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Effects of Relativistic Maxwellian Distribution on the Dust Grain Electrical Potential

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Abstract. The effects of relativistic on the dust charging process and the dust grain electrical potential are investigated by taking into account the cross section of relativistic by the OLM theory (Orbit Limited Motion), a kinetic model and the relativistic Maxwellian distribution function for currents carried by ions and electrons. The calculations are applied by the numerical analyses to find the electrical potential of dust grain in the charging process. It is shown that the electrical potential of dust grain is increased in the relativistic regime, and the slope of the transition region to zero is much more severe than the non-relativistic state and also, the possible values for dust density are shifted to the larger amounts. The comparison of the results of the relativistic and nonrelativistc Maxwellian distribution functions shows in the low dust to ion density ratio, only the relativistic Maxwell distribution function can indicate the dust charging process. As another result, the increase of the dust density shows the collective behavior, because of the dust grains behavior as a component from conventional multi-ionic plasma. Also, it is indicated that the role of mass is more colorful than the ion temperature in the light plasma such as hydrogen versus the heavy plasma such as oxygen in relativistic regime. Moreover, it is showed that, as ions are closer to the ultra-relativistic range, the dust grain electrical potential is increased and the difference between the dust grain electrical potential in oxygen, helium and hydrogen plasmas becomes more and more.

Keywords: dust electrical potential, dust charging, relativistic Maxwellian distribution, orbit limited motion theory (OLM), dusty plasma.

1 Introduction

One type of plasma is dusty plasma that is consisting of dust particles in ordinary plasma. The suspended dust particles have brought more complexity into the ordinary plasma, and thus the dusty plasmas are known as the complex plasmas. In fact, the study of dusty plasmas relates to the collective behavior of dust grains and theirs charging process [1, 2, 3, 4, 5]. These particles can range from tens of nanometers to hundreds of microns and they are usually much massive than electrons and ions of plasma [6, 7]. In astrophysics, it is known that the dust grains are very prevalent throughout the universe [8, 9]. They are present in the interplanetary spaces, circumsolar, the interstellar clouds, planetary nebulas, cometary tails, from the cosmos to the planets and from them to the near earth and also, in the plasma laboratory [10, 11, 12]. There are many active fields on the dusty plasma researches such as the process of dust charging and calculations the electrical potential of dust grains [13, 14, 15, 16]. For sample, the rockets and satellites in the Earth's atmosphere plasma are charging similar to the charging of spherical probe in plasma [17, 18]. A theory that takes into account the dust charging mechanisms is needed to evaluate the overall behavior of dust grains in the plasma that the Orbital-Limited Motion Theory (OLM) is the most commonly. The OLM theory was first proposed by Langmuir and Mat-Smith [19] and was completed in the 1960s [20, 21]. By this theory, the electrical potential of dust grain for a wide range of particle sizes is predicts accuracfully [22, 23, 24, 25].

In this work, we consider the effects of the temperature and gas of plasma on the process of dust charging and as a result the electrical potential of dust grain using the OLM theory and a kinetic model. The negative electrical potential for dust grains is assumed because the mass of the electrons is much lower than the mass of the ions and thus those will reach to the dust particles faster than the ions. The relativistic Maxwellian distribution for the charging currents of dust grain in equilibrium plasma is considered [26, 27]. The Maxwellian distribution in the Boltzmann-Gibbs statistics is believed to be universally valid for macroscopic ergodic equilibrium systems. When the free path is smaller than the plasma size, the plasma is in the thermodynamic equilibrium with the electrostatic columbic force. This is a scenario that usually occurs for many short-range interactions in the Hamiltonian systems [28, 29].

In another paper, we have previously investigated the effect of relativistic non-extensivity distribution in the non-equilibrium plasmas on the process of dust grain charging. It was showed that the relationship between the electrical potential of dust grain and dust density are very affected by the electron and ion nonextensive degree [30] .

We now want to consider the dust charging process in equilibrium situation for plasmas and compare the results. Also, it has been extracted that the velocity distribution function can changing from the non-relativistic to relativistic state in some laboratory and space plasmas. For example, in situitions that the particle velocities is approximately as close as the speed of light, the relativistic regime plays very important role especially for understanding of plasma behavior in the Van Allen radiation belts, interstellar plasma, earth magnetosphere and etcs. This may be widely helped in studing of complex plasma systems such as turbulences, chaoses, and solitons [12, 29, 30, 31, 32].

For the first time, the relativistic Maxwell distribution function on gas discharge has investigated by Jutter [33] many years ago. Moreover, Swisdak [34] has considered the generation of random varieties by the relativistic Maxwellian distribution in defferent temperature and drift velocity in the kinetic models on plasmas and gases. Recently, Kumari and Pandey [18] have studied whistler waves with relativistic Maxwell distributions in the Saturn magnetosphere. They have shown that the variation of growth rate in relativistic regime is different to non-relativistic regime. The most important applications of the relativistic regime are collisions involving heavy nuclei, the solar neutrino problem, anomalous diffusion of a quark in a quark-gluon plasma and and flux of cosmic rays on earth, the electron-positron annihilation [10, 11, 12, 35]. In solar physics, it is well known that neutrinos detected are arrived from sun and a relativistic Maxwellian distribution function is convenient for statistical distribution of neutrinos in the solar plasma [36]. In around of the earth orbit, it is observed that because of the solar wind, the electron distributions from thermal energies to modestly relativistic energies are isotropic approximately. These electrons eclectrons contain a thermal core population $\sim 95\%$ of the number density and suprathermal halo $\sim 5\%$ with higher energies that suprathermal halo is formed from hot $\sim 100eV$ electrons originating in the solar corona.

