

## A Simplified Solution for Advection Dominated Accretion Flows with Outflow

Seyede Tahere Kashfi · Shahram Abbassi

Department of Physics, Ferdowsi University of Mashhad, Mashhad, Iran

**Abstract.** The existence of outflow in the advection dominated accretion flows has been confirmed by both numerical simulations and observations. The outflow models for ADAF have been investigated by several groups with a simple self similar solution. But this solution is inaccurate at the inner regions and can not explain the emitted spectrum of the flow; so, it is necessary to obtain a global solution for ADAFs with outflow. In this paper, we use a simplified global solution to study the structure of ADAF in the presence of outflow. In this method which is proposed by Yuan et al (2008, hereafter YMN08), the radial momentum equation is replaced by a simple algebraic relation between angular velocity and Keplerian angular velocity to avoid the difficulty of the calculation of global solution. We consider the radial dependence for mass accretion rate  $\dot{M} = \dot{M}_{out}(r/r_{out})^s$  where  $s$  is a constant and we do not change the other dynamical equations. We investigate the variation of physical quantity of accretion flow which is caused by outflow. The results that we obtained comply with our expectations from the influence of outflow on the structure of accretion flow.

*Keywords:* accretion; accretion disks; black hole; global solution; outflow

### 1 Introduction

Advection dominated accretion flow (ADAF) is an important model for black hole accretion. In this model which is introduced by Narayan & Yi (1994, Abramowicz et al. 1995, Chen et al. 1995), most of the energy is stored in the accretion gas and advected to the central black hole. The ADAFs have high temperature and thermal stability and can explain the hard X-ray emission from X-ray sources and AGNs. Also, because the advective nature of these flows, they are especially good to explaining low luminous systems.

On the one hand, as Narayan & Yi (1994, 1995a) noted, this type of accretion flow have an interesting feature that is the positive value for Bernoulli parameter. This parameter is the sum of the gravitational potential energy, kinetic energy and the enthalpy of the flow. When this parameter have a positive value, the gas can escape to infinity, and on the another hand, for a given density at a large distance from the black hole (e.g., measured by Chandra on 1'' scales), the density close to the black hole is much less than the ADAF or Bondi prediction (Yuan et al. 2003). Also, numerical simulations (Stone et al. 1999; Hawley & Balbus 2002; Igumenshchev et al. 2003) indicate that a large fraction of accretion material of the flow can not reach the central black hole and it is lost in the form of outflow. For these reasons, the existence of outflows should be considered in ADAF models.

Outflow models for ADAF have been investigated by several group (Xu & Chen 1997; Blandford & Begelman 1999; Beckert 2000; Xue & Wang 2005; Bu et al. 2009), in the frame of a self similar solution. Self similar solution is only valid for the regions away from the boundaries and does not match the boundary conditions of the flow; also, it is too simplified for calculating the emitted spectrum; so, we need to obtain the global solution.

Quataert & Narayan (1999) presented the global solution for ADAF with outflow for the first time. They use the form  $\dot{M} \propto r^{-s}$  to describe the accretion rate. In their investigation, all dynamical equations of the flow do not change, except the continuity equation. They calculated spectral models of ADAFs with outflow and compared their results with observations of several astrophysical systems. Also, Yuan et al. (2003; 2009) adopted the same approach for RIAFs and hot accretion flows.

However, the global solution is a two points boundary value problem and technically, it is difficult to solve. Because the difficulty of calculating the global solution, the effect of existence of outflow on the structure of ADAFs has not been investigated any more.

In this paper, we focus on the structure of ADAFs in the presence of outflow rather than the calculation of the emitted spectrum. By following the simplified global solution which is introduced by YMN08, we replace the radial momentum equation by a simple algebraic relation between the angular velocity and Keplerian angular velocity of the flow. In this way, we avoid the two points boundary value problem and the consequently, the approximate global solution for ADAF with outflow can be obtained.

We present the basic equations in section 2, and numerical results in section 3. Section 4 is devoted to a short summary. We should note that the effect of outflow on the angular momentum and energy equations is ignored here. Since the most important effect of outflow is to modify the density profile, we do not expect this treatment to cause a remarkable change on our results.

