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Spin and Isospin Asymmetry, Equation of State and Neutron Stars

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Abstract. In the present work, we have obtained the equation of state for neutron star matter considering the influence of the ferromagnetic and antiferromagnetic spin state. We have also investigated the structure of neutron stars. According to our results, the spin asymmetry stiffens the equation of state and leads to high mass for the neutron star.

Keywords: Neutron stars, Spin asymmetry, Equation of state

1 Introduction

Neutron stars are hyper-dense and magnetized laboratories for investigating strange phenomena in the nuclear and particle physics. Pulsars and magnetars are two kinds of neutron stars with strong surface magnetic field. Actually the exact origin of this magnetic field is not yet known. In the interior of magnetars, the magnetic field strength may be even larger according to virial theorem [6] and such strong field may cause spin asymmetry. The occurrence of such strange phenomena can affect the equation of state (EOS) of neutron star matter. Theoretically, the equation of state has been applied to determine the maximum mass of a neutron star which should be in agreement with the precise observations. The accurate measurement of neutron star mass $M = 1.97 \pm 0.04 M_\odot$ in the system PSR J1614-2230 was one of the most important development in observational data [9]. This precise measurement is based on Shapiro delay in neutron star-white dwarf binary [12]. Another well-measured massive neutron star is PSR J0348+0432, with mass about $M = 2.01 \pm 0.04 M_\odot$ [1]. Next, there is an evidence that the black widow pulsar PSR B1957+20 might have even larger masses approximately $M_{PSR} = 2.4 M_\odot$ [17]; however, one have to consider the uncertainties in this mass estimation. Finally, the largest mass $2.1 M_\odot \leq M_{NS} \leq 2.7 M_\odot$ has been given for the gamma-ray black widow pulsar PSR J1311-3430 by simple heated light curve fits [16]. These massive neutron stars require the equation of state of the system to be rather stiff. Therefore, theoretical approaches should confirm these observational data.

Recently, several studies used different theoretical approaches showed the stiff EOS for the neutron star matter. Gandolfi et al. [10] have used quantum Monte Carlo techniques and calculated the equation of state of neutron star matter with realistic two- and three-nucleon interactions. Their calculation resulted $M_{max} < 2.2 M_\odot$ for neutron star mass. They have also used Auxiliary Field Diffusion Monte Carlo technique by incorporating semi-phenomenological Hamiltonian including a realistic two-body interaction and many-body forces [11]. They found the maximum mass of neutron star lies in the range 2.2-2.5 times of solar mass. Some other attempts by Partha Roy Chowdhury showed the rotating star mass is around (1.93-1.95)$M_\odot$ [7] . They have applied a pure nucleonic equation of state for neutron star matter. Shen et al. [14] have constructed a new equation of state for a wide range of temperatures, densities and proton fractions to be used in astrophysical simulations of neutron stars. They have predicted that the maximum mass of neutron star is about $2.77M_{\odot}$ with a radius of about 13.3 km. Sun et al. [15] have investigated neutron star structure using EOS which has provided by density dependent relativistic Hartree-Fock theory. Their results showed that maximum mass of neutron stars lies in the range $(2.45-2.49)M_{\odot}$. More recently, we gained $M_{NS}=1.991M_{\odot}$ by applying the Lowest Order Constrained Variational (LOCV) method and using UV_{14}+TNI potential [4].

In this article, we investigate some physical properties of polarized neutron star matter using the LOCV method and the AV_{18} potential. This modern equation of state is derived from an accurate many-body calculation and is based on the cluster expansion of the energy functional. Moreover, we obtain the particles abundance, equation of state and the structure of neutron stars. Finally, we compare our results by experimental data.

2 Formalism

We assume the neutron star matter as a charge neutral infinite system that is a mixture of leptons and interacting nucleons. The energy density of this system can be obtained as follows,

$$\varepsilon = \varepsilon_N + \varepsilon_l, \tag{1}$$

where $\varepsilon_N(\varepsilon_l)$ is the energy density of nucleons (leptons). In the following, we determine these energy densities in more details.

