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Spinning Magnetic Brane of Quartic Quasi-Topological Gravity in the Presence of Nonlinear Electrodynamics

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Abstract. In this paper, we obtain a new class of $(n+1)$-dimensional magnetic brane solutions of quasi-topological gravity in the presence of exponential and logarithmic nonlinear electrodynamics by a spinning magnetic branes with one or more rotation parameters. For the spinning brane, the brane has a net electric charge, when one or more rotation are non zero, and also this electric charge is proportional to the magnitude of the rotation parameter. However, when all the rotation parameters are zero (static brane), the electric field vanishes and the brane has no net electric charge. In the class of solutions, we have a spacetime with an angular magnetic field. These solutions are horizonless and have no curvature, but there is a conic singularity with a deficit angle. In addition, these two forms of nonlinear electrodynamics theory have the same behaviors for the obtained solutions. Finally, we use the counterterm method and compute conserved quantities of these spacetimes.

Keywords: Quasi-topological gravity; rotating magnetic brane; Thermal stability

1 Introduction

Since the universe is expanding rapidly, generalized theories of gravity and nonlinear electrodynamics theory are very important because they can analyze and solve many problems. Modified theories of gravity introduces new degrees of freedom which interprets the different great distance behavior. Given that cosmic acceleration can arise due to tiny corrections to generalized gravitation and also since models of modified gravities have the ability to early-time inflation, there is the possiblity to find a way to unification of the early and late time inflation [1, 2, 3]. These models also describe cosmological phases and hierarchical problems [4, 5, 6, 7]. In another model of modified gravities, the relationship between matter and geometry is examined [8, 9]. We now consider a new gravitational action that has both some of the simplicity and useful properties of Lovelock actions, and also its field equations in lower dimensions (for example, five dimensions) are not obvious. Hence, quasi-topological gravity contains cubic and quartic terms of Riemann tensor and there are no limitation on dimensions above five dimensions [10, 11, 12, 13, 14].
Nonlinear electrodynamics theory is expressed for important purposes: the most important ones are the destroying infinite self- energy of point charges such as electrons, application in cosmological models, string theory, astrophysical issues, AdS/CFT correspondence and systems in nature that are inherently nonlinear [15, 16, 17, 18, 19, 20]. Born-Infeld theory is

the first form of nonlinear electrodynamics theory [21, 22, 23, 24, 25] and also expoonential [26, 27] and logarithmic [28] Lagrangians as other types of this theory and express as follows

$$\mathcal{L}(F) = \begin{cases} 4\beta^2[\exp(-\frac{F}{4\beta^2}) - 1], & EN \\ \\ -8\beta^2 \ln[1 + \frac{F}{8\beta^2}]. & LN \end{cases} \tag{1}$$

β is the nonlinear parameter with dimension of mass and $F = F_{\mu\nu}F^{\mu\nu}$, where $F_{\mu\nu}$ is the electromagnetic field tensor that is defined as $F_{\mu\nu} = \partial_\mu A^\nu - \partial_\nu A^\mu$ and A_μ is the vector potential. On the other hand, these two Lagrangians reduce to the linear Maxwell Lagrangian as $\beta \to \infty$. Like the Born - Infeld theory, logarithmic nonlinear electrodynamics theory eliminates the infinity of the electric field [29, 30, 31, 32], while the exponential form is unable to do, but this theory causes a weaker singularity than the one in Einstein-Maxwell theory [33].
Modified gravities in the presence of nonlinear electrodynamics theory is the subject of many papers that many studies have been carried out on. Dilaton black holes and black branes in the presence of nonlinear electrodynamics in [29, 30, 31, 32, 33], Topological and AdS black holes in Lovelock-Born-Infeld gravity in [34, 35], Magnetic brane solutions of Lovelock gravity in [36], have been studied.
In recent years, quasi-topological gravity in the presence of nonlinear electrodynamics has been the subject of many papers. Quartic quasi-topological black holes in the presence of nonlinear electromagnetic Born-Infeld theory in [37], Magnetic brane of cubic quasi-topological in the presence of Maxwell and Born-Infeld electromagnetic field in [38], have been studied. The solutions of magnetic branes are horizonless and have a conical geometry with a deficit angle. In this paper, we have studied the $(n+1)$-dimensional magnetic branes solutions of quartic quasi-topological gravity in the presence of exponential and logarithmic nonlinear electrodynamics.
Hence, the structure of this paper is as follows:
In section 2, Rotating metric of a horizonless spacetime and an action including nonlinear electrodynamics and quartic quasi-topological theories are defined. Then, we obtain equations and solutions. In section 3, we interpret the physical properties and behavior of the solutions obtained and in section 4, by using the counterterm method, we obtain conserved quantities. Section 5 is a conclusion of the obtained data from this magnetic brane.

2 Quasi-topological action and rotating metric

We want to abtain the solutions of the quartic quasi-topological gravity in the presence of a nonlinear electromagnetic field. Therefore, instead of $(g_{\rho\rho})^{-1} \propto g_{tt}$ and $g_{\phi\phi} \propto -\rho^2$, we use the metric like $(g_{\rho\rho})^{-1} \propto g_{\phi\phi}$ and $g_{tt} \propto -\rho^2$. Hence, we will work with the following rotating metric

$$\begin{aligned} ds^2 &= (-\frac{\rho^2}{l^2}\Xi^2 + g(\rho)\Xi^2 - g(\rho))dt^2 + 2(\frac{\rho^2}{l}\Xi\sqrt{\Xi^2-1} - lg(\rho)\sqrt{\Xi^2-1}\Xi)dtd\phi + \frac{d\rho^2}{f(\rho)} \\ &\quad +(-\rho^2\Xi^2 + \rho^2 + l^2 g(\rho)\Xi^2)d\phi^2 + \frac{\rho^2}{l^2}dX^2, \end{aligned} \tag{2}$$

