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Dynamics of entangled quantum optical system in independent media

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Abstract. We study the dynamics of two three-level atoms interacting with independent bosonic Lorentzian reservoirs at zero temperature. Such systems can be created in far astronomical objects. Quantum mechanical behaviour of these particles can produce detectable effects on the spectroscopic identifications of these objects, if such behaviour remain stable during the interaction with their media. It is shown that detuning increases the robustness of negativity and geometric discord as the measures of entanglement and quantum correlation beyond entanglement, respectively against interaction with reservoirs. In separable and bound entangled regions, negativity is zero while the geometric discord remains not zero. We also study the evolution of geometric discord and negativity for an isotropic state.

Keywords: Entanglement, Quantum discord, Negativity, qutrit

1 Introduction

The entangled states of two distant particles can be established through quantum mechanical effects. In this situation, measuring the properties of one particle can instantly change some properties of the other particle. Recent experiments have strongly verified the existence of quantum entanglement and it is beginning to have practical applications in several branches of science as well [1, 2].
Recently some investigations have been done for using the entanglement concepts in astrophysical situations [1, 2, 3]. Interaction of entangled particles with black holes and studying the thermodynamics of such situations is currently receiving great attentions. Teleportation analogy of the black hole at its final state [4], controllability of black hole evaporation [5], observation of thermal Hawking radiation and its entanglement in an analogue black hole [3] are some examples of entanglement in astrophysical objects. Recently some works have been done in order to detect entangled two-photon systems in astrophysical processes [3].
Most available naturally produced quantum entangled objects between two distant particles in astrophysical regions is two-photon spontaneous transition of the hydrogen $2^2S_{1/2}$ metastable level. Two-photon emission rate of two nearby planetary nebulae IC 2149 and NGC 7293 has been calculated in [6]. It is estimated that production of entangled pairs per second is an order of $10^{44} - 10^{48}$.
There are also other entangled situations which can be investigated in the optical spectroscopy of astronomical objects. Emerged photon due to energy transition of electron on Calcium atoms is another interesting situation which can be modeled as a qutrit system. Therefore we have to know the entanglement evolution of a qutrit-qutrit system, surrounding by their environments as an open quantum system. Effects of environment on the evolution of quantum entanglement can be considered too. Interaction with environment is responsible for decoherence and destroying fragile quantum properties [7, 8]. Discord as the measure

of quantum correlation, negativity as the measure of entanglement and geometric quantum discord as the measure of quantum correlation have been studied for qubit-qutrit systems in a classical dephasing environment [9]. Recently, Liang Qiu et al. [16] have studied the evolution of lower bound of geometric quantum discord for two-qutrit system under depolarization.

In this paper, we study the dynamics of quantum correlations and entanglement of qutrit-qutrit system to understand the robustness of quantum correlations in comparison to entanglement against interaction with independent reservoirs and their evolution. Stability of such correlations is very important. Indeed we can observe effects of entanglement in distant astronomical objects if these correlations are sufficiently strong and long lasting. The main aim of this paper is investigating the effects of medium on the entanglement and geometric discord between far particles. The qutrit-qutrit is strong system against the medium effects, so we can extend our results to the other entangles systems such as photon-photon interactions. We will use the geometric discord and negativity as measures of quantum correlation and entanglement respectively.

We study the effect of a simple local unitary transformation on the evolution of geometric discord and negativity. Peres–Horodecki have provided a separability criterion for bipartite systems [10]. The criterion is the positive partial transpose (PPT) of a bipartite system which can be considered as a signal for separability [10]. The PPT is sufficient and necessary condition for separability in 2×2 (qubit –qubit) and 2×3 (qubit –qutrit) systems while for bipartite systems with dimension of 3×3 (qutrit–qutrit) this condition is necessary but not sufficient. It means that, there are states which eigenvalues of partial transpose are positive but they are entangled [10, 11]. Negativity is defined as:

$$N(\rho^{ab}) = \frac{\sum_i (|\eta_i| - \eta_i)}{2} \tag{1}$$

Where the η_is are eigenvalues of partial transpose of ρ^{ab} with respect to subsystem a , $(\rho^{ab})^{T_a}$ is partial transpose of ρ^{ab} [11].

Dakic et al. [12] introduced the geometric quantum discord, as $D(\rho) = min_\chi ||\rho - \chi||^2$ where χ is a set of zero-discord states and $||A|| = \sqrt{tr(A^\dagger A)}$ is Hilbert-Schmidt norm [12]. The Hilbert space $H^A \otimes H^B$ describes a bipartite system with dim $H^A = m$ and dim $H^B = n$. $L(H^A)$ and $L(H^B)$ are spaces that contain all linear operators on H^a and H^b respectively. $X_i (i = 1, 2, ..., m^2)$ and $Y_j (j = 1, 2, ..., n^2)$ are set of Hermitian operators which contain orthonormal bases for space $L(H^A)$ and $L(H^B)$ respectively. The set $X_i \otimes Y_j$ constitutes an orthonormal base for $L(H^A \otimes H^B)$. Any bipartite state in $H^A \otimes H^B$ can be written as $\rho = \sum_{ij} c_{ij} X_i \otimes Y_j$ and

$$\rho = \frac{1}{mn}(I_m \otimes I_n + \sum_i x_i \lambda_i \otimes I_n + \sum_j y_j I_m \otimes \lambda_j + \sum_i t_{ij} \lambda_i \otimes \lambda_j) \tag{2}$$

