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Simultaneous sudden change in the exact values of geometric quantum discord and measurement induced nonlocality

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Abstract. Recent studies show that two completely distinct fields as quantum information and string theory realization can be related and share some of the same properties. So, the knowledge on quantum information of some special quantum systems provides new insights on the sting theory. We analytically study geometric discord and measurement induced nonlocality for a Horodeckis' $3 \otimes 3$ bound entangled state to find quantum correlation. We find that there is a simultaneous sudden change point in the geometric discord and the measurement-induced nonlocality in the region of the bound entangled states, which is considerable. Moreover, we investigate the lower bound of geometric discord and the upper bound of measurement-induced nonlocality for this system. This study leads to some new and interesting novel results as well.

Keywords: geometric quantum discord, measurement induced nonlocality, quantum correlations, qutrit-qutrit system

1 Introduction

The relationship between black hole entropy in string theory and the quantum correlation of qubits and qutrits in quantum information theory has been considered a lot. Also, it is shown that the entanglement measure of two qutrits is related to the entropy of the 9-charge black hole of D = 5 supergravity [1, 2].

The theory of hidden variables and locality satisfy Bell inequalities, but the quantum mechanics violates these inequalities [3–7]. The violation of Bell inequalities shows that the quantum mechanics is a nonlocal theory [8]. There are different ways to quantify nonlocality reasonably. Luo and Fu introduced that it can be quantified by measurement induced nonlocality [9].

Every quantum measurement changes the state of a physical system generally. Moreover, if a local measurement is performed on one part of a multipartite system, this measurement may change the state of that part and also entire system. But Lu and Fu defined a special measurement so that it does not change the state of the subsystem which is measured [9]. They quantify nonlocality in the physical system by determining the maximal distance of multipartite system before and after these measurements. This distance is called "measurement induced nonlocality".

Correlations play an outstanding role in multipartite systems. One way to study the quantum correlations is the geometric measure of quantum discord. It defines as minimal distance between a given state and a classical state [10, 12].

In quantum information theory, projective measurement of a system is represented by a complete orthonormal set of states $\{|r\rangle\}$, the measurement operator corresponding to that set becomes $\{|r\rangle\langle r|\}$ and they are known as a projection valued measure (PVM). These orthogonal measurement operators are Hermitian. The number of PVM operators is equal to the dimension of the Hilbert space of the system. Positive operator valued measure (POVM) is a more generalized type of measurement which these measurement operators are not necessarily orthogonal and commutative. It is considerable that the number of POVM operators can be more than the dimension of the Hilbert space. In addition, a POVM is a set of Hermitian positive operators that sum to the identity operator [13].

This paper organized as follows, we study geometric discord in section 2. Then, we introduce measurement induced nonlocality in section 3. In the following, we introduce the physical model of a two-qutrit system and then analytically examine geometric discord and measurement induced nonlocality for that system in section 4. Finally, section 5 is devoted for conclusions.

2 Geometric discord

The geometric discord is a measure to quantify quantum correlations in geometric perspective for every physical system [14, 15]. Lu and Fu introduced geometric discord as

$$D(\rho) = \min_{\Pi^A} |\rho - \Pi^A(\rho)||^2, \tag{1}$$

where $\Pi^A = \{\Pi^A_{r'}\}$ is the POVM operators set with respect to subsystem "A" [10, 11]. We note that for any operator, Hilbert-Schmidt norm is defined as $||O|| = \sqrt{tr(O^\dagger O)}$. The Hilbert space of a bipartite system is $H^A \otimes H^B$ where H^A (H^B) is Hilbert space of subsystem $A(B)$, with $dim(H^A) = m$ and $dim(H^B) = n$. $L(H^A)$ and $L(H^B)$ are the spaces that have all linear operators on H^A and H^B respectively. The orthonormal bases of $L(H^A)$ and $L(H^B)$ are the sets of Hermitian operators as $X_i (i = 1, 2, ..., m^2)$ and $Y_j (j = 1, 2, ..., n^2)$ respectively. It means that $Tr(X_i X'_i = \delta_{ii'})$. The set of $X_i \otimes Y_j$ contains an orthonormal base for $L(H^A \otimes H^B)$. A general bipartite state in $H^A \otimes H^B$ can be written [10, 12]

$$\rho = \frac{1}{mn} 1^A \otimes 1^B + \sum_{i=2}^{m^2} x_i X_i \otimes \frac{1^B}{\sqrt{n}} + \sum_{j=2}^{n^2} \frac{1^A}{\sqrt{m}} \otimes y_j Y_j + \sum_{i=2}^{m^2} \sum_{j=2}^{n^2} t_{ij} X_i \otimes Y_j. \tag{2}$$