2.1 Energy density of leptons

The energy density of leptons, which are considered as noninteracting Fermi gas, is given by,

$$\varepsilon_{lep} \quad = \sum_{l=e,\ \mu}\ \sum_{k\leq k_l^F}(m_l^2c^4+\hbar^2c^2k^2)^{1/2}\ . \tag{2}$$

In this equation, $k_l^F=(6\pi^2\rho_l/\nu)^{1/3}$ is Fermi momentum of leptons and ν is degeneracy. For fully spin polarized matter, degeneracy is $\nu=1$.

2.2 Energy density of spin polarized nucleon matter

The nucleonic part of neutron star matter is composed of neutrons and protons with densities ρ_n and ρ_p, respectively. The total number density of the system is

$$\begin{aligned}\rho &= \rho_p+\rho_n, \\ &= (\rho_p^{(\uparrow)}+\rho_p^{(\downarrow)})+(\rho_n^{(\uparrow)}+\rho_n^{(\downarrow)}).\end{aligned} \tag{3}$$

The labels $(\uparrow)$ and $(\downarrow)$ are used for spin-up and spin-down nucleons, respectively. The following parameters can be used to identify a given spin-polarized state for the asymmetric nuclear matter,

$$\delta_p=\frac{\rho_p^{(\uparrow)}-\rho_p^{(\downarrow)}}{\rho_p},\quad \delta_n=\frac{\rho_n^{(\uparrow)}-\rho_n^{(\downarrow)}}{\rho_n} \tag{4}$$

δ_p and δ_n are proton and neutron spin asymmetry parameters, respectively. In the fully ferromagnetic (FM) polarized nuclear matter, spin of all neutrons and protons are parallel,

$\delta_n = \delta_p = 1.0$, and in the antiferromagnetic (AFM) spin state, we have $\delta_n = \pm 1.0, \delta_p = \mp 1.0$. The asymmetry parameter which describes the isospin asymmetry of the system is defined as,

$$\beta = \frac{\rho_n - \rho_p}{\rho} = 1 - 2x_p \tag{5}$$

where $x_p = \rho_p/\rho$ is the proton fraction. Pure neutron matter is totally an asymmetric nuclear matter with $x_p = 0$, while for the symmetric nuclear matter $x_p = 1/2$. The energy density of spin-polarized asymmetrical nuclear matter, ε_{nucl} can be determined as,

$$\varepsilon_N = \rho(E + m), \tag{6}$$

where $m = 938.92$ MeV is the nucleon mass and E is the total energy per nucleon which is calculated by using the LOCV method as follows.

We adopt a trial many-body wave function of the form

$$\psi = \mathcal{F}\phi, \tag{7}$$

where ϕ is the uncorrelated ground state wave function of A independent nucleons (simply the Slater determinant of the plane waves) and $\mathcal{F} = \mathcal{F}(1 \cdots A)$ is an appropriate A-body correlation operator which can be replaced by a Jastrow form i.e.,

$$\mathcal{F} = \mathcal{S} \prod_{i>j} f(ij), \tag{8}$$

in which $\mathcal{S}$ is a symmetrizing operator. Now, we consider the cluster expansion of the energy functional up to the two-body term [8],

$$E_{nuc}([f]) = \frac{1}{A} \frac{\langle \psi | H | \psi \rangle}{\langle \psi | \psi \rangle} = E_1 + E_2 \cdot \tag{9}$$

The one-body term E_1 for an asymmetrical nuclear matter is

$$E_1 = \sum_{\tau=n,p} \sum_{\sigma=\uparrow,\downarrow} \sum_{k \le k_{F}{}_{\tau}^{\sigma}} \frac{\hbar^2 k^2}{2m_\tau}, \tag{10}$$

where $k_{F}{}_{\tau}^{\sigma} = (6\pi^2 \rho_\tau^\sigma)^{1/3}$ is the fermi momentum of each component of spin-polarized asymmetric nuclear matter. The two-body energy E_2 is

$$E_2 = \frac{1}{2A} \sum_{ij} \langle ij \left| \nu(12) \right| ij - ji \rangle, \tag{11}$$

where

$$\nu(12) = -\frac{\hbar^2}{2m} [f(12), [\nabla_{12}^2, f(12)]] + f(12) V(12) f(12). \tag{12}$$