Here $\{\lambda_i, i = 1, 2, ..., m^2 - 1\}$ and $\{\lambda_j, j = 1, 2, ..., n^2 - 1\}$ are defined as generators of $SU(m)$ and $SU(n)$, respectively [13].

$$x_i = \frac{m}{2} tr(\rho \lambda_i \otimes I_n), y_j = \frac{n}{2} tr(\rho I_m \otimes \lambda_j) \;\; and \;\; t_{ij} = \frac{mn}{4} tr(\rho \lambda_i \otimes \lambda_j) \tag{3}$$

$\vec{x} = [x_i]$, $\vec{y} = [y_j]$, $T = [t_{ij}]$ and $C = [c_{ij}] = [tr(\rho X_i \otimes Y_j)]$ is equivalent to:

$$C = \begin{bmatrix} \frac{1}{\sqrt{mn}} & \frac{\sqrt{2}\vec{y^t}}{n\sqrt{m}} \\ \frac{\sqrt{2}\vec{x}}{m\sqrt{n}} & 2\,\frac{T}{mn} \end{bmatrix} \tag{4}$$

The lower bound of geometric discord is:

$$D(\rho) \geq tr(CC^t) - \sum_{i=1}^{m} \epsilon_i = \sum_{i=m+1}^{m^2} \epsilon_i \tag{5}$$

The set $\{\epsilon_i, i = 1, 2, ..., m^2\}$ contains eigenvalues of CC^t which have been ordered decreasingly [13]. This paper is organized as follows. Section 2 introduces the dynamics of non-Markovian qutrit-qutrit system in independent reservoirs. Sections 3 and 4 study the evolution of negativity and geometric discord in different conditions respectively. Finally, conclusion and remarks will be presented in the section 5.

2 Dynamics of qutrit-qutrit system

2.1 The "V" type three-level atomic system

Figure (1) presents energy levels of a three level V type atom. We have used such V type atom as a qutrit system. Electric dipole moments are $\vec{\mu_1}$ and $\vec{\mu_2}$ and the allowed dipole transitions in this system are: $|3\rangle \leftrightarrow |1\rangle$, with atomic transition frequency ω_1 and $|3\rangle \leftrightarrow |2\rangle$ with atomic transition frequency ω_2. The spontaneous emission is responsible for decaying the excited states of three level atom into its ground state, and therefore there is not a direct transition between excited states, i.e., the dipole transition $|1\rangle \leftrightarrow |2\rangle$ is forbidden. When the dipole moments of two transitions are parallel, an indirect transition between excited states occurs, because of the interaction with vacuum. The couplings between levels $|1\rangle$ and $|2\rangle$ and reservoirs are $g_k^{(1)}$ and $g_l^{(2)}$ respectively [14, 15].

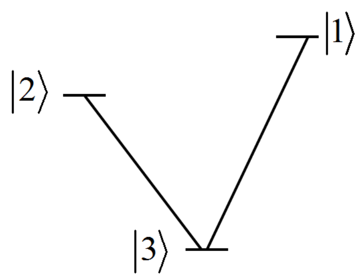


Figure 1: Three-level atom in V type

Now the Hamiltonian of two three-level atoms interacting with independent bosonic Lorentzian reservoirs at zero temperature under the RWA can be written as follows:

$$H_{IV} = \sum_k g_k^1 \,(a_k^\dagger |3_a\rangle\langle 1_a|) + \sum_l g_l^2 (\, b_l^\dagger |3_a\rangle\langle 2_a|) + \sum_m g_m^1 \,(c_m^\dagger |3_b\rangle\langle 1_b|) + \sum_n g_n^2 (\, d_n^\dagger |3_b\rangle\langle 2_b|) + h.c \tag{6}$$

first and second parts of our bipartite system are represented by a and b respectively and $a^\dagger, b^\dagger, c^\dagger$ and $d^\dagger$ are creation operators . Equation (6) shows that there is not any direct interaction between two qutrits.

2.2 Dynamics of atoms

We first write Hamiltonian in the interaction picture and then use the time-convolutionless (TCL) master equation to get dynamics of two three-level atoms [15]. We find:

$$\frac{d\rho_S(t)}{dt} = -i\lambda_1(t)\Big[|1_a\rangle\langle 1_a|, \rho_s(t)\Big] - i\lambda_2(t)\Big[|2_a\rangle\langle 2_a|, \rho_s(t)\Big] - i\lambda_3(t)\Big[|1_b\rangle\langle 1_b|, \rho_s(t)\Big] \quad (7)$$

