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Evolution of Information and Complexity in an Ever-Expanding Universe

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Abstract. Using the usual definitions of information and entropy in quantum gravity and statistical mechanics and the existing views about the relation between information and complexity, we examine the evolution of complexity in an ever expanding universe.

Keywords: cosmic expansion, entropy of the universe, rise of complexity

1 Introduction

Bekenstein and Hawking discovered a deep relation between gravity and thermodynamics. In studying the mathematical properties of black holes, they found that the following correspondence can be made between black hole quantities and thermodynamical quantities[1][2] (Table 1).

black hole quantity	thermodynamic quantity
surface gravity	temperature
surface area of the horizon	entropy
mass	Internal energy

Table 1. Correspondence between black hole and thermodynamical quantities as discovered by Bekenstein and Hawking.

Since then, it has become clear that the dynamical processes governing black holes obey relations which are exactly similar to the laws of thermodynamics: 1) The vanishing surface gravity is never achieved in a natural process 2) First law: Increase in the mass of the black hole equals surface gravity times increase in surface area 3) Second law: Surface area of the horizon never decreases in a natural process. Since in classical general relativity there is no element of randomness which can be attributed to the black hole horizon it soon became clear that the origin of black hole entropy should be quantum. Quantum black holes are, in fact, random objects with enormous amounts of entropy, obeying the relation

$$dS = \frac{dM}{T}, \tag{1}$$

in which S, M, and T are the entropy, mass, and temperature of the black hole, respectively. If we express the surface area in Planck units, the entropy of the black hole is given by

$$S = \frac{A}{4\ell_{pl}^2}, \tag{2}$$

where ℓ_{pl} is the Planck length (Eq. 8). From the statistical mechanical point of view, the entropy is known to be proportional to the logarithm of the number of microstates. By the number of microstates, we mean the number of ways a system can be in, while its macroscopic description remains the same. The Bekenstein bound is the maximum amount of information required to completely describe a physical system down to the quantum level[1]:

$$S_{max} = \frac{2\pi k_B R E}{\hbar c}, \tag{3}$$

where S_{max} is the maximum entropy (Bekenstein bound), k_B is the Boltzmann constant, R is the radius of the system, E is the total energy (including rest masses), $\hbar = h/2\pi$ is the Planck's constant, and c is the speed of light. According to modern ideas of information theory, entropy can be interpreted as information (this is known as information entropy). Therefore, a natural unit to measure it is in bits and the total number of bits is related to the total number of degrees of freedom of a system. Consider a finite volume V. This volume contains a number N of (really) fundamental particles. These really fundamental (and yet unknown) particles define the bits of information in the system. The Bekenstein entropy bound of a system, therefore, defines the maximum number of bits inside V,

$$I_{max} = \frac{2\pi R E}{\hbar c \ln 2}. \tag{4}$$

According to Bekenstein, this bound comes from the second law of thermodynamics as applied to a black hole. It is surprising that this information is imprinted on the surface area of V, which is A. This is known as the holographic principle which is of considerable interest in quantum gravity[3][4]. The holographic concept and the idea of entropy imprinted on the boundary of a volume (the so-called holographic screen), has lead to interesting suggestions on the origin of the gravitational force as an entropic force[5]. Also note that the black hole entropy (2) exactly saturates the Bekenstein's bound.

We now return to our main question: What is the relation between the information content of the expanding universe and the rise of complexity in various stages of the evolution of the universe? In order to answer this question, we must first review some basic concepts from cosmology, and then try to link them with the concept of information and complexity.

2 Basic ideas from cosmology

The metric describing the large scale geometrical structure of the universe is the so-called Robertson-Walker metric:

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right], \tag{5}$$

where t is the cosmic time, $a(t)$ is the cosmic scale factor, (r, θ, ϕ) are the co-moving co-ordinates and $k = 0, \pm 1$ represent the flat, closed, and open geometries of the universe, respectively. WMAP data indicates a flat universe[6] ($k = 0$) and we restrict ourselves to this case, from now on. The evolution of the universe is governed by the Einstein's equations or specifically Friedmann equations, which upon assuming an equation of state $p = w\rho$ between the pressure and density lead to

$$a(t) \propto t^{\frac{2}{3(1+w)}}. \tag{6}$$

Important eras in the evolution of the universe are summarized in Table 2.