Here X_is are defined as

$$X_1 = \frac{1}{\sqrt{m}} 1^A, X_i = \frac{1}{\sqrt{2}} \lambda_{i-1} : i = 2, ..., m^2 \tag{3}$$

and similarly Y_js are

$$Y_1 = \frac{1}{\sqrt{n}} 1^B, Y_j = \frac{1}{\sqrt{2}} \lambda_{j-1} : j = 2, ..., n^2, \tag{4}$$

that $\{\lambda_i\}$ and $\{\lambda_j\}$ are generators of $SU(m)$ and $SU(n)$ respectively. The $\vec{x}$ and $\vec{y}$ vectors and also the T and C matrices are defined as

$$\begin{aligned}
\vec{x} &= [x_i] = \frac{1}{\sqrt{n}}[tr(\rho X_i \otimes 1^B)], \\
\vec{y} &= [y_j] = \frac{1}{\sqrt{m}}[tr(\rho 1^A \otimes Y_j)], \\
T &= [t_{ij}] = [tr(\rho X_i \otimes Y_j)] \text{ and} \\
C &= \begin{bmatrix} \frac{1}{\sqrt{mn}} & \vec{y}^t \\ \vec{x} & T \end{bmatrix}.
\end{aligned} \tag{5}$$

For calculating the geometric quantum discord of Eq. (1), we should apply the POVM operators. In general, POVM operators performed on subsystem "A" can be displayed as $\Pi^A_{r'} = |r'\rangle\langle r'|$, which

$$|r'\rangle = \sum_r U_{r'r}|r\rangle; \; r, r' = 1, 2, 3. \tag{6}$$

Here, $U_{r'r}$ are elements of the $SU(3)$ unitary transformation matrix. They are presented as

$$\begin{aligned}
U_{11} &= cos\theta_1 cos\theta_2 e^{i\phi_1}, \\
U_{12} &= sin\theta_1 e^{i\phi_2}, \\
U_{13} &= cos\theta_1 sin\theta_2 e^{i\phi_3}, \\
U_{21} &= (sin\theta_2 sin\theta_3 - sin\theta_1 cos\theta_2 cos\theta_3 e^{i\phi_4})e^{-i(\phi_3+\phi_5)}, \\
U_{22} &= cos\theta_1 cos\theta_3 e^{i\phi_2}, \\
U_{23} &= -(cos\theta_2 sin\theta_3 + sin\theta_1 sin\theta_2 cos\theta_3 e^{i\phi_4})e^{-i(\phi_1+\phi_5)}, \\
U_{31} &= -(sin\theta_1 sin\theta_3 cos\theta_2 + sin\theta_2 cos\theta_3 e^{-i\phi_4})e^{i(\phi_1-\phi_2+\phi_5)}, \\
U_{32} &= cos\theta_1 sin\theta_3 e^{i\phi_5} \text{ and} \\
U_{33} &= (cos\theta_2 cos\theta_3 e^{-i\phi_4} - sin\theta_1 sin\theta_2 sin\theta_3)e^{i(-\phi_2+\phi_3+\phi_5)},
\end{aligned} \tag{7}$$

where $0 \le \theta_k \le \frac{\pi}{2}$ $(k = 1, 2, 3)$ and $0 \le \phi_l \le 2\pi$ $(l = 1, 2, \cdots, 5)$ are eight independent parameters of $SU(3)$ [16]. Luo and Fu showed that the geometric discord of Eq. (1) can be redefined as follows [10]

$$D(\rho^{AB}) = Tr(CC^t) - \max_O \; Tr(O^t OCC^t). \tag{8}$$

That O is a 3×9 matrix with $O_{r'i} = Tr_A(\Pi^A_{r'} X^A_i)$ as

$$O = \frac{1}{\sqrt{6}} \begin{bmatrix} \sqrt{2} & 0 & 0 & O_{14} & 0 & 0 & O_{17} & 0 & 0 \\ \sqrt{2} & 0 & 0 & O_{24} & 0 & 0 & O_{27} & 0 & 0 \\ \sqrt{2} & 0 & 0 & O_{34} & 0 & 0 & O_{37} & 0 & 0 \end{bmatrix}, \tag{9}$$

where $O_{r'4} = \sqrt{3}\left(|U_{r'1}|^2 - |U_{r'2}|^2\right)$ and $O_{r'7} = 1 - 3\,|U_{r'3}|^2$.