$$-i\lambda_4(t)\Big[|2_b\rangle\langle 2_b|, \rho_s(t)\Big] + \gamma_1(t)(|3_a\rangle\langle 1_a|\rho_s(t)|1_a\rangle\langle 3_a| - \tfrac{1}{2}\Big\{\rho_s(t), |1_a\rangle\langle 1_a|\Big\}) + \gamma_2(t)(|3_a\rangle\langle 2_a|\rho_s(t)|2_a\rangle\langle 3_a| - \tfrac{1}{2}\Big\{\rho_s(t), |2_a\rangle\langle 2_a|\Big\}) + \gamma_3(t)(|3_b\rangle\langle 1_b|\rho_s(t)|1_b\rangle\langle 3_b| - \tfrac{1}{2}\Big\{\rho_s(t), |1_b\rangle\langle 1_b|\Big\}) + \gamma_4(t)(|3_b\rangle\langle 2_b|\rho_s(t)|2_b\rangle\langle 3_b| - \tfrac{1}{2}\Big\{\rho_s(t), |2_b\rangle\langle 2_b|\Big\}).$$

Where $\lambda_i(t)$ and $\gamma_j(t)$ in equation (7) are defined as follows:

$$\lambda_i(t) = \int_0^t ds \int_0^\omega d\omega J(\omega) sin(\omega - \omega_i)s \quad (8)$$

and

$$\gamma_i(t) = \int_0^t ds \int_0^\omega d\omega J(\omega) cos(\omega - \omega_i)s \qquad i = 1, 2, 3, 4$$

In continuous field frequency limit we change $\sum_\nu |g_\nu|^2$ by $\int d\omega J(\omega)\delta(\omega_\nu - \omega)$ where $\nu = k, l, m, n$. The Lorentzian spectral density is defined as [15]:

$$J(\omega) = \frac{1}{2\pi}\frac{\gamma_0\lambda^2}{(\lambda^2 + (\omega_{cav} - \omega)^2)} \qquad i = 1, 2, 3, 4 \quad (9)$$

where $\omega_{cav} = \Delta_i - \omega_i$, λ is the spectral width, γ_0 is the coupling strength and ω_{cav} is the resonance frequency of the cavity in the Jaynes-Cummings model and Δ_i is detuning between the level of atom and the central cavity mode. We assume that two atoms are identical, so $\lambda_1(t) = \lambda_3(t), \lambda_2(t) = \lambda_4(t)$, $\gamma_1(t) = \gamma_3(t)$ and $\gamma_2(t) = \gamma_4(t)$.

Initial density matrix $\rho(0)$ is taken as [16, 17] :

$$\rho(0) = \frac{2}{7}|\phi\rangle\langle\phi| + \frac{\alpha}{7}\sigma_+ + \frac{5-\alpha}{7}\sigma_- \quad (10)$$

with $|\phi\rangle = \frac{|11\rangle + |22\rangle + |33\rangle}{\sqrt{3}}$, $\sigma_+ = \frac{|12\rangle\langle 12| + |23\rangle\langle 23| + |31\rangle\langle 31|}{3}$ and $\sigma_- = \frac{|21\rangle\langle 21| + |32\rangle\langle 32| + |13\rangle\langle 13|}{3}$.
The density matrix equation (10) is an interesting selection for initial condition, because it describes a separable state for $2 \le \alpha \le 3$, bound entangled for $3 < \alpha \le 4$ and free entangled state for $4 < \alpha \le 5$, The equation (7) could be solved analytically as:

$$\begin{aligned}
\rho_{11}(t) &= \frac{2}{21}exp(-G_1(t) - G_3(t)); \rho_{15}(t) = \frac{2}{21}exp(i(-L_1(t) + L_2(t) - L_3(t) + L_4(t)) - G_1(t) - G_3(t)) \quad (11)\\
\rho_{19}(t) &= \frac{2}{21}exp(i(-L_1(t) - L_3(t)) - G_1(t) - G_3(t)); \rho_{22}(t) = \frac{\alpha}{21}exp(-G_1(t) - G_4(t));\\
\rho_{44}(t) &= \frac{(5-\alpha)}{21}exp(-G_2(t) - G_3(t)); \rho_{55}(t) = \frac{2}{21}exp(-G_2(t) - G_4(t));\\
\rho_{59}(t) &= \frac{2}{21}exp(-i(L_2 + L_4) - (G_2 + G_4));
\end{aligned}$$

Entries $\rho_{33}(t)$, $\rho_{66}(t)$, $\rho_{77}(t)$, $\rho_{88}(t)$ and $\rho_{99}(t)$ will be calculated numerically and the rest of the elements are zero. where $G_i(t) = \int_0^t \gamma_i(s)ds$ and $L_i(t) = \int_0^t \lambda_i(s)ds \;\; i = 1, 2, 3, 4$.