where l is a scale factor related to the cosmological constant Λ, $\Xi^2 = 1 + \frac{a^2}{l^2}$ which a is the rotation parameter and $dX^2 = \sum_{i=1}^{n-2}(dx^i)^2$ is the Euclidean metric. ϕ is the angular coordinate that it ranges in $0 \leq \phi < 2\pi$ and it is dimensionless and also ρ is the radial

coordinate.
The action of quartic quasi-topological in $(n+1)$-dimensions in the presence of nonlinear electrodynamics theory is

$$I_{bulk} = \frac{1}{16\pi}\int d^{n+1}x\sqrt{-g}\big\{-2\Lambda + \mathcal{L}_1 + \hat{\lambda}\mathcal{L}_2 + \hat{\mu}\mathcal{L}_3 + \hat{c}\mathcal{L}_4 + \mathcal{L}(F)\big\}, \tag{3}$$

where g is the determinant of the metric 2 and $\Lambda = -n(n-1)/2l^2$. Just the Einstein-Hilbert lagrangian, second order Lovelock or Gauss-Bonnet lagrangian, cubic and quartic quasi-topological Lagrangians are respectively defined as

$$\mathcal{L}_1 = R \tag{4}$$

$$\mathcal{L}_2 = R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2 \tag{5}$$

$$\begin{aligned}\mathcal{L}_3 \;=\;& R_a{}^c{}_b{}^d R_c{}^e{}_d{}^f R_e{}^a{}_f{}^b + \frac{1}{(2n-1)(n-3)}\bigg(\frac{3(3n-5)}{8}R_{abcd}R^{abcd}R \\ & -3(n-1)R_{abcd}R^{abc}{}_eR^{de} + 3(n+1)R_{abcd}R^{ac}R^{bd} + 6(n-1)R_a{}^bR_b{}^cR_c{}^a \\ & -\frac{3(3n-1)}{2}R_a{}^bR_b{}^aR + \frac{3(n+1)}{8}R^3\bigg),\end{aligned} \tag{6}$$

$$\begin{aligned}\mathcal{L}_4 \;=\;& c_1R_{abcd}R^{cdef}R^{hg}{}_{ef}R_{hg}{}^{ab} + c_2R_{abcd}R^{abcd}R_{ef}{}^{ef} + c_3RR_{ab}R^{ac}R_c{}^b + c_4(R_{abcd}R^{abcd})^2 \\ & +c_5R_{ab}R^{ac}R_{cd}R^{db} + c_6RR_{abcd}R^{ac}R^{db} + c_7R_{abcd}R^{ac}R^{be}R^d{}_e + c_8R_{abcd}R^{acef}R^b{}_eR^d{}_f \\ & +c_9R_{abcd}R^{ac}R_{ef}R^{bedf} + c_{10}R^4 + c_{11}R^2R_{abcd}R^{abcd} + c_{12}R^2R_{ab}R^{ab} \\ & +c_{13}R_{abcd}R^{abef}R_{ef}{}^c{}_gR^{dg} + c_{14}R_{abcd}R^{aecf}R_{gehf}R^{gbhd},\end{aligned} \tag{7}$$

where

$$\begin{aligned}
c_1 &= -(n-1)(n^7 - 3n^6 - 29n^5 + 170n^4 - 349n^3 + 348n^2 - 180n + 36)\\
c_2 &= -4(n-3)(2n^6 - 20n^5 + 65n^4 - 81n^3 + 13n^2 + 45n - 18)\\
c_3 &= -64(n-1)(3n^2 - 8n + 3)(n^2 - 3n + 3)\\
c_4 &= -(n^8 - 6n^7 + 12n^6 - 22n^5 + 114n^4 - 345n^3 + 468n^2 - 270n + 54)\\
c_5 &= 16(n-1)(10n^4 - 51n^3 + 93n^2 - 72n + 18)\\
c_6 &= -32(n-1)^2(n-3)^2(3n^2 - 8n + 3)\\
c_7 &= 64(n-2)(n-1)^2(4n^3 - 18n^2 + 27n - 9)\\
c_8 &= -96(n-1)(n-2)(2n^4 - 7n^3 + 4n^2 + 6n - 3)\\
c_9 &= 16(n-1)^3(2n^4 - 26n^3 + 93n^2 - 117n + 36)\\
c_{10} &= n^5 - 31n^4 + 168n^3 - 360n^2 + 330n - 90\\
c_{11} &= 2(6n^6 - 67n^5 + 311n^4 - 742n^3 + 936n^2 - 576n + 126)\\
c_{12} &= 8(7n^5 - 47n^4 + 121n^3 - 141n^2 + 63n - 9)\\
c_{13} &= 16n(n-1)(n-2)(n-3)(3n^2 - 8n + 3)\\
c_{14} &= 8(n-1)(n^7 - 4n^6 - 15n^5 + 122n^4 - 287n^3 + 297n^2 - 126n + 18).
\end{aligned} \tag{8}$$

Also $\hat{\lambda}$, $\hat{\mu}$ and $\hat{c}$ are respectively the parameters of Gauss-Bonnet, cubic and quartic quasi-topological Lagrangians. Actually, these parameters are arbitrary coupling constants, and rescaling is done to simplify the field equations. So, gives [39, 40, 41]

$$\hat{\lambda} = \frac{\lambda L^2}{(n-2)(n-3)}, \tag{9}$$

$$\hat{\mu} = \frac{8\mu(2n-1)l^4}{(n-2)(n-5)(3n^2-9n+4)}, \tag{10}$$

$$\hat{c} = \frac{cl^6}{n(n-1)(n-3)(n-7)(n-2)^2(n^5-15n^4+72n^3-156n^2+150n-42)} \tag{11}$$

For the static magnetic brane, the vector potential has only one component A_ϕ, while for the spinning magnetic brane, add angular momentum to the spacetime. So, in this class, the vector potential includes two components A_ϕ and A_t. Therefore, the vector potential for the rotating solutions as

$$A_\mu = h(\rho)(\frac{\sqrt{\Xi^2-1}}{l}\delta^t_\mu - \Xi\delta^\phi_\mu). \tag{12}$$

