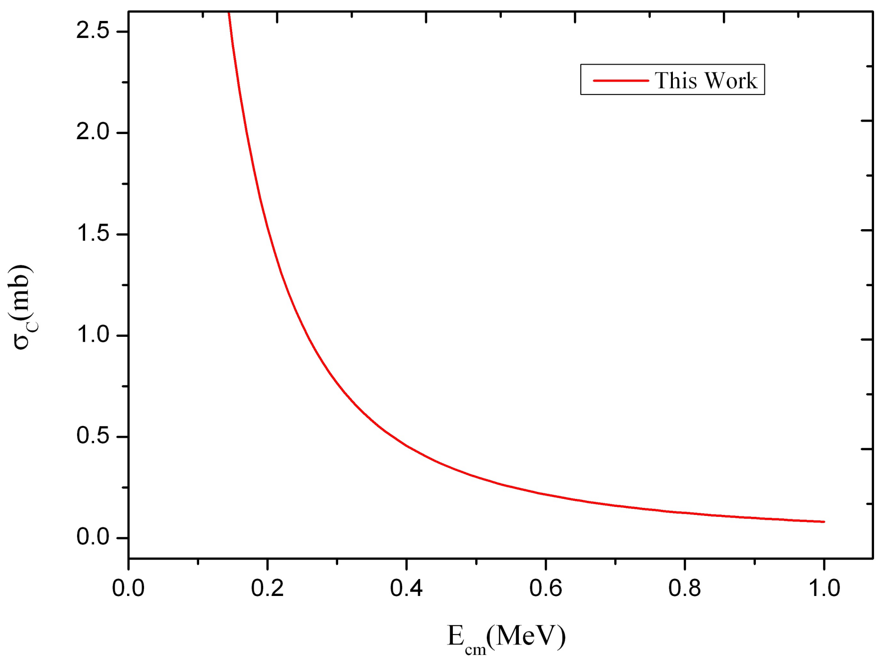

Figure 4: The demonstration of σ_C for this reaction with respect to the energy in the center of mass frame($\ell = 0$).

T-matrixes are calculated by considering the halo Effective Field Theory for this scattering at low energy. The relation for the differential cross section is [23]

$$\frac{d\sigma}{d\Omega} =\mid f(\theta) \mid^2 \tag{24}$$

where $f(\theta)$ is the scattering amplitude that here is equal to s-wave T-matrix. The momentum of the center of mass of $\alpha - \alpha$ system is [22, 23, 24].

$$p = \sqrt{ME}. \tag{25}$$

Using below relation [25, 26, 27]

$$\sigma_{tot} = \int \frac{d\sigma}{d\Omega} d\Omega. \tag{26}$$

and calculation of this integral using previous relation, we can calculate the total cross section of the low energy $\alpha - \alpha$ s-wave elastic scattering. In the s-wave state, T- matrix is

$$T_C^{\ell=0} = -\frac{4\pi}{M}[\frac{e^{2i\sigma_0} - 1}{2ip}]. \tag{27}$$

$$T_{SC}^{\ell=0} = -\frac{4\pi}{M} e^{2i\sigma_0}[\frac{e^{2i\delta_0} - 1}{2ip}]. \tag{28}$$

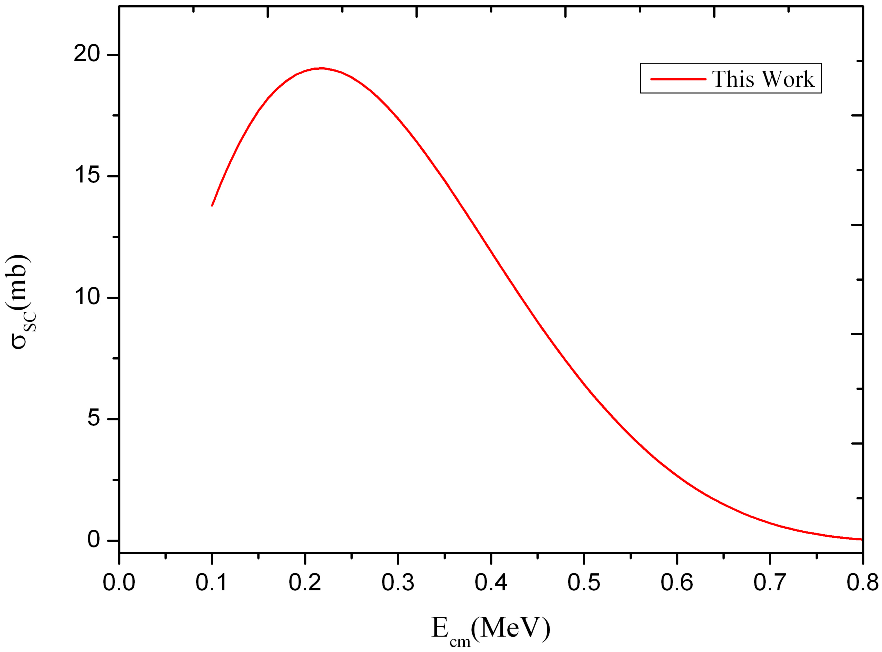

Figure 5: The demonstration of σ_{SC} for this reaction with respect to the energy in the center of mass frame($\ell = 0$).