3 Evolution of negativity

In this paper, we have calculated the evolution of negativity for the system with initial state equation (10). Figure (2) shows negativity in $\alpha = 4.3$ for different values for the $\Delta_i s$. It shows that the negativity would be preserved and is more robust against reservoir effects, when $\Delta_i, i = 1, 2$ are increased. For simplicity, we assume that $\Delta_i = 0$, and study the effects of initial state (α), t, γ_0 and λ on the evolution of negativity. Results have been presented in figures (3), (4.a) and (4.b). Figure (3) demonstrates the negativity as functions of α and $\gamma_0 t$. Figure (4.a) shows negativity for $\lambda = 0.01\gamma_0$ and $\Delta_i = 0, i = 1, 2$ in different $\gamma_0 t$ as a function of α and (4.b) shows the negativitiy in different α as a function of $\gamma_0 t$. We see from figures (3), (4.a) and figure (4.b) that negativity is increasing monotonically with increasing value of α and has a decreasing trend with increasing of time in free entangled region. In separable and bound entangled regions, negativity is zero, as expected. Several sections of negativity has been plotted in the figure (4.a). Specific values of $\gamma_0 t$ can be found phase transition like, sudden change occurs in the profile of negativity. For selected initial value of λ and $\alpha \simeq 4.3$ this sudden change happens at $\gamma_0 t = 3$. In support of figure (4.a), the figure (4.b) indicates sudden change occurs for every values of initial state in free entangled region. Figure (4.b) also shows that whatever α be greater the sudden change of entanglement phenomenon happens in greater values of $\gamma_0 t$.

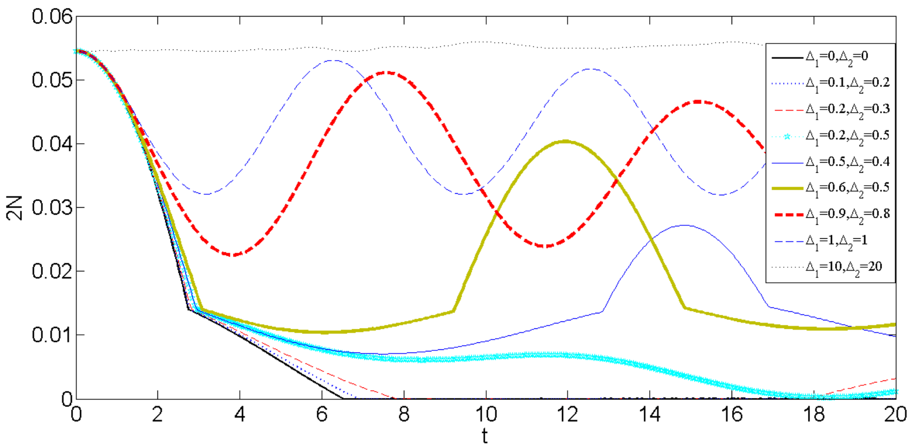


Figure 2: Evolution of negativity for $\alpha = 4.3, \lambda = 0.01$ and $\gamma_0 = 1$ in different Δ_1 and Δ_2

Figure (5.a) shows negativity as a function of α for different values of spectral width λ at $t = 3$ with fixed value $\gamma_0 = 1$. This figure shows that negativity is zero in the separable and bound entangled regions for all values of λ. Negativity decreases as λ increases such that negativity every where is almost zero for $\lambda > \gamma_0$. On the other hand, non-zero region of negativity becomes smaller and also shifts toward larger values of α. In the figure (5.b) negativity has been plotted with respect to α with different values of coupling γ_0 with $\lambda = 0.01\gamma_0$. Therefore the system is completely in non-Markovian situation. This figure shows that negativity appears in larger values of α as γ_0 increases, while decreases with increasing values of γ_0 . The greater values of coupling γ_0 causes deeper interaction between qutrit and its environment.

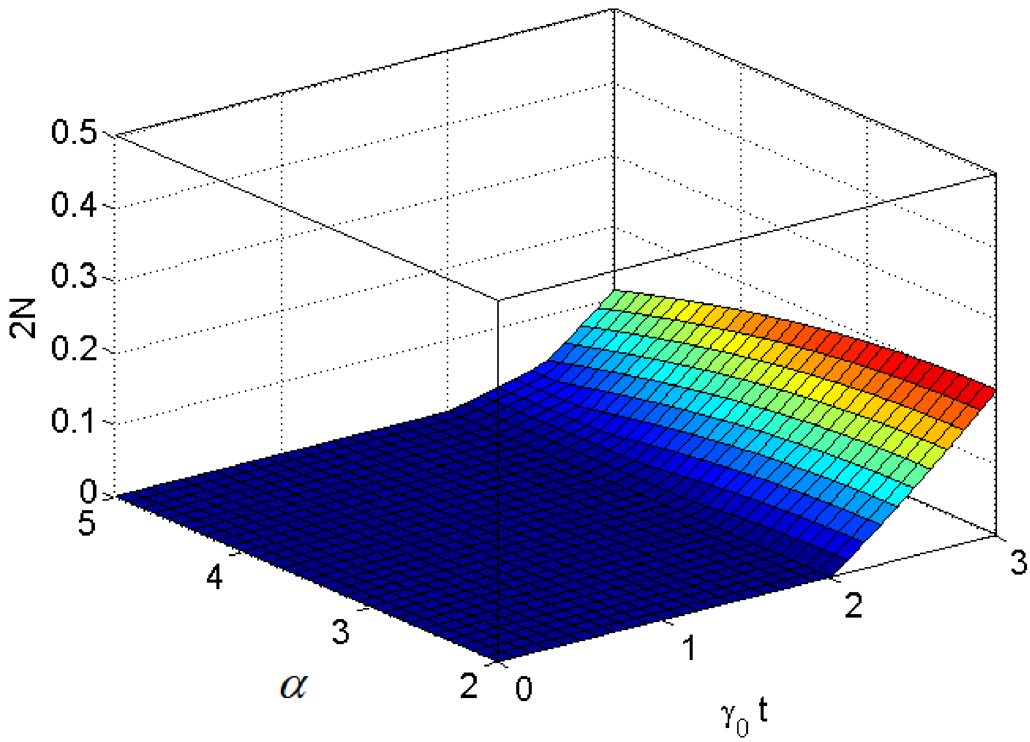
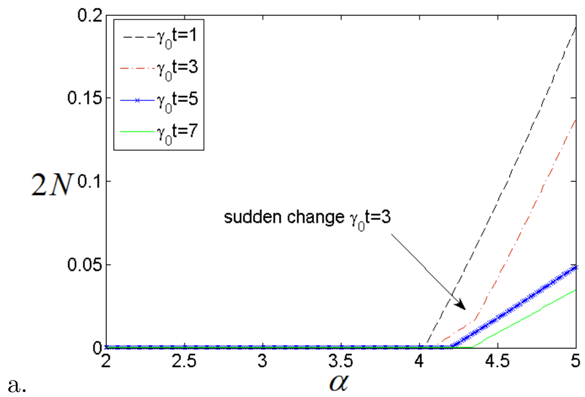