## 2 Basic Equations

We consider an advection dominated accretion flow which is axisymmetric and steady state ( $\frac{\partial}{\partial t} = \frac{\partial}{\partial \varphi} = 0$ ). The Paczyński & Witta potential  $\phi = -\frac{GM}{(r-r_g)}$  is adopted to mimic the geometry of a Schwarzschild black hole where  $M$  is the mass of black hole and  $r_g = \frac{2GM}{c^2}$  is the Schwarzschild radius. Also, the hydrostatic equilibrium in the vertical direction is assumed; so, we have a height-integrated set of equations. In this formulation, all physical quantities are functions of only the cylindrical radius.

To take into account the role of outflow, the continuity equation must be modified. So, we consider the dependency of the mass accretion rate on radius as follows (Blandford & Begelman 1999)

$$\dot{M} = -4\pi r \rho H v_r = \dot{M}_{out} \left(\frac{r}{r_{out}}\right)^s \quad (1)$$

where  $\dot{M}_{out}$  is the mass accretion rate at the outer radius and  $s$  is a parameter that indicates the strength of outflow. All the other quantities have their usual meanings.

The other equations that describe the structure of flow are

$$v_r \frac{dv_r}{dr} = (\Omega^2 - \Omega_K^2)r - \frac{1}{\rho} \frac{dp}{dr}, \quad (2)$$

$$v_r(\Omega r^2 - j) = -\alpha r c_s^2, \quad (3)$$

$$\rho v_r \left( \frac{d\epsilon_i}{dr} - \frac{p_i}{\rho^2} \frac{d\rho}{dr} \right) = (1 - \delta)q^+ - q_{ie}, \quad (4)$$

$$\rho v_r \left( \frac{d\epsilon_e}{dr} - \frac{p_e}{\rho^2} \frac{d\rho}{dr} \right) = \delta q^+ + q_{ie} - q^-. \quad (5)$$

In these equations,  $\epsilon_i(\epsilon_e)$  is the internal energy of ions (electrons) per unit mass of gas and  $p_i(p_e)$  is the pressure due to ions (electrons) which is defined as

$$\epsilon_e = \frac{1}{\gamma_e - 1} \frac{kT_e}{\mu_e m_H}, \quad \epsilon_i = \frac{1}{\gamma_i - 1} \frac{kT_i}{\mu_i m_H}, \quad (6)$$

$$p_i = \frac{\rho}{\mu_i} \frac{k}{m_H} T_i, \quad p_e = \frac{\rho}{\mu_e} \frac{k}{m_H} T_e \quad (7)$$

The  $q^+$  is the net turbulent heating rate and the quantity  $\delta$  determines the fraction heating rate that directly heats electrons.  $q_{ie}$  describes the energy transfer rate from ions to electrons by Coulomb collision and  $q^-$  is the radiative cooling rate that includes bremsstrahlung and synchrotron emission and their Comptonization (see Manmoto et al. 1997 for detail).

The set of dynamical equations includes two algebraic equations (1) and (3) and three first order differential equations (2), (4) and (5). To solve these differential equations we need to have the outer boundary conditions for  $v_r$ ,  $T_i$  and  $T_e$ . Also, because the transonic nature of ADAF, the sonic point conditions have to be considered and the most difficult part of this solution is to apply these conditions in the radial momentum equation.

By following YMN08, we use a simple relation between angular velocity and Keplerian angular velocity;

$$\Omega = f\Omega_k \quad (8)$$

where

$$f = \begin{cases} f_0 & r > 3r_g \\ 3f_0 \frac{r-r_g}{2r_g} & r < 3r_g. \end{cases}$$

In this relation,  $f_0$  is an adjustable parameter that YMN08 defined it as  $f_0 = 0.33$  for a wide rang of  $\dot{M}$  and a large value of  $\alpha$ .

With considering this simple relation, the radial velocity become

$$v_r = \frac{-\alpha r c_s^2}{(f\Omega_k r^2 - j)}. \quad (9)$$

By substituting the equations (1) and (9) into the energy equations and expressing the isothermal sound speed  $c_s$  in terms of ions and electrons temperature, we can obtain two first order differential equations for two unknown variables  $T_i$  and  $T_e$ . If we have  $T_i$  and  $T_e$  at the outer boundary, we can integrate these differential equations inward for a given value of  $\alpha$  and  $\dot{M}$ . The other physical quantities such as  $\rho$ ,  $v_r$  and  $c_s$  will be calculated from the relations which they have with temperature. We use the the standard fourth order Runge-Kutta method and solve the differential equations with initial conditions.