Here, $f(12)$ and $V(12)$ are the two-body correlation and potential. In our calculations, we use the AV_{18} two-body potentials [20]. Now, we minimize the two-body energy, Eq. (11), with respect to the variations in the correlation functions $f^{(k)}$, but subject to the normalization constraint [13, 2],

$$\frac{1}{A} \sum_{ij} \langle ij \left| h_{S_z,T_z}^2 - f^2(12) \right| ij \rangle_a = 0, \tag{13}$$

where in the case of spin polarized asymmetrical nuclear matter, the Pauli function $h_{S_z,T_z}(r)$ is as follows,

$$h_{S_z,T_z}(r) = \begin{cases} \left[1-9\left(\frac{J_J^2(k_{F_\tau}^{(\sigma)}r)}{k_{F_\tau}^{(\sigma)}r}\right)^2\right]^{-1/2} & S_z=\pm1, T_z=\pm1 \\ 1 & otherwise \end{cases} \quad (14)$$

From the minimization of the two-body cluster energy, we get a set of coupled and uncoupled Euler-Lagrange differential equations [5]. We can calculate the correlation functions by numerically solving these differential equations and then, using these correlation functions, the two body energy is obtained. Finally, we can compute the energy of the system.

2.3 URCA processes

Now, we investigate direct URCA processes in the spin polarized neutron star matter. In fully polarized ferromagnetic spin state, the nature of chemical equilibrium is mainly dominated by the following weak interaction processes,

$$\begin{array}{l} n(\uparrow) \;\rightarrow\; p(\uparrow) \;+\; l(\uparrow) \;+\; \bar{\nu}_l(\downarrow) \\ p(\uparrow) \;+\; l(\uparrow) \;\rightarrow\; n(\uparrow) \;+\; \nu_l(\downarrow) \end{array} \quad (15)$$

Here, ν_l stands for the leptons neutrinos which leave the system without delay. In this case, the β-equilibrium conditions and charge neutrality of neutron star matter impose the following coupled constraints on our calculations,

$$\begin{aligned} \mu_e(\uparrow)=\mu_\mu(\uparrow) &= \mu_n(\uparrow)-\mu_p(\uparrow) \\ &= 4(1-2x_p)S_2(\rho,\delta_n=\delta_p=1)+8(1-2x_p)^3S_4(\rho,\delta_n=\delta_p=1) \end{aligned} \quad (16)$$

$$\rho_p(\uparrow)=\rho_e(\uparrow)+\rho_\mu(\uparrow) \quad (17)$$

where S_2 and S_4 are given by [3],

$$\begin{aligned} S_2(\rho,\delta_n,\delta_p) &= \frac{1}{2}\left(\frac{\partial^2 E(\rho,\delta_n,\delta_p)}{\partial\beta^2}\right)_{\beta=0} \\ S_4(\rho,\delta_n,\delta_p) &= \frac{1}{24}\left(\frac{\partial^4 E(\rho,\delta_n,\delta_p)}{\partial\beta^4}\right)_{\beta=0}. \end{aligned} \quad (18)$$

Similarly, The β-equilibrium and the charge neutrality conditions for fully anti-ferromagnetic spin polarized are,

$$\begin{aligned} \mu_e(\downarrow)=\mu_\mu(\downarrow) &= \mu_n(\downarrow)-\mu_p(\uparrow) \\ &= 4(1-2x_p)S_2(\rho,\delta_n=-\delta_p=1)+8(1-2x_p)^3S_4(\rho,\delta_n=-\delta_p=1) \end{aligned} \quad (19)$$

$$\rho_p(\uparrow)=\rho_e(\downarrow)+\rho_\mu(\downarrow). \quad (20)$$

We find the abundance of the particles by solving the coupled equations of charge neutrality and β-equilibrium conditions. Finally, we calculate the total energy and the equation of state of the neutron star matter.