Figure 3: Evolution of negativity as a functions of α and $\gamma_0 t$ when $\lambda = 0.01\gamma_0$ and $\Delta_i = 0, i = 1, 2$.



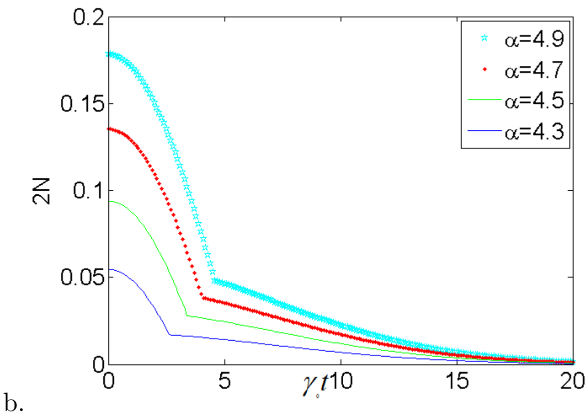
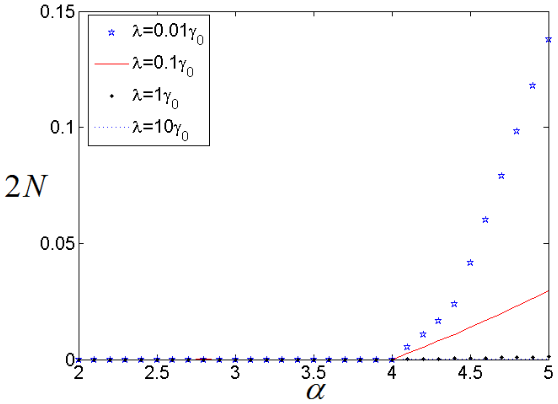
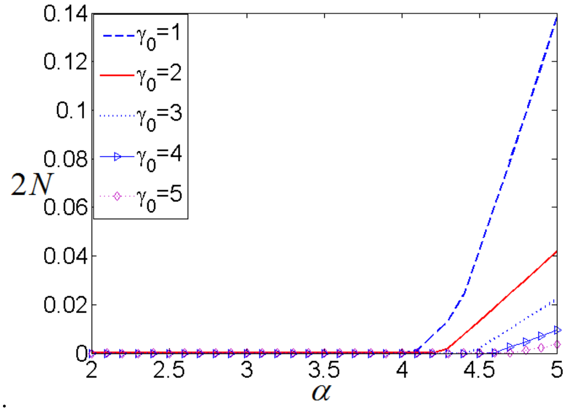


Figure 4: Negativity for $\lambda = 0.01\gamma_0$ and $\Delta_i = 0, i = 1, 2$ (a) shows the negativitiy in different $\gamma_0 t$ as a function of α , (b) shows the negativitiy in different α as a function of $\gamma_0 t$.



a.



b.

Figure 5: (a) shows the negativitiy in different λ when $\gamma_0 = 1$ and $\Delta_i = 0, i = 1,2$ as a function of α , (b) shows the negativitiy in different γ_0 when $\lambda = 0.01\gamma_0$ and $\Delta_i = 0, i = 1,2$ as a function of α.

4 Geometric discord

Let us study the effects of interactions on the behavior of geometric discord. The lower bound of geometric discord has been calculated numerically using the equation (5). Figure (6) shows geometric discord of a system with $\alpha = 4.3$ and different values for detunings, Δ_i. The geometric discord is more robust against reservoirs when detunings $\Delta_i, i = 1,2$ are increased.