### 3 Results

In our calculations, we set  $M = 10M_\odot$  and  $\delta = 0.3$ . We examine our approach for different values of  $s$ ,  $\alpha$  and  $\dot{M}$ . The conditions that we imposed at the outer boundary are the same as in YMN08.

Figure 1 shows the variation of the accretion rate as a function of radius for  $s = 0$ ,  $s = 0.2$ ,  $s = 0.4$  and  $s = 0.6$ . In this figure, the solid line indicates the case without outflow. In the presence of outflow, as radius decreases, the accretion rate declines and for a powerful outflow  $s = 0.6$ , only a few of material can be accreted on the black hole.

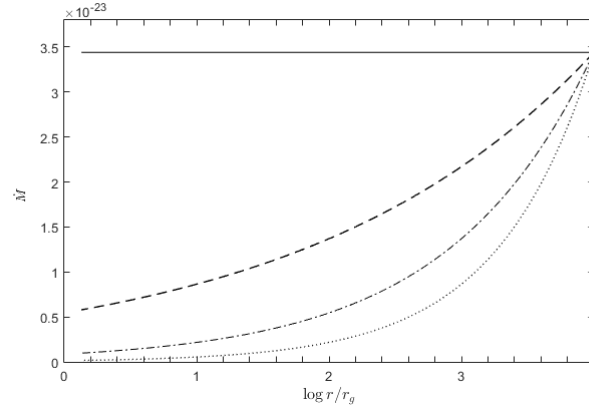


Figure 1: Accretion rate as a function of radius. The solid line, dashed line, dash dotted line and dotted line are related to  $s = 0$ ,  $s = 0.2$ ,  $s = 0.4$  and  $s = 0.6$  respectively.

Figure 2 indicates the radial variation of cooling rate.  $q^-$  includes three radiation mechanisms, bremsstrahlung radiation, synchrotron radiation and their Comptonization. In the outer regions of the flow, the bremsstrahlung emission is dominated while the synchrotron and Comptonization are the main emissions in the hot inner regions [8]. The bremsstrahlung emission is almost insensitive to the existence of outflow. As the figure 2 demonstrates, the cooling rate in the outer radii dose not change with the increasing the value of  $s$  because in these regions, the bremsstrahlung radiation is important and the effects of synchrotron and Comptonization emissions are negligible.

From the studies on the emitted spectrum [12], we know that the synchrotron emission decreases strongly with increasing  $s$  because when the value of  $s$  increases, the density and gas pressure decrease, so it is produced a weaker magnetic field. With increasing  $s$  the Compton power decreases strongly more than the synchrotron. In figure 2, we can see that the total cooling rate significantly reduces with large value for  $s$ .

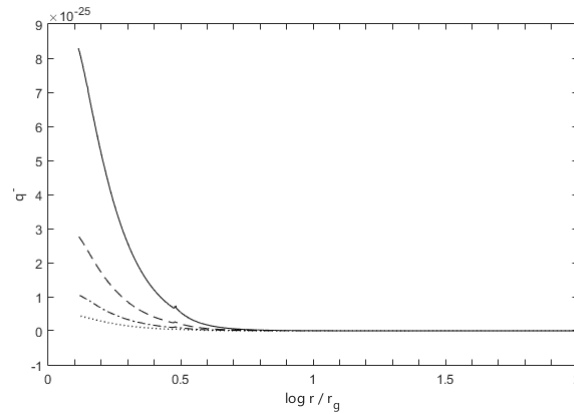


Figure 2

In figures 3 and 4, the profiles of ions and electrons temperature, radial velocity and density for different values of  $\alpha$  and  $\dot{M}$ , are shown. In figure 3,  $\alpha = 0.1$  and  $\dot{M}_{out} =$

$10^{-3}\dot{M}_{Edd}$ , and in figure 4,  $\alpha = 0.3$  and  $\dot{M}_{out} = 10^{-3}\dot{M}_{Edd}$ . In figure 3, the radius of outer boundary is  $r_{out} = 10^4 r_g$  and boundary conditions are  $T_i = 0.2T_{vir}$ ,  $T_e = 0.19T_{vir}$  where  $T_{vir} = 3.6 \times 10^{12}(\frac{r_g}{r})$ .