We demonstrate σ_C in the FIG. 4 for this reaction with respect to the energy in the center of mass frame. In this figure, it has been shown that cross section is decreasing with increase energy. Also, we demonstrate σ_{SC} in the FIG. 5 for this reaction with respect to the energy in the center of mass frame.

In this figure, has been shown T-matrix and cross section of strong interaction are maximum at 0.29 MeV and then is decreasing. Eventually, in the FIG. 6 we demonstrated σ_t using $T^{\ell=0} = T_C^{\ell=0} + T_{SC}^{\ell=0}$ relation for this reaction and because strong interaction is bigger than coulomb interaction then total cross section is similar to strong cross section. As seen in FIG. 6 we obtain using the strong and the Coulomb forces that leading to a narrow resonance at an energy of about 0.29 MeV that in this energy the total cross section is equal to (see TABLE. 1)

$$\sigma_t \simeq 16.0226 \quad mb. \tag{29}$$

This approach plainly implements also to the other processes like $\pi^+ p$ or $\pi^+\pi^+$ elastic scattering at not high energies where repulsive Coulomb interactions are prominent. The only adaptation needed is to replace the mass $M/2$ with twice the reduced mass $m = m_1 m_2/(m_1 + m_2)$ where the scattered particles have not the same masses. Correspondingly , for elastic scattering in channels like $\pi^- p$ or $\pi^-\pi^+$ where the Coulomb force is attractive, it also takes the same type [23].

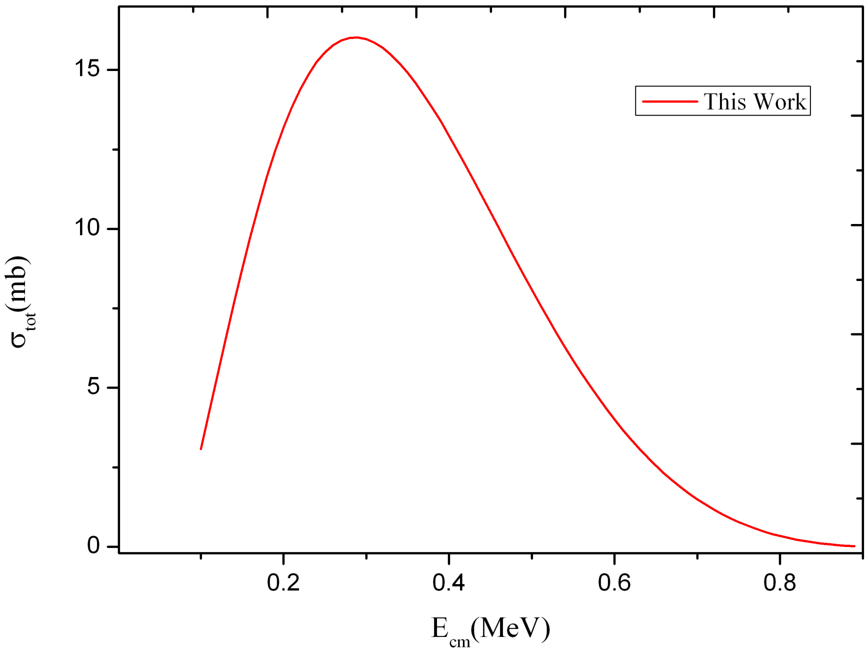

Figure 6: The demonstration of σ_t for this reaction with respect to the energy in the center of mass frame($\ell = 0$).

4 Discussion

Procedures including alpha particles and alpha-like nucleuses comprise a major part of stellar nucleosynthesis and hypothesized mechanisms for thermonuclear supernovae. We investigate ($\alpha\alpha$), the two-alpha-particle system, in halo effective field theory for halolike systems. The approximation is uncomplicated and can also be implemented in other non-relativistic field theories. When the captured nucleus is a Alpha, the cross section is exponentially repressed at the very low energies due to the propulsive coulomb barrier. For this reason, low-energy exprimental investigations of these types of reactions are extremely difficult.