EOS parameter	$a(t)$	cosmological epoch
$w = 1/3$	$t^{1/2}$	radiation-dominated era
$w = 0$	$t^{2/3}$	matter-dominated era
$w = -1$	e^{Ht}	inflation/dark-energy-dominated era

Table 2. Equations of state, corresponding to important stages of the evolution of the universe ($p = w\rho$).

The particle horizon L_p is defined[6] as the largest spatial co-moving distance a particle could have whose light signals could have reached us if it was emitted at time $t = t_{min} < t_0$. This definition means that all particles within the particle horizon could have been in causal contact with us in the history of the universe. For the equation of state $p = w\rho$ which leads to $a(t) = At^{2/3(1+w)}$, the particle horizon becomes

$$L_p = c \int_0^{t_0} \frac{dt}{a(t)} = ct_0, \tag{7}$$

where c is the velocity of light and t_0 is the present age of the universe $t_0 = 13.7 \times 10^9$yrs. Note that L_p remains finite only for $w > -1/3$ or $w < -1$ (the range $w < -1$ has no physical significance). The ever increasing horizon size of the universe means that more and more particles become causally connected and the information content of the universe increases with time. During the inflationary era $t \sim 10^{-35}s$, the horizon size increase extremely rapidly (while the event horizon remains almost constant) and the information content of the universe within the particle horizon increases explosively, too.

3 Information and complexity

John Archibald Wheeler suggested that the concept of information in the universe is more fundamental than its matter and energy content. Bekenstein, in his 2003 article in Scientific American, referred to this view as he wrote[2] "... regard the physical world as made of information, with energy and matter as incidentals". The concept of information content of a system goes back to Claude E. Shannon's seminal papers[7] in 1948. While entropy is commonly described as a measure of the disorder in a physical system, the Austrian physicist Ludwig Boltzmann described it more precisely in terms of the total number of microstates a system can have while showing the same macrostate. Shannon tried to find a link between the concept of information and entropy. Shannon proposed that information can be measured as a distribution, which he called the entropy[7]. This entropy measures the inherent uncertainty in the system, equivalently it assesses, how much information is gained when an outcome of the system (distribution) is observed. According to Bekenstein "the number of arrangements that are counted by Boltzmann entropy reflects the amount of Shannon information one would need to implement any particular arrangement...". Note that the physical units of the ordinary entropy (energy/temperature) and Shannon entropy (bits, dimensionless) are different, but this does not seem to be a fundamental difficulty. Putting all these together, the entropy of a system is linked to its information content and that is inscribed at the surface area of the system's boundary. Since the Planck length is a very small quantity

$$\ell_{pl} \equiv= \sqrt{\frac{\hbar G}{c^3}} = 1.616 \times 10^{-35} m, \tag{8}$$

the amount of information which can be contained in an even modest volume in enormous. For example, a spherical volume of radius 1cm contains $\sim 10^{67}$ bits of information. The most natural volume to consider in an expanding universe is the volume contained inside its horizon. There are various definitions for the horizon in an expanding universe, among which we choose the particle horizon which is most relevant to our discussion. What is the relation between information and complexity? Among various definitions of complexity, let us adopt the one proposed by Kolmogorov[8]. According to Kolomogorov, "complexity can be measured as the minimum of bits necessary to reconstruct an information file". Consider, for example a large file containing "...01010101...". This is evidently a simple file, capable of being compressed to a file of a few bits. On the contrary, a file containing information about the digits appearing in the number $\pi = 3.14159...$ should be a much more complex file. Let us quote a few sentences from Kolomogorov[9]: "Our definition of the quantity of information has the advantage that it refers to individual objects and not to objects treated as members of a set of objects with a probability distribution given on it. The probabilistic definition can be convincingly applied to the information contained, for example, in a stream of congratulatory telegrams. But it would not be clear how to apply it, for example, to an estimate of the quantity of information contained in a novel or in the translation of a novel into another language relative to the original. I think that the new definition is capable of introducing in similar applications of the theory at least clarity of principle."

Armed with the above concepts from quantum gravity and information theory, we are now ready to see what happens to the complexity of the matter content of the universe as it expands. As noted before, just after the inflation, we have a tremendous amount of entropy and particles generated during reheating. The particles produced during reheating are rapidly thermalised and their distributions are given by the usual Fermi-Dirac (for fermions like electrons, quarks and neutrinos) and Bose-Einstein (like photons and other gauge bosons). A system in thermodynamic equilibrium, although having a large amount of entropy, is essentially a simple system, according to the Kolomogorov's conception of complexity. The temperature, particle type, chemical potential and total volume are the only information you need to describe a system of identical fermions or bosons in thermodynamic equilibrium. As the universe expands, particles go out of equilibrium one after each other. For example, neutrinos decouple around $t \simeq 1s$, and neutral atoms decouple from the ambient radiation field at $t \simeq 380000$yrs, leading to the extensively studied cosmic microwave background radiation (CMB)[10]. As the universe becomes an out-of-equilibrium system and structures form due to the attractive force of gravity on large scales and electromagnetic forces on small scales, the complexity of the material content of the universe rises.