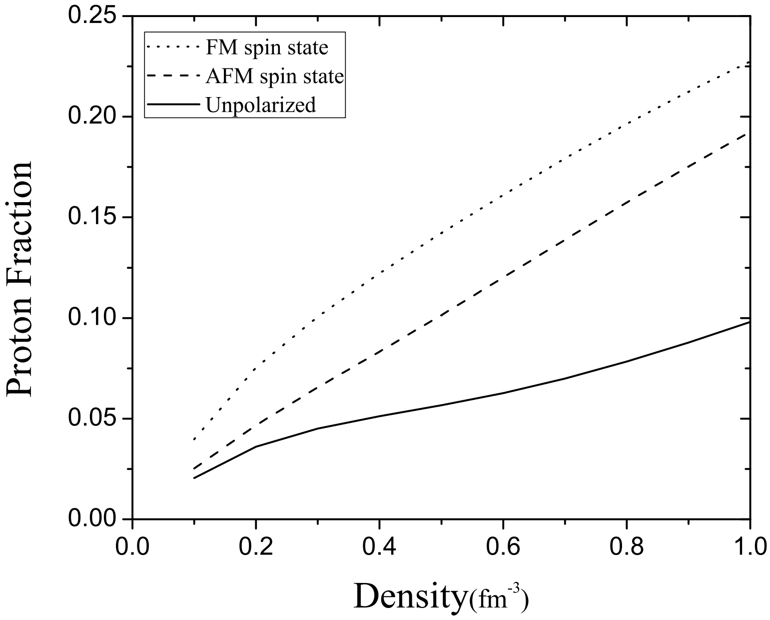

Figure 1: The proton fraction in the neutron star matter for different spin states.

3 Results and discussion

Figure 1 shows the proton fraction, x_p, versus the baryon number density, ρ, for unpolarized, ferromagnetic and antiferromagnetic spin state. It can be seen from this figure that the abundance of protons is an increasing function of both spin polarization and baryon density. Therefore, we can conclude that nuclear portion of spin polarized neutron star matter tend to be symmetric matter. It is also seen that for a given density, the highest value of proton fraction is gained for the ferromagnetic spin state.

In Figs. 2 and 3, we have presented the energy density, ε, and pressure of neutron star matter as a function of baryon number density, ρ, for unpolarized, ferromagnetic and antiferromagnetic spin state, respectively. Here, we have not considered the contribution of magnetic field. In these figures, we have also plotted the energy density and pressure of the fully polarized neutron matter (PNM), i.e. $\beta = 1, \delta_n = 1$. As we can see, the energy density and pressure increase by increasing both of spin and isospin asymmetry parameters. we have concluded that the spontaneous phase transition to ferromagnetic and antiferromanetic spin state does not occur. If such a transition existed, a crossing of the energies of different polarizations would have been observed at some density, indicating that the ground state of the system would be ferromagnetic or antiferromagnetic from that density on. As can be seen in these figures,there is no sign of such a crossing. Our results can be compared with those of Vidana's [18, 19]. Also, it is clear from these figures that the EOS of spin polarized neutron star matter is stiffer than unpolarized matter.

Now, we can investigate the structure of neutron star by using the equation of state and integrating the TOV equation. A summary of our results for the maximum mass, radius, central energy density and central baryon density of neutron star predicted from different