In addition, Lu and Fu have also obtained a lower bound for geometric quantum discord as [10, 17, 18]

$$D(\rho) \ge Tr(CC^t) - \sum_{i=1}^{m} \mu_i = \sum_{i=m+1}^{m^2} \mu_i, \tag{10}$$

where μ_is are eigenvalues of CC^t matrix (in descending order).

3 Measurement induced nonlocality

Lu and Fu defined the measurement induced nonlocality which is the maximum distance between a given bipartite state before and after a local measurements which does not disturb the local system "A" [9]. For a general bipartite state in Eq. (2), MIN is introduced as

$$N(\rho) = \max_{\Lambda^A} ||\rho - \Lambda^A(\rho)||^2 \tag{11}$$

The $\Lambda^A = \{\Lambda^A_{r'}\}$ is a set of local orthogonal projective operators (PVM) with summing to the identity. For a non-degenerate ρ^A, the maximization is not required in Eq.(11), while for a m-degenerate ρ^A any $\Lambda^A_{r'}$ can be a combination of these degenerate states generally. To construct PVM, we could use of Eq. (6) with these extra two conditions:
1- They should be orthogonal, so these POVM operators (Π^A) go to PVM operators (Λ^A).
2- These local measurements should not disturb the subsystem (e.g., $\Lambda^A(\rho^A) = \rho^A$).

An equivalent definition of MIN is

$$N(\rho^{AB}) = Tr(TT^t) - \min_{O'} \ Tr(O'^t O' T T^t). \tag{12}$$

That O' is a 3×8 matrix with $O'_{r' i-1} = Tr_A(\Lambda^A_{r'} X^A_i); i = 2, 3, \cdots, 9$

Moreover, an upper bound for MIN could be defined as [9, 19]

$$N(\rho) \leq Tr(TT^t) - \sum_{i=m^2-m+1}^{m^2-1} \beta_i = \sum_{i=1}^{m^2-m} \beta_i, \tag{13}$$

where β_is are eigenvalues of TT^t matrix (in descending order).

4 GQD and MIN for a qutrit-qutrit physical system

We assume that a qutrit-qutrit state is described by [6, 20]:

$$\rho = \frac{2}{7} |\psi_+\rangle \langle\psi_+| + \frac{\alpha}{7}\sigma_+ + \frac{5-\alpha}{7}\sigma_-. \tag{14}$$

Here, $|\psi_+\rangle = \frac{|11\rangle+|22\rangle+|33\rangle}{\sqrt{3}}$, $\sigma_+ = \frac{|12\rangle\langle 12|+|23\rangle\langle 23|+|31\rangle\langle 31|}{3}$ and $\sigma_- = \frac{|21\rangle\langle 21|+|32\rangle\langle 32|+|13\rangle\langle 13|}{3}$. This equation has simple characterization for $2 \leq \alpha \leq 5$ e.g., where ρ is (i) separable for $2 \leq \alpha \leq 3$; (ii) bound entangled for $3 < \alpha \leq 4$ and (iii) free entangled for $4 < \alpha \leq 5$.