Using the vector potential and action 3 and also integrating by parts as

$$\begin{aligned} S &= \frac{n-1}{16\pi l^2} \times \int d^n x \int d\rho N(\rho)\Big\{\Big[\rho^n\Big(1+\Psi+\lambda\Psi^2+\mu\Psi^3+c\Psi^4\Big)\Big]' \\ &+ \begin{cases} \frac{4\beta^2 l^2 \rho^{n-1}}{n-1} \Big[\exp\Big(-\frac{h'^2}{2l^2\beta^2N^2(\rho)}\Big)-1\Big]\Big\}, & EN \\ -\frac{8\beta^2l^2\rho^{n-1}}{n-1} \ln\Big[1+\frac{h'^2}{4\beta^2l^2N^2(\rho)}\Big]\Big\}, & LN \end{cases} \end{aligned} \tag{13}$$

where $g(\rho) = N^2(\rho)f(\rho)$, $\Psi(\rho) = -\frac{l^2}{\rho^2}f(\rho)$ and prime represents the first derivative with respect to ρ. Varying this action with respect to $\Psi(\rho)$ yields

$$\{1+2\lambda\Psi(\rho)+3\mu\Psi^2(\rho)+4c\Psi^3(\rho)\}N'(\rho) = 0. \tag{14}$$

which shows that $N(\rho)$ must be a constant and we choose $N(\rho) = 1$. Varying the action 13 with respect to $N(\rho)$ and $h(\rho)$ and substituting $N(\rho) = 1$(or $f(\rho) = g(\rho)$) yields

$$\Big\{(n-1)\rho^n\Big(1+\Psi+\lambda\Psi^2+\mu\Psi^3+c\Psi^4\Big)\Big\}' + \begin{cases} 4\rho^{n-1}(l^2\beta^2+h'^2)\times \exp\Big(-\frac{h'^2}{2l^2\beta^2}\Big) - 4l^2\beta^2\rho^{n-1} = 0, & EN \\ -8\beta^2l^2\rho^{n-1}\ln(1+\frac{h'^2}{4\beta^2l^2}) +4\rho^{n-1}h'^2(1+\frac{h'^2}{4\beta^2l^2})^{-1} = 0, & LN \end{cases} \tag{15}$$

and

$$\begin{cases} \left(\rho^{n-1}h^{'}\exp\left[-\frac{h^{'2}}{2l^2\beta^2}\right]\right)^{'}=0, & EN \\ \left(\rho^{n-1}h^{'}(1+\frac{h^{'2}}{4\beta^2l^2})^{-1}\right)^{'}=0. & LN \end{cases} \tag{16}$$

If we solve the equation 16, we get to the nonvanishing components of the electromagnetic field tensor

$$F_{\phi\rho}=\Xi h^{'}=\begin{cases} \Xi l\beta\sqrt{-L_W(-\eta)}, & EN \\ \Xi\frac{2ql^{n-2}}{\rho^{n-1}}(1+\sqrt{1-\eta})^{-1}, & LN \end{cases} \tag{17}$$

and

$$F_{t\rho}=-\frac{\sqrt{\Xi^2-1}}{l\Xi}F_{\phi\rho}=-\frac{\sqrt{\Xi^2-1}}{l}h^{'}=\begin{cases} -l\frac{\sqrt{\Xi^2-1}}{l}\beta\sqrt{-L_W(-\eta)}, & EN \\ -\frac{\sqrt{\Xi^2-1}}{l}\frac{2ql^{n-2}}{\rho^{n-1}}(1+\sqrt{1-\eta})^{-1}, & LN \end{cases} \tag{18}$$

where $\eta=\frac{q^2l^{2n-6}}{\beta^2\rho^{2n-2}}$, also q and L_W are respectively the constant of integration and the Lambert function. We get to the relation $F_{\phi\rho}=-\partial_\rho A_\phi$ and $F_{t\rho}=-\partial_\rho A_t$.

$$A_\phi=\begin{cases} -\Xi\frac{n-1}{n-2}l\beta\left(\frac{l^{n-3}q}{\beta}\right)^{\frac{1}{n-1}}\left(-L_W(-\eta)\right)^{\frac{n-2}{2(n-1)}}\Big\{{}_2F_1\Big(\left[\frac{n-2}{2(n-1)}\right],\left[\frac{3n-4}{2(n-1)}\right], \\ -\frac{1}{2(n-1)}L_W(-\eta)\Big)-\frac{n-2}{n-1}\exp(-\frac{1}{2(n-1)}L_W(-\eta))\Big\}, & EN \\ \Xi\frac{ql^{n-2}}{(n-2)\rho^{n-2}}\,{}_3F_2([\frac{n-2}{2(n-1)},\frac{1}{2},1],[\frac{3n-4}{2(n-1)},2],\eta). & LN \end{cases} \tag{19}$$

Using equation 17 in equation 15 leads to the relation

$$c\Psi^4+\mu\Psi^3+\lambda\Psi^2+\Psi+\kappa=0, \tag{20}$$

where κ is

$$\begin{aligned}\kappa &= 1-\frac{M}{(n-1)\rho^n} \\ &+\begin{cases} -\frac{4l^2\beta^2}{n(n-1)}-\frac{4(n-1)\beta ql^{n-1}}{n(n-2)\rho^n}(\frac{l^{n-3}q}{\beta})^{\frac{1}{n-1}}(-L_W(-\eta))^{\frac{n-2}{2(n-1)}}\times \\ {}_2F_1([\frac{n-2}{2(n-1)}],[\frac{3n-4}{2(n-1)}],-\frac{1}{2(n-1)}L_W(-\eta)) \\ +\frac{4\beta ql^{n-1}}{(n-1)\rho^{n-1}}[-L_W(-\eta)]^{\frac{1}{2}}\times[1+\frac{1}{n}(-L_W(-\eta))^{-1}], & EN \\ \frac{8(2n-1)}{n^2(n-1)}\beta^2l^2[1-\sqrt{1-\eta}]-\frac{8(n-1)q^2l^{2n-4}}{n^2(n-2)\rho^{2n-2}}\,{}_2F_1([\frac{n-2}{2(n-1)},\frac{1}{2}],[\frac{3n-4}{2(n-1)}],\eta) \\ -\frac{8}{n(n-1)}l^2\beta^2\ln[\frac{2-2\sqrt{1-\eta}}{\eta}], & LN \end{cases}\end{aligned} \tag{21}$$