The phenomenological important quantities in these systems are the scattering lengths and effective ranges. This approximation should be put on a firmer basis or replaced by a more direct method, possibly in coordinate space [23]. In this work, we consider the low energy $\alpha - \alpha$ elastic scattering in s-wave state. Because of the unnaturally large scattering length, $a_{\alpha\alpha}$, for a such system, it is considered as a halo system. We do it using halo Effective Field theory. First, we define the lagrangian of this system and then using Feynman rules and diagrams calculate the T-matrix; finally, calculate the cross section for elastic scattering in a such system.

As seen in FIG. 5, we obtain using the strong and the coulomb forces that arrives to a narrow resonance at an energy of about 0.29 MeV that in this energy the total cross section is equal to 16.0226 mb. This problem deserves further investigation.

Table 1: Comparison between results by considering the coulomb and short range interaction and the total cross section.

E_{cm}(MeV)	σ_c(mb)	σ_{sc}(mb)	σ_t(mb)
0.20	1.5349	19.3394	13.1819
0.29	0.8122	17.7986	16.0226 (Max)
0.40	0.4563	11,8955	12.9337
0.60	0.2152	2.6688	4.001
0.80	0.1248	0.05321	0.3370
1.00	0.0813	0.2625	0.0530

References

[1] Peng, R. B. 1999, ApJ, 194, 802

[2] Beane, S. R., Bedaque, P. F., Haxton, W. C., Phillips, D. R., & Savage, M. J., in: M. Shifman (Ed.), At the Frontier of Particle Physics, vol. 1, World Scientific, Singapore, 2001, pp. 133-269

[3] Weinberg, S. 1991, Nucl. Phys. B, 363, 3

[4] van Kolck, U. 1994, Phys. Rev. C, 49, 2932

[5] Kaplan, D. B., Savage, M. J., & Wise, M. B., 1996, Nucl. Phys. B, 478, 629

[6] Kaplan, D. B., Savage, M. J., & Wise, M. B., 1998, Phys. Lett. B, 424, 390

[7] Ajzenberg-Selove, F. & Lauritsen, T. 1974, Nucl. Phys. A, 227, 1

[8] Woo, W., Kwiatkowski, K., Zhou, S. H., & Viola, V. E. 1985, Phys. Rev. C, 32, 706

[9] Elhatisari,S., Lee, D., Rupak, G., & et al. 2015, Nature, 528, 111

[10] Warburton, E. K. 1986, Phys. Rev. C, 33, 303

[11] Gegelia, J. 1998, Phys. Lett. B, 429, 227

[12] Mehen, T., & Stewart, I. W. 1999, Phys. Lett. B, 445, 378

[13] Kaplan, D. B., Savage, M. J., & Wise, M. B. 1999, Phys. Rev. C, 59, 617

[14] Chen, J. -W., Griesshammer, H. W., Savage, M. J. & Springer, R. P. 1998, Nucl. Phys. A, 644, 245

[15] Savage, M. J., Scaldeferri, K. A., & Wise, M. B. 1999, Nucl. Phys. A, 652, 273

[16] Bedaque, P. F., & van Kolck, U. 1998, Phys. Lett. B, 428, 21

[17] Kong, X., & Ravndal, F. 1999, Phys. Lett. B, 450, 320

[18] Kong, X., & Ravndal, F. 1999, Nucl. Phys. A, 656, 421

[19] For clear presentation of the theoretical and experimental situation see Bethe, H. & Morrison, P. 1956, Elementary Nuclear Theory, John Wiley and Sons, New York

[20] Landau, L. D., & Smorodinski, J. 1944, Phys. Acad. Sci. USSR, 8, 219

[21] Bethe, H. A. 1949, Phys. Rev., 76, 38

[22] Jackson, J. D., & Blatt, J. M. 1950, Rev. Mod. Phys., 22, 77

[23] Kong, X., & Ravndal, F. 1999, Phys. Rev. D, 54, 014031

[24] Holstein, B. 1999, Phys. Rev. D, 60, 114030

[25] Chen, J. -W., Rupak, G., & Savage, M. J. 1999, Nucl. Phys. A, 653, 386

[26] Miller, G. A., Nefkens, B. M. K., & Slaus, I. 1990, Phys. Rep., 194, 1

[27] Henley, E. M., in: Isospin in Nuclear Physics, ed. Wilkinson, D. H. 1969, North-Holland Publishing Company, Amsterdam

[28] Heydenburg, N. P., & Temmer, G. M. 1956, Phys. Rev., 104, 123

[29] Nilson, R. W., Jentschke, K., Briggs, G. R., Kerman, R. O., & Snyder, J. N. 1958, Phys. Rev., 109, 850

[30] Tombrello, T. A., & Senhouse, L. S. 1963, Phys. Rev. 129, 2252

[31] Afzal, S. A., Ahmad, A. A. Z., & Ali, S. 1969, Rev. Mod. Phys., 41, 247