After applying SU(3) generators, one can find all $\vec{x}$ and $\vec{y}$ matrices of Eq. (5) for this physical system, which follow as

$$\vec{x} = \mathbf{0} \ \text{ and } \ \vec{y} = \mathbf{0}. \tag{15}$$

Also, all nonzero of T matrix will be found

$$t_{11} = -t_{22} = t_{44} = -t_{55} = t_{66} = -t_{77} = \frac{2}{21} \tag{16}$$

$$t_{33} = t_{88} = -\frac{1}{42} \ \text{ and } \ t_{38} = -t_{83} = -\frac{\sqrt{3}}{42}(5 - 2\alpha)$$

By using Eqs. (5, 15, 16), the C matrix will be found, so we have

$$\begin{aligned} Tr(CC^t) &= \frac{1}{147}(2Q(\alpha) + 27), \\ Tr(O^tOCC^t) &= \frac{1}{147}\Big(Q(\alpha)\big(2 - F_1(\theta_1, \theta_2, \theta_3, \phi_4)\big) + 19\Big). \end{aligned} \tag{17}$$

With

$$\begin{aligned} Q(\alpha) &= \alpha^2 - 5\alpha^2 + 5, \\ F_1(\theta_1,\theta_2,\theta_3,\phi_4) &= -8\sin(\theta_1)^2\cos(\theta_2)^2\sin(\theta_2)^2\sin(\theta_3)^2\cos(\theta_3)^2\cos(2\phi_4) \\ &- \frac{1}{4}\sin(2\theta_2)^2(\sin(2\theta_3)^2(6\sin(\theta_1)^2+\cos(\theta_1)^4) - 4\sin(\theta_1)^2 - 4\cos(\theta_1)^4) \\ &+ \frac{1}{4}\sin(\theta_1)(1+\sin(\theta_1)^2)\sin(4\theta_2)\sin(4\theta_3)\cos(\phi_4) \\ &- 4(\sin(\theta_1)^2+\cos(\theta_1)^4)(\sin(\theta_3)^2+\cos(\theta_3)^4) + 4. \end{aligned} \tag{18}$$

From Eq. (8) and with application of Eq. (17), the GQD can be worked out as follows

$$D(\rho^{AB}) = \min_{\theta_1,\theta_2,\theta_3,\phi_4}\frac{1}{147}\Bigg(8 + Q(\alpha)F_1(\theta_1,\theta_2,\theta_3,\phi_4)\Bigg) \tag{19}$$

Analytical optimization is lead to exact and considerable relation for the geometric quantum discord as

$$D(\rho^{AB}) = \begin{cases} \frac{1}{2352}(31\alpha^2 - 155\alpha + 283) & 2 \le \alpha < \frac{5+\sqrt{5}}{2} \\ \frac{8}{147} & \frac{5+\sqrt{5}}{2} \le \alpha \le 5 \end{cases} \tag{20}$$

This equation shows the dependence of geometric discord on the value of α parameter. It is worth noting that numerical calculations verify this analytical expression. In addition, form Eq. (10) the lower bound of geometric discord will be

$$D(\rho^{AB}) \ge \begin{cases} \frac{1}{147}(2\alpha^2 - 10\alpha + 18) & 2 \le \alpha < \frac{5+\sqrt{5}}{2} \\ \frac{8}{147} & \frac{5+\sqrt{5}}{2} \le \alpha \le 5 \end{cases} \tag{21}$$

The first step to find the MIN for this system, the O' matrix should be calculated. It is notable that the reduced density is 3-degenerate ($\rho^A = \frac{1}{3}I^A$), so in Eq. (11) optimization process is required. The calculations show that for achieving two above conditions (POVM goes to PVM and $\Lambda^A(\rho^A) = \rho^A$), we should choose $\theta_1 = \frac{\pi}{2}$ or $\theta_2 = \frac{\pi}{2}$ or $\theta_3 = \frac{\pi}{2}$ in Eq. (7). So, the O' matrix will be found and now by using Eqs. (16), we have

$$\begin{aligned} Tr(TT^t) &= \frac{1}{441}(6Q(\alpha) + 32), \\ Tr(O'^tO'TT^t) &= \frac{1}{441}\Bigg(Q(\alpha)\big(6 - \frac{3}{2}F_2(\theta_2,\theta_3,\phi_4)\big) + 8\Bigg). \end{aligned} \tag{22}$$

Where

$$\begin{aligned} F_2(\theta_2,\theta_3,\phi_4) &= -\sin(2\theta_3)^2\sin(2\theta_2)^2\cos(2\phi_4) + (-3\sin(2\theta_3)^2+2)\sin(2\theta_2)^2 \\ &+ \sin(4\theta_2)\sin(4\theta_3)\cos(\phi_4) + 2\sin(2\theta_3)^2, \end{aligned} \tag{23}$$

by replacing Eq. (22) in Eq. (12), the MIN can be calculated as

$$N(\rho^{AB}) = \max_{\theta_2,\theta_3,\phi_4}\frac{1}{294}\Bigg(16 + Q(\alpha)F_2(\theta_2,\theta_3,\phi_4)\Bigg). \tag{24}$$