where M is the integration constant which is related to the mass of the space-time. In the obtained solution, we use the following relation for the Lambert function

$$L_W(x)e^{L_W(x)}=x. \tag{22}$$

To have real solutions for the equation 20, when

$$\Delta = \frac{H^2}{4} + \frac{P^3}{27} > 0, \tag{23}$$

should be satisfied, where P and H are defined as

$$P = -\frac{\alpha^2}{12} - \gamma \quad , \quad H = -\frac{\alpha^3}{108} + \frac{\alpha\gamma}{3} - \frac{\beta^2}{8}, \tag{24}$$

which α, β and γ are

$$\alpha = \frac{-3\mu^2}{8c^2} + \frac{\lambda}{c} \quad , \quad \beta = \frac{\mu^3}{8c^3} - \frac{\mu\lambda}{2c^2} + \frac{1}{c}$$
$$\gamma = \frac{-3\mu^4}{256c^4} + \frac{\lambda\mu^2}{16c^3} - \frac{\mu}{4c^2} + \frac{\kappa}{c}. \tag{25}$$

Then, we define

$$U = \left(-\frac{H}{2} \pm \sqrt{\Delta} \right)^{\frac{1}{3}}, \tag{26}$$

$$y = \begin{cases} -\frac{5}{6}\alpha + U - \frac{P}{3U}, & U \neq 0, \\ -\frac{5}{6}\alpha + U - \sqrt[3]{H}, & U = 0, \end{cases} \tag{27}$$

$$W = \sqrt{\alpha + 2y}, \tag{28}$$

finally, we get the solution $f(\rho)$ below

$$f(\rho) = \frac{-\rho^2}{l^2}\left(-\frac{\mu}{4c} + \frac{\pm_s W \mp_t \sqrt{-(3\alpha + 2y \pm_s \frac{2\beta}{W})}}{2} \right). \tag{29}$$

In this relation, two $\pm_s$ should have both the same sign, while the sign of $\pm_t$ is independent. it is necessary to mention that in order to have the cubic quasi-topological or Gauss-Bonnet solutions, we should replace $\mu = 0$ or $\lambda = 0$ in the relation 20 and find the solutions not in the above relations because we gives the vague values [39, 42].

3 Studing the general properties of the solutions

To analyze the physical properties of the obtained solutions, it is necessary to examine the singularity and horizons. Therefore, one might think that there is a curvature singularity located at $\rho = 0$. However, as we will see below, the spacetime will never achieve $\rho = 0$. So, as we can see in Figs. 1 and 2, the function $f(\rho)$ is negative for $\rho < r_+$ and positive for $\rho > r_+$, where r_+ is the largest real root of $f(\rho) = 0$. In the range of $\rho < r_+$, $g_{\rho\rho}$ and followed by $g_{\phi\phi}$ are negative. So the results that the signature of the metric changes from $(-++++)$ to $(---++)$ that is not acceptable and the analysis is not true. As a result, the function $f(\rho)$ is limited to the acceptable range $\rho > r_+$. Therefore, it shows that we use an incorrect extension and we do the following suitable transformation as

$$r = \sqrt{\rho^2 - r_+^2} \Rightarrow d\rho^2 = \frac{r^2}{r^2 + r_+^2} dr^2, \tag{30}$$

as a result

$$\begin{aligned} ds^2 &= (-\frac{r^2+r_+^2}{l^2}\Xi^2 + g(r)\Xi^2 - g(r))dt^2 + 2(\frac{r^2+r_+^2}{l}\Xi\sqrt{\Xi^2-1} - lg(r)\sqrt{\Xi^2-1}\Xi)dtd\phi \\ &+ \frac{r^2dr^2}{(r^2+r_+^2)f(r)} + (-(r^2+r_+^2)\Xi^2 + (r^2+r_+^2) + l^2g(r)\Xi^2)d\phi^2 + \frac{r^2+r_+^2}{l^2}dX^2. \end{aligned} \quad (31)$$

In this metric, the range of r is $0 \le r < \infty$. So, the electrodynamic field and the metric functions are real for $r \ge 0$. In addition, the function $f(r)$ is positive in the whole space time and is zero at $r = 0$. The Kretschmann scalar does not diverge in the range $0 \le r < \infty$, but we can show that there is a conical singularity at $r = 0$ with a deficit angle $\delta\phi$. So the functions $F_{\phi r}$ and F_{tr} and also κ becomes

$$F_{\phi\rho} = \begin{cases} \Xi l\beta\sqrt{-L_W(-\eta)}, & EN \\ \Xi\frac{2ql^{n-2}}{(r^2+r_+^2)^{\frac{n-1}{2}}}(1+\sqrt{1-\eta})^{-1}, & LN \end{cases} \quad (32)$$

and

$$F_{t\rho} = \begin{cases} -l\frac{\sqrt{\Xi^2-1}}{l}\beta\sqrt{-L_W(-\eta)}, & EN \\ -\frac{\sqrt{\Xi^2-1}}{l}\frac{2ql^{n-2}}{(r^2+r_+^2)^{\frac{n-1}{2}}}(1+\sqrt{1-\eta})^{-1}, & LN \end{cases} \quad (33)$$