Figures (7) and (8) demonstrate results of calculation which they show geometric discord is non-zero for all values of α which proves the existence of quantum correlations even in separable state region ($2 \leq \alpha \leq 3$). Figure (8.a) presents geometric discord as a function of α. This figure indicates that geometric discord has a minimum value in separable region ($2 \leq \alpha \leq 3$). The minimum occurs due to differences in the nature and properties of separable and bound entangled states. It may be noted that the negativity is zero in this region. Figure (8.b) demonstrates time evolution of geometric discord for different initial states of the system. The time evolution of the geometric discord almost is not sensitive to α parameter in free entangled region of initial state. Figure (8.a) explains that the geometric discord is almost constant in free entangled region, while the negativity increases in this situation. Figure (8.a) also indicates that a sudden change in geometric discord occurs but with respect to α. It is in agreement with results of [19, 16]. Finally, figures (7) and (8.b) demonstrate that geometric discord, unlike negativity, decreases monotonically in time for all values of parameter α, while evolution of negativity has a sudden change in free entangled region. The evolution of geometric discord in free entangled region is almost independent of α, while for negativity the point of sudden change happens in greater $\gamma_0 t$ as α increases. A sudden change point also is observed in the evolution of geometric discord for qutrit-qutrit system under multilocal or global depolarising noises, while for collective depolarizing noise, such a sudden change has not been occurred [16]. We have not observed any kind of sudden change in our non-Markovian qutrit-qutrit system too.

Figures (9) show discord with respect to α for different values of λ with $\gamma_0 = 1$ (9.a) and different values of γ_0 with $\lambda = 0.01\gamma_0$ (9.b), both at $t = 3$. Figure (9.a) shows that point of sudden change moves toward greater values of α, while maximum value of the quantum discord decreases. For $\lambda > \gamma_0$ (Markovian regime) discord becomes almost zero. Figure (9.b) indicates that same situation occurs for coupling parameter γ_0. Therefore, we can

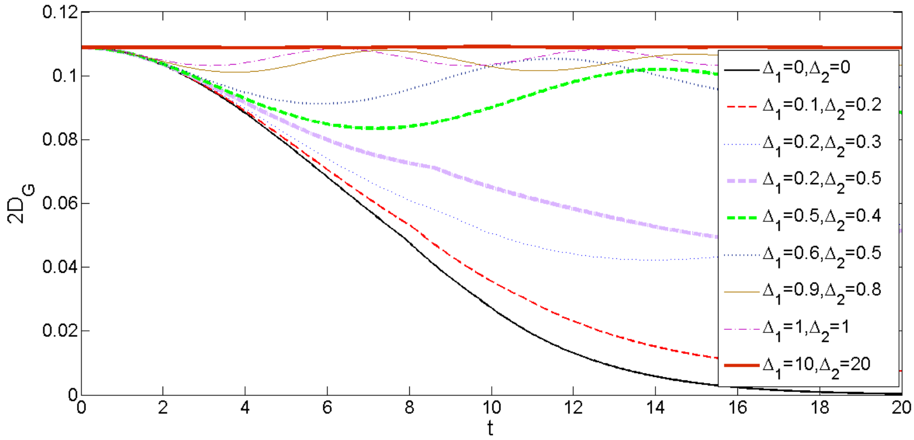


Figure 6: Evolution of geometric discord for $\alpha = 4.3, \lambda = 0.01$ and $\gamma_0 = 1$ in different Δ_1 and Δ_2

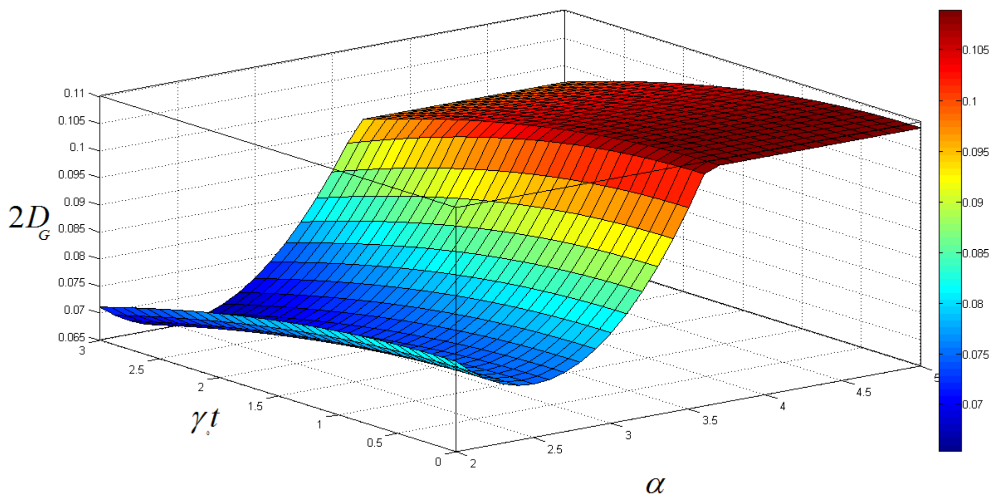
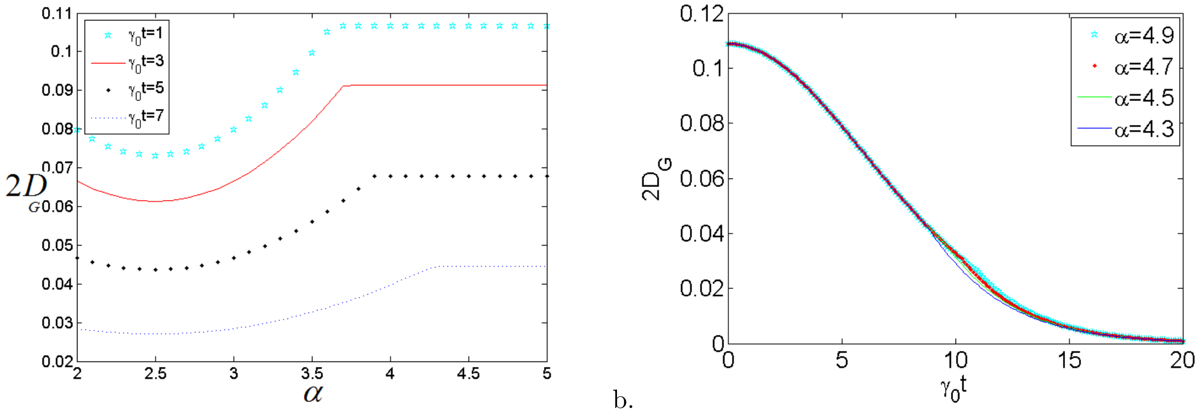