With a complicated optimization process, the MIN can be described by the following formula

$$N(\rho^{AB}) = \begin{cases} \frac{8}{147} & 2 \leq \alpha < \frac{5+\sqrt{5}}{2} \\ \frac{1}{147}(\alpha^2 - 5\alpha + 13) & \frac{5+\sqrt{5}}{2} \leq \alpha \leq 5 \end{cases} \tag{25}$$

We note that, this analytical expression is in agreement with the result of numerical calculations. Also, upper bound for MIN from Eq. (13) could be obtained as

$$N(\rho^{AB}) \leq \begin{cases} \frac{8}{147} & 2 \leq \alpha < \frac{5+\sqrt{5}}{2} \\ \frac{1}{441}(2\alpha^2 - 10\alpha + 18) & \frac{5+\sqrt{5}}{2} \leq \alpha \leq 5 \end{cases} \tag{26}$$

5 Conclusions

In this paper, we studied the case of qutrit-qutrit system where the density operator of this system depends on α parameter and we novelty demonstrated that without entanglement, there is the MIN and also the geometric discord. This is an obvious fact that one can observe in Fig. (1). In our physical system, the geometric discord and also MIN are never zero.

The lower bound of the geometric discord and upper bound of MIN are demonstrated in Fig. (1) accompanied with their exact analytical values. The behavior of exact geometric discord and lower bound of it are the same, and this behavior could been seen also for MIN.

It is worth noting that MIN has a non-zero constant value in the separable states region and a part of the bound entangled states region. However, in these regions the geometric discord is variable and non-zero, as Fig. (1) shows. In the point of $\alpha = \frac{5+\sqrt{5}}{2}$ (that this point is in the bound entanglement region), there is a sudden change in the geometric discord and the MIN. After this sudden change point, the geometric discord has a non-zero constant value with respect to the α parameter, while the MIN increases with an increase in this parameter.

As we mentioned analytically in previous section, when the geometric discord is a function of the α parameter, the MIN is constant and vice-versa. Physical interpretation of this happening is an open question. Another point that should be considered is the increasing behavior of the MIN in free entangled region.

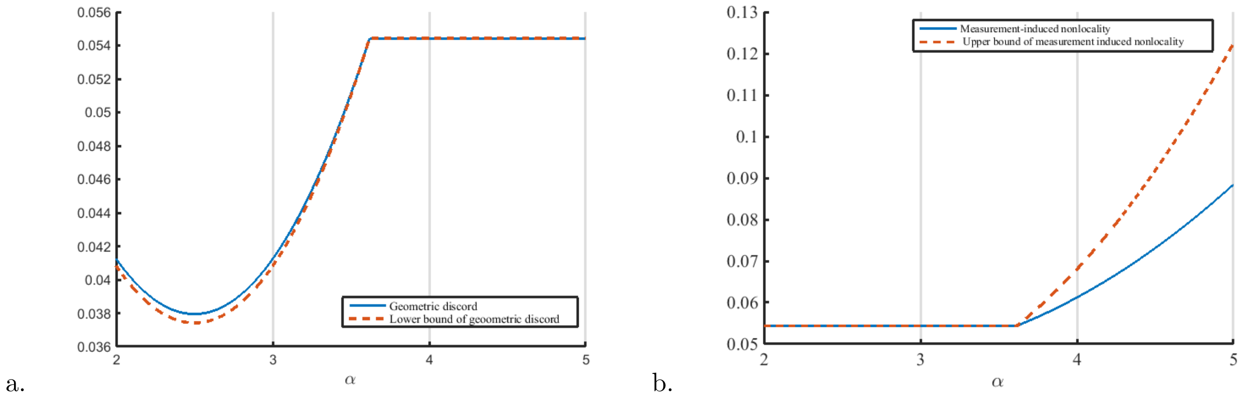

Figure 1: a: Geometric discord and lower bound geometric discord b: MIN and upper bound of MIN

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