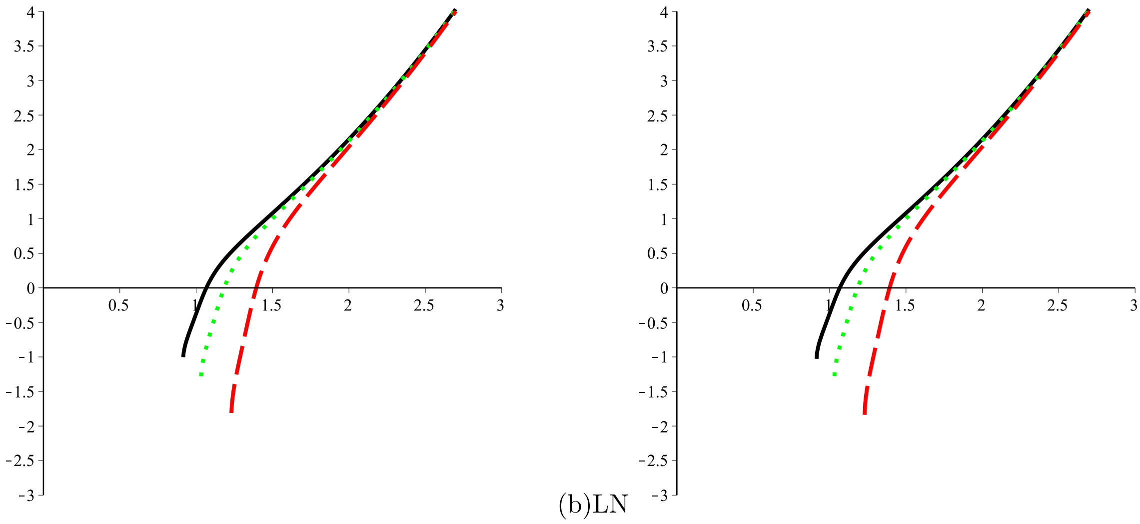
$$\begin{aligned} \kappa &= 1 - \frac{M}{(n-1)(r^2+r_+^2)^{\frac{n}{2}}} \\ &+ \begin{cases} -\frac{4l^2\beta^2}{n(n-1)} - \frac{4(n-1)\beta ql^{n-1}}{n(n-2)(r^2+r_+^2)^{\frac{n}{2}}}(\frac{l^{n-3}q}{\beta})^{\frac{1}{n-1}}(-L_W(-\eta))^{\frac{n-2}{2(n-1)}} \times \\ {}_2F_1([\frac{n-2}{2(n-1)}],[\frac{3n-4}{2(n-1)}],-\frac{1}{2(n-1)}L_W(-\eta)) \\ +\frac{4\beta ql^{n-1}}{(n-1)(r^2+r_+^2)^{\frac{n-1}{2}}}[-L_W(-\eta)]^{\frac{1}{2}} \times [1+\frac{1}{n}(-L_W(-\eta))^{-1}], & EN \\ \frac{8(2n-1)}{n^2(n-1)}\beta^2l^2[1-\sqrt{1-\eta}] - \frac{8(n-1)q^2l^{2n-4}}{n^2(n-2)(r^2+r_+^2)^{n-1}} \\ {}_2F_1([\frac{n-2}{2(n-1)},\frac{1}{2}],[\frac{3n-4}{2(n-1)}],\eta) - \frac{8}{n(n-1)}l^2\beta^2\ln[\frac{2-2\sqrt{1-\eta}}{\eta}], & LN \end{cases} \end{aligned} \quad (34)$$

where $\eta = \frac{q^2l^{2n-6}}{\beta^2(r^2+r_+^2)^{n-1}}$. To determine the singularity of the solutions, we examine the Kretschmann scalar,

$$\mathcal{K} = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} = f''^2 + \frac{2(n-1)}{\rho^2}f'^2 + \frac{2(n-1)(n-2)}{\rho^4}f^2, \quad (35)$$

double prime is the second derivative of the function f with respect to ρ. Since the Kretschmann scalar diverges at this point, obtained solutions have a singularity at $\rho = 0$, but, as we see that, the point $\rho = 0$ is not in the acceptable range of ρ. Therefore, this magnetic brane has no singularity.

In this section, we examine the behavior of $f(\rho)$ in Figs. 1 and 2. So, we have considered $l = 1$ without losing the issue. In Fig. 1, $f(\rho)$ versus ρ for different values of q and for Exponential and Logarithmic nonlinear electrodynamics is plotted. As mentioned, there is a r_+ which $f(\rho) < 0$ for $\rho < r_+$ and unacceptable. In Fig. 1, for constant values of parameters



(a)EN (b)LN

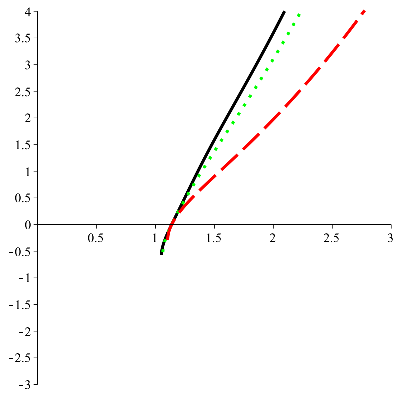
Figure 1: $f(\rho)$ versus ρ for $q=1$(solid), $q=2$(dotted) and $q=4$(dashed) with $M=3$, $\beta=5$, $n=4$, $\lambda=-0.6$, $\mu=1.1$ and $c=-0.6$.