To see how the maximum information evolves in an expanding universe, we calculate

$$S_{max} = \frac{4\pi L_p^2}{\ell_{pl}^2} = \left(\frac{4\pi c^5}{\hbar^2 G}\right) t^2. \tag{9}$$

It is seen that the maximum entropy (information) content of the universe increases as $\sim t^2$ in an expanding universe from the quantum gravity point of view. This of course does not include the entropy of the horizon. Recently, people have used two criteria: 1) Increase in total entropy and 2) Decrease in the first derivative of entropy to reach interesting limits on the equation of state parameter of the comic matter [13, 14]. But what does all this information content do with the rise of complexity in the universe? As pointed out earlier, it is not necessarily so. Therefore, we have to explore the relation between information and complexity in the expanding universe in more detail. As the universe goes out of equilibrium after the early hot big bang period, structures start to form and we need more and more information to describe the structures formed. Formation of large structures in the universe (say galaxies and galaxy clusters) happen due to the action of force of gravity. A fairly simple analysis initiated by J. Jeans happens that a density perturbation of size L grows due to the force of gravity, if L is larger than the so-called Jeans's length[11]:

$$L_J = \sqrt{\frac{\pi}{G\rho}} c_s, \tag{10}$$

where ρ is the density of the medium and c_s is the speed of sound in the medium. In an expanding universe, the same criterion holds, but while the rate of growth of a perturbation in an initially static medium is exponential, the growth rate in an expanding universe is a power law in time. Therefore, density perturbations grow slower in an expanding universe as compared to a an initially static medium. Detailed calculations and computer simulations show that dark matter is needed if cosmic structures are to be formed in due time in the expanding universe. These simulations indicate that first galaxies and stars formed approximately 500 million years after the big bang[12]. Employing the Kolomogrov's idea of complexity, we need only a few bits of information to describe a universe in thermal equilibrium (say temperature, chemical potential of various particles and so), while much more information is needed to describe the universe when galaxies are formed (distribution and types of galaxies, types of stars, distribution of mass and angular momentum in galaxies etc...). When smaller structures like the planets are formed, this information content and the corresponding complexity rises even more. As any astronomer knows, the structure of a planet (like the Earth) is much more complex than a typical star (like the Sun) and we know the physics of the Sun much better than what we know about the physics of the Earth. The existence of biosphere adds to this complexity. From a quantum point of view, complexity is somehow related to the number of available states of the system. For example, a single spin $\hbar/2$ system can be described by a single bit (spin up or down), while a system of two such particles have $2^2 = 4$ states (the singlet and triplet states) and we therefore need two bits of information to describe in which quantum state the system is. In order to find a simple model for the evolution of quantum states in the expanding universe, we approximate the universe as a 3D box of size $L_p = ct$, where L_p is the size of the particle horizon discussed earlier. From elementary quantum mechanics, we know that number of states in such a box is infinite and as the quantum numbers (n, m, p) increase the states become more and more degenerate

$$E_{nmp} = \frac{\pi^2 \hbar^2}{2mL^2}(n^2 + m^2 + p^2) \tag{11}$$

From cosmology, however, we know that at any time t, the energy scale of a typical particle in the universe is given by

$$E \sim k_B T, \tag{12}$$

where T is the cosmic temperature obeying

$$\frac{T}{T_0} = \frac{a(t_0)}{a(t)} = \left(\frac{t_0}{t}\right)^{\frac{2}{3(1+w)}}. \tag{13}$$

where we have used Equation (6). In the above equation, t_0 is a reference cosmic time (e.g. the present age of the universe $t_0 = 13.7$Gyr). Consider a quantum state in the three dimensional space represented by quantum numbers (n, m, p). The states relevant to our problem extend from $r_{nlm} = 0$ in this space up to the radius where $E_T \simeq kT$. The total number of quantum states in this volume is therefore

$$N = \frac{4\pi}{3}(n^2 + m^2 + p^2)^{3/2} = \left(\frac{2mL_p^2 E_T}{\pi^2 \hbar^2}\right)^{3/2}, \tag{14}$$

where m is the particle mass. We therefore have

$$\frac{N(t)}{N(t_0)} = \left(\frac{L_p^2(t)T(t)}{L_p^2(t_0)T(t_0)}\right)^{3/2} = \left(\frac{t}{t_0}\right)^{\frac{2+3w}{1+w}} \tag{15}$$