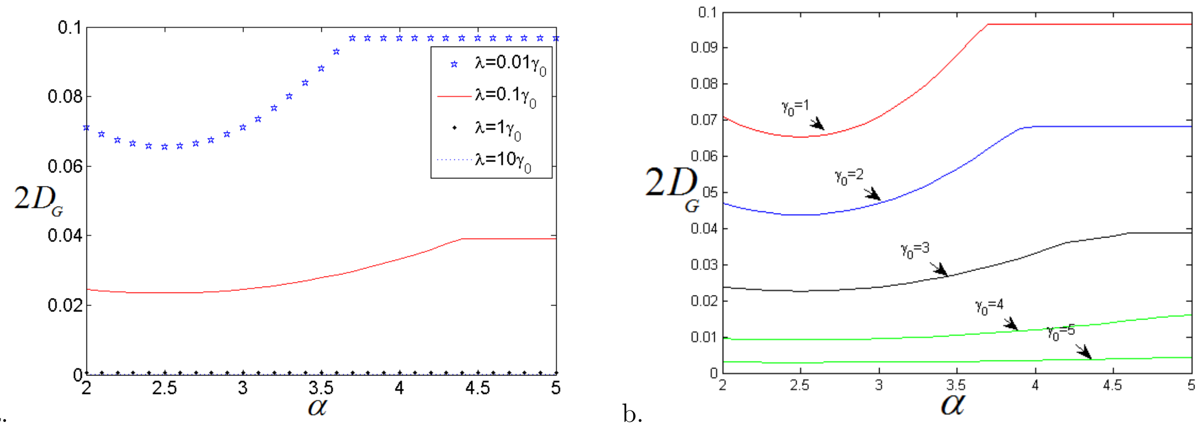
Figure 7: Evolution of geometric discord for $\lambda = 0.01\gamma_0$ and $\Delta_i = 0, i = 1, 2$ as a functions of α and $\gamma_0 t$



a. b.

Figure 8: Geometric quantum discord for $\lambda = 0.01\gamma_0$ and $\Delta_i = 0, i = 1, 2$. (a) shows the Geometric discord in different $\gamma_0 t$ as a function of α , (b) shows the Geometric discord in different α as a function of $\gamma_0 t$.

say that point of sudden change moves toward greater values of α and quantum discord is decreasing functions of γ_0 and also λ in Markovian and non-Markovain regimes.



a. b.

Figure 9: (a) shows the geometric discord in different λ when $\gamma_0 = 1$ and $\Delta_i = 0, i = 1, 2$ as a function of α , (b) shows the geometric discord in different γ_0 when $\lambda = 0.01\gamma_0$ and $\Delta_i = 0, i = 1, 2$ as a function of α.

The initial state (10) under the local unitary transformation $U = I_3 \otimes O$ with $O = |0\rangle\langle 1| + |1\rangle\langle 0| + |2\rangle\langle 2|$ changes to:

$$\sigma(0) = U\rho(0)U^\dagger = \frac{2}{7}|\phi'\rangle\langle\phi'| + \frac{\alpha}{7}\sigma'_+ + \frac{5-\alpha}{7}\sigma'_- \tag{12}$$

with $|\phi'\rangle = \frac{|01\rangle+|10\rangle+|22\rangle}{\sqrt{3}}$, $\sigma'_+ = \frac{|00\rangle\langle 00|+|12\rangle\langle 12|+|21\rangle\langle 21|}{3}$ and $\sigma'_- = \frac{|11\rangle\langle 11|+|20\rangle\langle 20|+|02\rangle\langle 02|}{3}$. Figure (10) shows that the geometric discord has not been changed considerably under this local unitary transformation. Other studies indicate that this local transformation on a qutrit-qutrit system evolved in depolarizing noise shifts the sudden change point [16]. Under local unitary transformation equation (12) negativity decreases and the point of sudden change appears in smaller values of time $\gamma_0 t$.