M, β, n, λ, μ, c, when we increase the value of q, the value of r_+ increases and we have the same behavior for both kinds of Exponential and Logarithmic nonlinear electrodynamics. The function f has a constant value for each value of ρ, but in the region near r_+, as q increases it decreases.
In Fig. 2, we examine the behaviors of $f(\rho)$ versus ρ for different values of λ, μ and c for Exponential nonlinear electrodynamics. We see that the value of r_+ is independent of the values of λ, μ and c. Also, we can find this subject by deriving the constant M using $f(r_+)=0$,

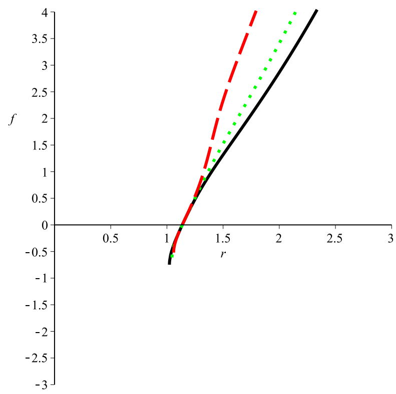
$$M=(n-1)r_+^n+\begin{cases} -\frac{4l^2\beta^2}{n}r_+^n-\frac{4(n-1)^2\beta q l^{n-1}}{n(n-2)}(\frac{l^{n-3}q}{\beta})^{\frac{1}{n-1}}(-L_W(-\eta_+))^{\frac{n-2}{2(n-1)}}\times \\ {}_2F_1([\frac{n-2}{2(n-1)}],[\frac{3n-4}{2(n-1)}],-\frac{1}{2(n-1)}L_W(-\eta_+)) \\ +4\beta q l^{n-1}r_+[-L_W(-\eta_+)]^{\frac{1}{2}}\times[1+\frac{1}{n}(-L_W(-\eta_+))^{-1}], & EN \\ \frac{8(2n-1)}{n^2}\beta^2l^2r_+^n[1-\sqrt{1-\eta}]-\frac{8(n-1)^2q^2l^{2n-4}}{n^2(n-2)r_+^{n-2}}{}_2F_1([\frac{n-2}{2(n-1)},\frac{1}{2}] \\ ,[\frac{3n-4}{2(n-1)}],\eta)-\frac{8}{n}l^2\beta^2r_+^n\ln[\frac{2-2\sqrt{1-\eta}}{\eta}], & LN \end{cases} \tag{36}$$

where $\eta_+=\eta(r=0)=\frac{q^2l^{2n-6}}{\beta^2r_+^{2n-2}}$. According to the above equation, the value of r_+ does not depend on the values of λ, μ and c. The same way, for ρ near r_+, f is independent of parameters mentioned. However, for larger ρ and parameters q, M, n and β, f depends on the values of parameters λ, μ and c.

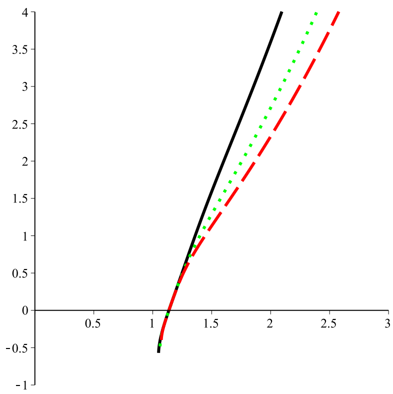
This spacetime has a conical singularity at $r=0$, however, the Kretschmann scalar does not diverge in the range $r=[0,\infty)$. When the radius r goes to zero, the limit of the ratio



[$M = 5$, $q = .1$, $\beta = 10$, $n = 4$, $\mu = -0.6$ and $c = -0.6$, $\lambda = -0.05$(solid), $\lambda = -0.3$(dotted) and $\lambda = -1.8$(dashed)]



[$M = 5$, $q = .1$, $\beta = 10$, $n = 4$, $\lambda = -0.05$ and $c = -0.6$, $\mu = -0.01$(solid), $\mu = -0.5$(dotted) and $\mu = -1$(dashed)]



[$M = 5$, $q = .1$, $\beta = 10$, $n = 4$, $\lambda = -0.05$ and $\mu = -0.6$, $c = -0.6$(solid), $c = -1.8$(dotted) and $c = -3.6$(dashed)]

Figure 2: $f(\rho)$ versus ρ for EN electrodynamics

"circumference/radius" is not 2π, so,

$$\left(\lim_{r\to 0}\left(\frac{1}{r}\sqrt{\frac{g_{\phi\phi}}{g_{rr}}}\right)\right)^{-1}=\left(\lim_{r\to 0}\left(\frac{lf(r)}{r^2}\sqrt{r^2-r_+^2}\right)\right)^{-1}\neq 1, \tag{37}$$

where we have used Taylor expansion for $f(r)$ at $r=0$ (or r_0)

$$f(r)=f(r)\mid_{r_0}+r\frac{df(r)}{dr}\mid_{r_0}+\frac{r^2}{2}\frac{d^2f(r)}{dr^2}\mid_{r_0}+\mathcal{O}(r^3), \tag{38}$$

that $f(r_0)=\frac{df(r)}{dr}\mid_{r_0}=0$. We can remove this conical singularity at $r=0$, if we identify the coordinate ϕ with the period

$$Period_{\phi}=2\pi\left(\lim_{r\to 0}\left(\frac{1}{r}\sqrt{\frac{g_{\phi\phi}}{g_{rr}}}\right)\right)^{-1}=2\pi(1-4\tau), \tag{39}$$

where τ is obtained by using equations 37 and 20.

$$\tau=\frac{1}{4}\left[1-\frac{2l}{r_+^3}\left(\frac{d^2\kappa}{dr^2}\mid_{r_0}\right)^{-1}\right]. \tag{40}$$

Hence, metric 31 describes a spacetime that is locally flat and has a conical singularity at $r=0$ with a deficit angle $\delta\phi=8\pi\tau$. Then, we are going to investigate the behavior of $\delta\phi$. According to the relation 40, the deficit angle of the spacetime is independent of the different coefficients of quasi-topological action and it is only dependent to the parameters q, β and n. Therefore, in figures 3 and 4, we have plotted $\delta\phi$ versus r_+ for different values of q, β and n. In Fig. 3, for different values of q, there is a minimum value for r_+ that $\delta\phi$ is real only for $r_+>r_{+\min}$ and the same way for $r_+>r_{+\max}$, $\delta\phi$ is independent of q and has a constant value for each value of r_+. While in the range of $r_{+\min}<r_+<r_{+\max}$, $\delta\phi$ depends on the value of q and as q increases it increases.
Even though, in Fig. 3, for the same parameters β and n, the values of $r_{+\min}$ in EN form are bigger than the ones in LN form, but $\delta\phi$ has a similar behavior in both of them.
For different values of β, the general behavior of $\delta\phi$ in Fig. 4 is almost similar to the ones in Figs. 3 with a small difference. The value of $\delta\phi$ in the range of $r_{+\min}<r_+<r_{+\max}$ for constant values of parameters q and n, is defendant on the parameter β and decreases as β increases. As well, the value of $r_{+\min}$ decreases as β increases.