In these equations, we have assumed $w \neq -1$. One observes that the number of available quantum states for a single particle increases with cosmic time, provided that the exponent in Eq. (15) is positive, or

$$w < -1 \quad \text{or} \quad w > -2/3. \tag{16}$$

This condition is certainly satisfied for known matter and energy forms (e.g. dust or radiation). For the vacuum or dark energy equation of state, $w = -1$, instead of (15) and we have

$$\frac{N(t)}{N(t_0)} = \left(\frac{t}{t_0}\right)^3 e^{\frac{3}{2}H(t_0 - t)}, \tag{17}$$

where

$$H \equiv \frac{\dot{a}}{a} = \sqrt{\frac{8\pi G \rho_{vac}}{3}}, \tag{18}$$

where ρ_{vac} is the constant vacuum energy density. Surprisingly, this leads to an early increase and late time exponential decrease in the number of states . The behavior of $N(t)$ with time is plotted in Figure 1 for the three equation of states $w = 0, 1/3$, and -1. The existence of a maximum information epoch in the dark-energy-dominated era is surprising. The time of maximum information can be calculated from Equation (17), by differentiating it with respect to t and equating it to zero. The result is:

$$t_{max} = \frac{2}{H}, \tag{19}$$

where H is the Hubble constant. Using the current estimates[6] of $H = 71^{+4}_{-3} kms^{-1}Mpc^{-1}$, we obtain $t_{max} = 27.5$Gyr, almost twice the present age of the universe.

This shows that we are still (and will be for quite a long time in future) in the phase of increasing information.

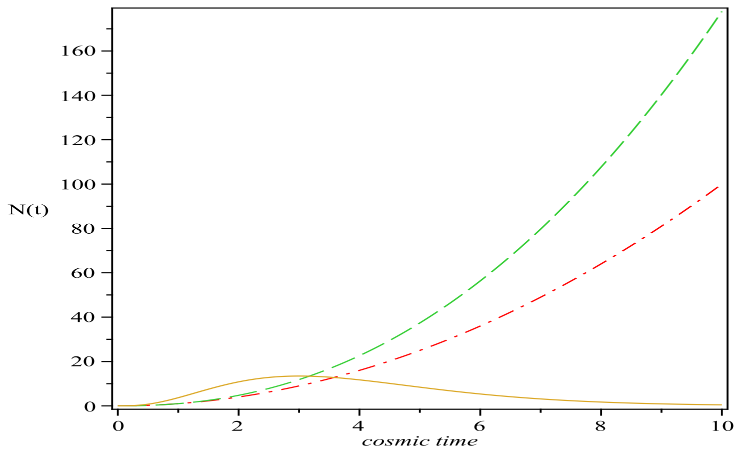

Figure 1: The time dependence of the number of quantum states within the particle horizon, in an expanding universe with $w = 1/3$ (dashed), $w = 0$ (dot-dashed), and $w = -1$ (solid line).

4 Concluding remarks

We reviewed the notions of information, entropy and complexity in the context of the standard FRW models of cosmology and latest ideas of holographic gravity. We estimated the evolution of the maximum bits of information in an expanding universe, as well as the evolution of the number of quantum states. We conclude that the capacity of the universe to hold information increases with the cosmic expansion. The apparent rise of complexity in the universe, in the form of large scale structures, galaxies, planetary systems and life is in harmony with this increasing capacity. The number of quantum states of the universe was estimated as a function of cosmic time and it was shown that this quantity, also, increases with cosmic time, except for the following range of the equation of state parameter w: $-1 \leq w < -2/3$, where the expansion of the universe is accelerating. This corresponds to the inflationary era and recent, dark-energy-dominated era. We already know that life (in its known form on our planet) originated about 4 billion years ago and evolved to more complex forms mainly during the dark-energy-dominated era. Once micro-structures are formed in the universe (i.e. structures which are mainly governed by local, short range interactions), their evolution is no longer affected by the dynamics of the universe in its largest scales. Therefore, the accelerating status of the expansion of the universe has effectively no influence on the micro-cosmos, which undergoes further evolution toward more complex structures.

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