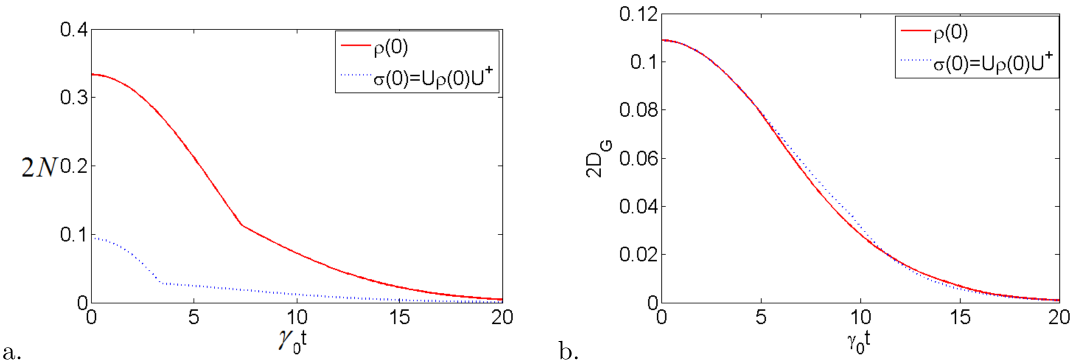


Figure 10: a: Comparison negativity for initial state $\rho(0)$ and its local unitary transformation $\sigma(0)$ and also b: geometric discord for $\lambda = 0.01\gamma_0$ and $\Delta_i = 0, i = 1, 2$.

The isotropic states are $d \times d$ dimensional bipartite states which are convex mixtures of a maximally entangled state with a maximally mixed state. They are invariant under unitary transformations of the form $U \otimes U^*$ [20]. Isotropic states for a qutrit-qutrit system are:

$$\rho = p|\phi\rangle\langle\phi| + \frac{(1-p)}{9} I_9 \tag{13}$$

with $0 \leq p \leq 1$. The isotropic state ρ is separable if and only if it is PPT i.e. $p \leq \frac{1}{4}$ [20]. We study the evolution of geometric discord and negativity for these states under amplitude damping when $p = 0.45$. Figure (11) shows both geometric discord and negativity decay in time. This figure also shows that a sudden change in the evolution of negativity happens as observed in the evolution of state (10).

5 Conclusion and remarks

The dynamics of qutrit-qutrit system in independent reservoirs, as an example of entangled particles at a distant produced in astronomical objects has been investigated. We have performed numerical simulations for the system to get the evolution of geometric discord and negativity and have examined the stability and robustness of their correlation. Our calculations show that, the geometric discord is non-zero even in separable region, because of the existence of a sort of quantum correlation in the system while negativity in separable and bound entangled regions is zero, as expected. The geometric discord has a minimum value in separable and bound entangled regions, this minimum is because of different properties of separable and bound entangled states. A sudden change in geometric discord with respect to α occurs because of approaching this point from bound to free entangled region. For free entangled states, geometric discord is almost constant with respect to α , while negativity is an increasing function of α. Simulations also show that both negativity and geometric discord decrease in time. We showed that the robustness of negativity and geometric discord against interaction with reservoir increases as detuning increases. A sudden change happens in the evolution of entanglement while such phenomenon does not occur in the evolution of geometric discord in free entangled region. It may be noted that the rate of change of negativity is greater than that for geometric discord. In conclusion, contrary to geometric discord, the evolution of negativity has a sudden change point.

It is interesting to investigate the behavior of negativity and geometric discord for other

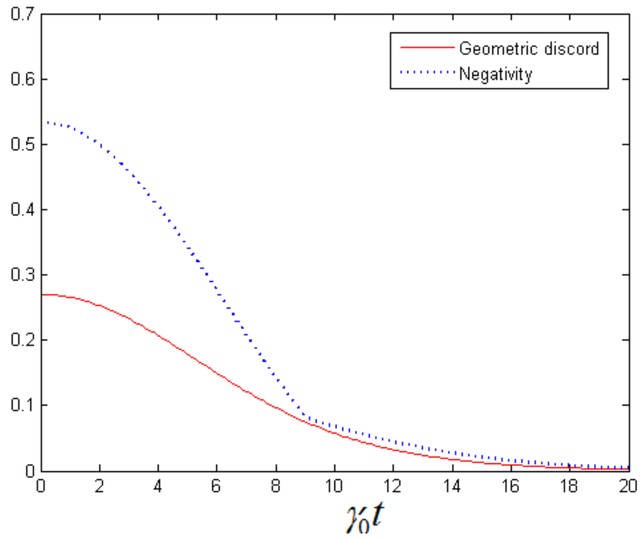


Figure 11: Negativity for $\lambda = 0.01\gamma_0$, $p = 0.45$ and $\Delta_i = 0, i = 1, 2$ as function of $\gamma_0 t$.

non-Markovian systems in astronomical objects. Also it is insightful to find entanglement indicator which able to provide deeper distinguish ability between the systems with different initial states.

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