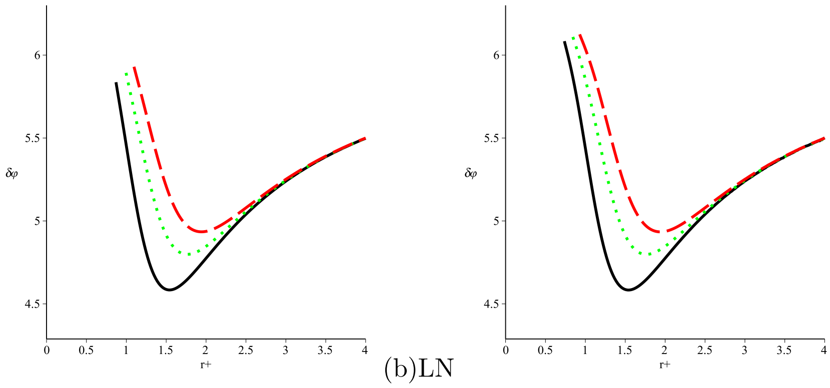
4 Conserved quantities of a magnetic rotating brane

Since the magnetic branes have no horizons, we can not define thermodynamic quantities for them. We want to obtain conserved quantities of this magnetic brane, including mass and electric charge density. According to AdS/CFT correspondence [43], we can extract the action and then the conserved quantities. So, we choose the following counterterm

$$I_1=I_{bulk}+I_b, \tag{41}$$

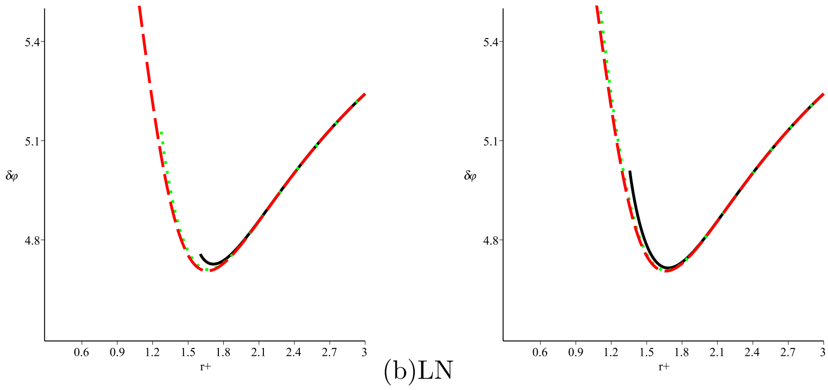
where I_b is a boundary term. I_b creats the variational principle well defined when we choose it as follows

$$I_b=I_b^{(1)}+I_b^{(2)}+I_b^{(3)}+I_b^{(4)} \tag{42}$$



(a)EN (b)LN

Figure 3: $\delta\phi$ versus r_+ for $q = 4$(solid), $q = 6$(dotted) and $q = 8$(dashed) with $\beta = 10$ and $n = 4$.



(a)EN (b)LN

Figure 4: $\delta\phi$ versus r_+ for $\beta = 2$(solid), $\beta = 4$(dotted) and $\beta = 8$(dashed) with $q = 1$ and $n = 4$.

where $I_b^{(1)}$, $I_b^{(2)}$, $I_b^{(3)}$ and $I_b^{(4)}$ are respectively the proper surface terms for Hilbert-Einstein [44], Gauss-Bonnet[45, 46, 47, 48], third order [49] and fourth order quasi-topological [50] gravities which are obtained as follows

$$I_b^{(1)} = \frac{1}{8\pi}\int_{\partial\mathcal{M}} d^n x\sqrt{-\gamma}K, \tag{43}$$

$$I_b^{(2)} = \frac{1}{8\pi}\int_{\partial\mathcal{M}} d^n x\sqrt{-\gamma}\frac{2\lambda l^2}{3(n-2)(n-3)}(3KK_{ac}K^{ac} - 2K_{ac}K^{cd}K_d^a - K^3), \tag{44}$$

$$\begin{aligned} I_b^{(3)} &= \frac{1}{8\pi}\int_{\partial\mathcal{M}} d^n x\sqrt{-\gamma}\Bigg\{\frac{3\mu l^4}{5n(n-2)(n-1)^2(n-5)}(nK^5 - 2K^3K_{ab}K^{ab} \\ &+4(n-1)K_{ab}K^{ab}K_{cd}K_e^dK^{ec} - (5n-6)KK_{ab}[nK^{ab}K^{cd}K_{cd} \\ &-(n-1)K^{ac}K^{bd}K_{cd}])\Bigg\}, \end{aligned} \tag{45}$$

$$\begin{aligned} I_b^{(4)} &= \frac{1}{8\pi}\int_{\partial\mathcal{M}} d^n x\sqrt{-\gamma}\frac{2cl^6}{7n(n-1)(n-2)(n-7)(n^2-3n+3)}\Bigg\{\alpha_1 K^3K^{ab}K_{ac}K_{bd}K^{cd} \\ &+\alpha_2 K^2K^{ab}K_{ab}K^{cd}K_c^eK_{de} + \alpha_3 K^2K^{ab}K_{ac}K_{bd}K6ceK_e^d \\ &+\alpha_4 KK^{ab}K_{ab}K^{cd}K_c^eK_d^fK_{ef} + \alpha_5 KK^{ab}K_a^cK_{bc}K^{de}K_d^fK_{ef} \\ &+\alpha_6 KK^{ab}K_{ac}K_{bd}K^{ce}K^{df}K_{ef} + \alpha_7 K^{ab}K_a^cK_{bc}K^{de}K_{df}K_{eg}K^{fg}\Bigg\}. \end{aligned} \tag{46}$$