As we expected, the profile of density indicates an obvious decline in the presence of outflow. When  $s = 0.6$ , the value of density in the inner radii is almost  $10^2$  times less than the case without outflow. Also, the radial velocity decreases with increasing the value of  $s$ . In the profile of radial velocity, the variation of sound speed with radius is illustrated too. By comparing the profiles related to different value of  $s$ , we can realize that as  $s$  increases, the sonic point moves in and comes close to the black hole.

The plot of ions temperature shows that when  $s$  increases, the ions temperature decreases. From equation (4), we know there are two terms that affect the value of  $T_i$ ,  $q^+$  and  $q_{ie}$  and from their definitions, we know  $q^+ \propto \dot{M}$  and  $q_{ie} \propto \dot{M}^2$ . Therefore, when the outflow exists, these two terms decrease and reduce the ions temperature.

In the outer regions of the electrons temperature profile, there is a decline in temperature when  $s$  increases. As we mentioned before, in these radii, the bremsstrahlung radiation dominates and this radiation is unaffected by the outflow. By return to the energy equation for electrons, we can see if  $q^-$  dose not change very much, the variation of two other terms  $q^+$  and  $q_{ie}$  reduces the electrons temperature because they are the heating processes for electrons and with increasing  $s$ , these terms decrease. By contrast, in the inner radii, the cooling rate strongly decreases and this causes an increasing in electrons temperature. So, for the region close to the black hole, we have a larger  $T_e$ .

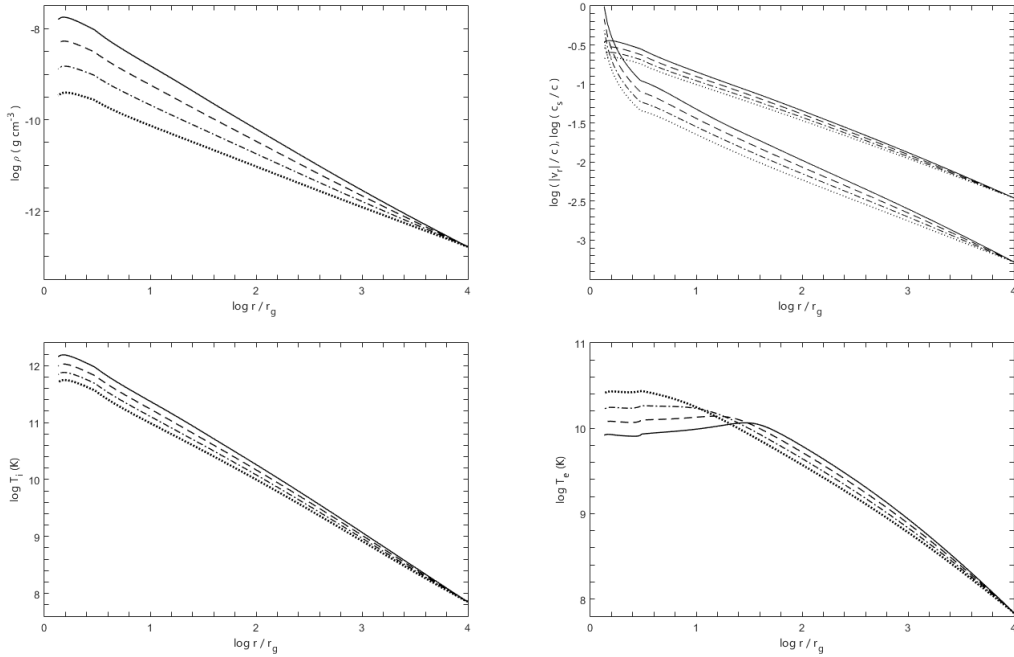


Figure 3: The variations with radii of density, radial velocity, ions temperature and electrons temperature for  $s = 0$ ,  $s = 0.2$ ,  $s = 0.4$  and  $s = 0.6$  are represented by solid, dashed, dash dotted and dotted lines respectively. The outer boundary is set at  $10^4 r_g$  with the boundary conditions of  $T_i = 0.2T_{vir}$  and  $T_e = 0.19T_{vir}$  and  $\alpha = 0.1$  and  $\dot{M}_{out} = 10^{-3}\dot{M}_{Edd}$  are fixed.