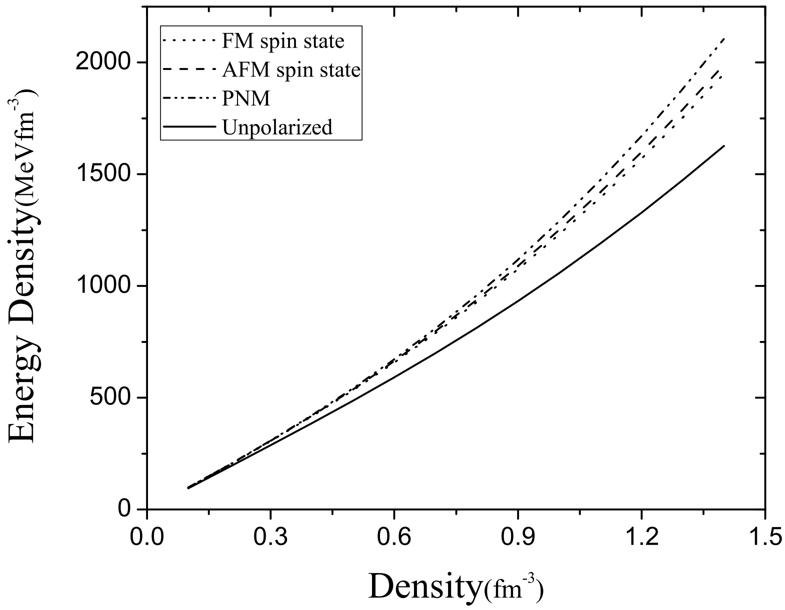

Figure 2: The energy density of neutron star matter versus baryon number density for for different spin states and fully polarized neutron matter.

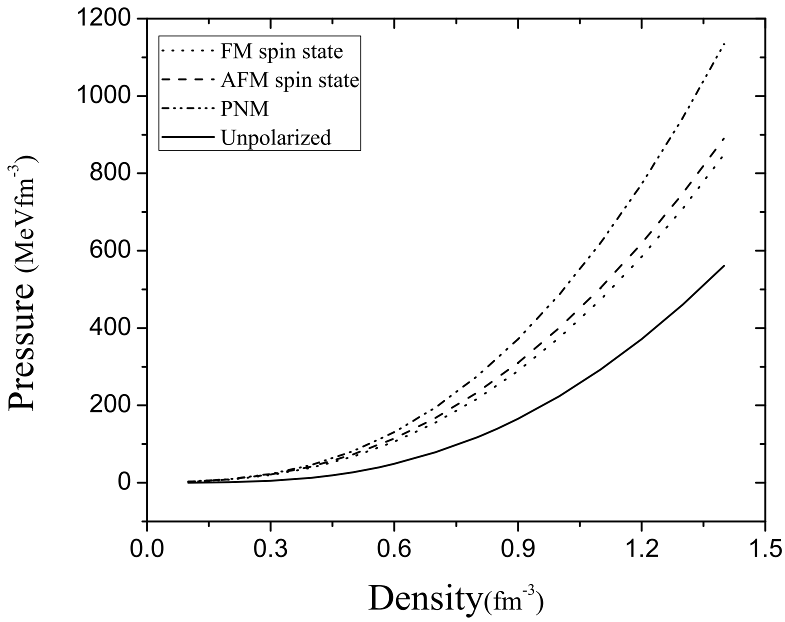

Figure 3: As a Fig. 2 but for pressure.

Table 1: Maximum mass, radius, central energy density and central baryon density of neutron star. The gravitational mass is given in solar mass ($M_{\odot}$).

EOS	M_{max}	R (km)	ϵ_c (10^{14} g/cm^3)	ρ_c (fm^{-3})
NSM [12]	1.63	8.04	-	-
FM-NSM	1.83	10.24	30.28	1.27
AFM-NSM	1.88	10.54	28.67	1.2
PNM	1.99	10.8	27.14	1.13

equations of state is given in table 1. We can conclude from this table that the more asymmetric is the neutron star matter, the higher maximum mass.

4 Summary and Conclusions

The purpose of this paper is investigating the influence of spin polarization on the equation of state of neutron star matter and, consequently, the structure of neutron star. We have used the lowest order constrained variational (LOCV) method by employing the AV_{18} potentials for nucleon-nucleon interaction. We conclude that the equation of state become stiffer by considering spin polarization, and it yields to high maximum mass for neutron stars.

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