Where, K^{ab} is the extrinsic curvature of the boundary with the trace K and $\gamma_{\mu\nu}$ is the induced metric on this boundary $\partial\mathcal{M}$ of the manifold $\mathcal{M}$.
Since the evaluated conserved quantities of the action 41 are divergent [51, 52]; so, to solve this problem and define a finite action for asymptotically AdS solutions with flat boundary, $\hat{R}_{abcd}(\gamma) = 0$, we use counterterm method inspired by AdS/CFT correspondence. However, in this method, we add a new term I_{ct} to the action 41 to have a divergence free stress-energy tensor [53, 54, 55] and we define I_{ct} which has only one term given as

$$I_{ct} = -\frac{1}{8\pi}\int_{\partial\mathcal{M}} d^n x\sqrt{-\gamma}\frac{(n-1)}{l_{eff}}, \tag{47}$$

where l_{eff} is a scale length factor that is related to the coefficients of quasi-topological gravities and l. Also, if these coefficients go to 0, it reduces to l. So, the total action can be written as a linear combination of the gravity term and the above counterterms.
The next step is to compute the conserved charges of the spacetime, we choose a spacelike surface $\mathcal{B}$ in $\partial\mathcal{M}$ with metric σ_{ij} and therefore, write the boundary metric in Arnowitt-Deser-Misner form

$$\gamma^{ab}dx^a dx^b = -N^2dt^2 + \sigma_{ij}(d\phi^i + V^idt)(d\phi^j + V^jdt). \tag{48}$$

where N is the lapse and V^i is the shift functions and also the coordinates ϕ^i are the angular variables parameterizing the hypersurface of constant r around the origin. On the other hand, we can obtain the quasilocal conserved quantities, when there is a Killing vector field ξ on the boundary,

$$\mathcal{Q}(\xi) = \int_{\mathcal{B}} d^{n-1}\phi\sqrt{\sigma}T_{ab}n^a\xi^b, \tag{49}$$

where the stress-energy tensor is

$$T_{ab} = g_{ab}\mathcal{L}(F) - 2\frac{\partial \mathcal{L}(F)}{\partial g^{ab}}, \tag{50}$$

and n^a and σ are respectively the timelike unit normal vector to the boundary $\mathcal{B}$ and determinant of the metric σ_{ij} and also ξ^b is a Killing vector field on the boundary. For our case of horizonless rotating spacetimes, the first Killing vector is $\xi = \partial/\partial t$. So, its associated conserved charge is the mass per unit volume, and gives

$$M_{total} = \int_{\mathcal{B}} d^{n-1}\phi\sqrt{\sigma}T_{ab}n^a\xi^b = \frac{1}{16\pi}M[n\Xi^2 - 1]. \tag{51}$$

After that, we can calculate the angular momentum per unit volume associated with the rotational Killing vectors $\zeta_i = \partial/\partial\varphi_i$ which actually is the second class of conserved quantities, and gives

$$J_i = \int_{\mathcal{B}} d^{n-1}\phi\sqrt{\sigma}T_{ab}n^a\zeta_i^b = \frac{n\Xi M}{16\pi}a_i. \tag{52}$$

For the rotational parameters of the spacetime $a_i = 0(\Xi = 1)$, the angular momentum per unit volume vanishes. Then, to obtain the electric charge of the spacetimes with a longitudinal magnetic field, we must consider the projections of the electromagnetic field tensors on special hypersurfaces with normal $u^0 = \frac{1}{N}$, $u^r = 0$ and $u^i = \frac{N^i}{N}$. Therefore, the electric field is defined as follows,

$$E^\mu = g^{\mu\rho}F_{\rho\nu}u^\nu. \tag{53}$$

then, the electric charge per unit volume by calculating the flux of the electromagnetic field at infinity is obtained [37, 39, 52].

$$\mathcal{Q} = \frac{q}{4\pi}\sqrt{\frac{(n-1)(n-2)}{2}}\Xi. \tag{54}$$

Given the above relation, we see that the electric charge is proportional to the rotation parameter and is zero for the case of static solutions. Compared to static magnetic brane, the above relations for electric charge and conserved quantities indicate that the metrics of static and rotating magnetic brane can be locally mapped but not globally.

5 Concluding Results

In this paper, we obtained magnetic solutions of quartic quasi-topological gravity in the presence of nonlinear electrodynamics exponential and logarithmic forms generated by a rotating magnetic brane. It should be noted that quasi-topological gravity is a higher derivative theory and has no limitations on dimensions. So, if we consider the parameters of quasi-topological gravity zero ($\lambda = \mu = c = 0$), this theory reduces to Einstein's theory and also reduces to linear Maxwell field, if the nonlinearity parameter β goes to infinity.
When the rotation parameters are nanzero, the brane has a net electric charge density which is proportional to the magnitude of the rotation parameter given by $\sqrt{\Xi^2 - 1}$. This solutions have no curvature singularity and no horizons, but have conical singularity with a deficit angle. The function f is defined in the range $r_+ < \rho < \infty$ and does not contain the point

$\rho = 0$. In the continue, we examine and analyze the behaviors of the function f for the various parameters. We see that the two forms of nonlinear electrodynamics theory contains exponential and logarithmic forms have similar effects on the function f.
Eventually, we applied the counterterm method in order to compute the conserved quantities.

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