Figure 4 shows the variations of physical quantities in the presence of outflow for an

another case. In this figure, the outer radius is  $r_{out} = 10^2 r_g$  and boundary conditions are  $T_i = 0.6T_{vir}$  and  $T_e = 0.08T_{vir}$ . The parameters  $\alpha$  and  $\dot{M}_{out}$  have a larger value in comparison with the case that was discussed in figure 3. So, the quantities  $q^+$  and  $q_{ie}$  have larger values and the electrons temperature increases more clearly in the inner regions.

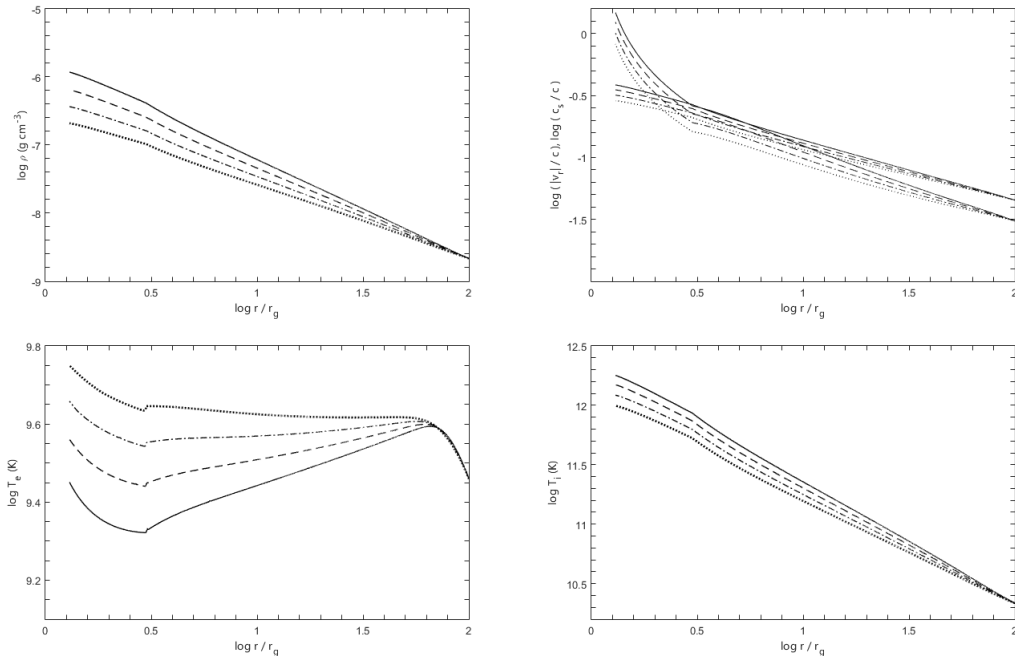


Figure 4: Radial dependency of density, radial velocity, ions temperature and electrons temperature for  $s = 0$ ,  $s = 0.2$ ,  $s = 0.4$  and  $s = 0.6$  are represented by solid, dashed, dash dotted and dotted lines respectively. The outer boundary radius is  $10^2 r_g$  and the boundary conditions are  $T_i = 0.6T_{vir}$  and  $T_e = 0.08T_{vir}$ .  $\alpha = 0.3$  and  $\dot{M}_{out} = 10^{-1} \dot{M}_{edd}$ .

## 4 Summary

The observational evidence indicates the outflow exists in the accretion flow systems. For this reason, the study of structure of accretion flow in the presence of outflow is important. The spectral models for accretion flows with outflow have been investigated before in the frame of global solution. In these studies, a power law function of radius has been considered for the mass accretion rate and the other dynamical equations of the flow remain unchanged. Also, in these works, the emitted spectra has been taken into account.

In this paper, we focus on the structure of accretion flow in the presence of outflow. To overcome to difficulty of calculation the global solution, we use a simplified global solution to obtain the dynamical structure of accretion flow. We consider an advection dominated accretion flow and use a power law function for mass accretion rate to take into account the role of outflow. In spite of the simplifications that we imposed, our results have a acceptable agreement with the physical predictions.

Although if we want to have a full investigation, the other equations of the flow must be modified and the effects such as the angular momentum and energy transfer should be

considered; as Xie & Yuan (2008) noted, the most important effect of outflow is on the density profile that is caused by the radial variation of mass accretion rate. If the strength of outflow that is determined by  $s$  is fixed, all other effects of outflow can only produce a difference for the density and temperature within a factor of  $\sim 2$  [14]. So, we hope the simple method we used does not cause the significant error.

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