In this area, the particle velocity is about $0.08 < v/c < 0.8$. In the range velocities $v/c > 0.1$ a plasma can be defined as a relativistic plasma [8]. Since the discovery of cosmic rays by Hess in the early $20th$ century, astrophysicists have made great efforts to identify the sources and mechanisms of generations of such energetic particles. Hess found out that reveal supernova $RXJ1713.7-3946$ such as the TeV cosmic rays source [37]. For these reasons, the distribution functions should be considered in relativistic state. In the process of dust grain charging into planetary rings, comets, interplanetary space and especially in magnetospheric and ionospheric plasma, the calculation the electrical potential of dust grain plays a very important role in understanding the dusty plasma physics [38, 39]. In this situation, we considered the ion and elexctron currents, and the effect of the temperature of the electrons and ions on the electrical potential of dust grain is evaluated. Moreover, for studying a more general case, the influence of gas of plasma is considered, and its role on the electrical potential of dust grain is calculated. Also, to find the relativistic effects on the process of dust charging, we compare the results when distribution function is non-relativistic Maxwellian.

This research is presented as follows: In section 2, the basic formula OLM theory and the cross section for collisions between the dust grains with ions or electrons in relativistic regime are presented. Moreover, the ion and electron currents in charging process of dust grains and the electrical potential of dust grain are obtained, by the relativistic Maxwellian distribution. In section 3, the numerical simulations of the nonlinear equations are discussed and the relativistic role on the electrical potential of dust grain by the changing of parameters such as temperature of electron to ion and gas of plasma is investigated. Finally, conclusions are peresented in Section 4.

2 Basic equations and charging process

To describe the charging process of dust grain, let us recall some basic facts about the collision cross-section in relativistic complex plasma. In relativistic non-magnetic plasma that contains dust grains, electrons, ions, from an infinite distance a plasma particle (j) is approaching a spherical dust grain with radius rd and charge qd. In conditions the Debye length very larger than dust grain of radius, $\varphi_d = q_d/r_d$ is the potential difference between the dust grain and plasma. When plasma particle enter in the Debye sphere and feel the charge of dust grain due to the electrostatic force, those will change direction. We assume that the collision cross section between the plasma particle and dust grains is $\sigma = \pi b_j^2$ in the relativistic regime where j is the ion or electron $(j = i\ or\ e)$ and b_j is the parameter of impact. In the Ref [30], we have considered the relativistic OLM theory and shown that by the conservation of momentum and energy in relativistic regime, the cross section is

$$\sigma = \pi r_d^2 (1 - \frac{\sqrt{1-\beta j^2}}{\beta_j} u_j)^2, \tag{1}$$

$$u_j = \frac{q_j \varphi_d}{m_j c^2}, \tag{2}$$

In these equations $\beta_j = v_j/c$ is the factor of relativistic, u_j is the ratio of potential energy to rest energy, v_j is the plasma particle speed before its grazing collision with the dust particle, m_j is the rest mass, q_j is the plasma particle charge and c is the ligth velocity.

Now, we find the charging currents of dust grain in plasma with the relativistic Maxwellian distribution. The charging current of dust grain Ij (carried by plasma species j) can be calculated using the OLM theory [16, 39]. Based on this theory, Ij results from plasma particles with the relativistic velocity distribution $f_j(v_j)$ and the charge q_j [18, 33, 34]

$$I_j = q_j \int\limits_{v_j^{\min}}^{c} v_j \sigma_j^d f_j(v_j) dv_j, \tag{3}$$

and

$$f_j(v_j) = n_j \Big(\frac{m_j}{2\pi k_B T_j}\Big)^{3/2} \Big(\frac{1}{\sqrt{1-\beta_j^2}}\Big)^{3/2} e^{-\frac{\alpha_j}{\sqrt{1-\beta_j^2}}}, \tag{4}$$

where $v_j^{\min}$ is the minimum of plasma particle velocity (j), k_B is the Boltzmann constant, T_j is the plasma particle temperature and $\alpha_j = m_j c^2/k_B T_j$ is the ratio of rest mass to thermal energy. In relativistic regime also, it is important that $v_j^{\min} \leq v_j \leq c$. There are two situations for $v_j^{\min}$; the attractive potential and the repulsive potential. The dust and plasma particle attract each other in an attractive potential, and then the initial velocity is not required for the incident particles, but the plasma particle and the dust grain repel each other in the repulsive potential, and hence, the existence of an initial velocity $v_j^{\min}$ is necessary. Thus, in this case, $v_j^{\min}$ becomes

$$v_j^{\min} = c\sqrt{1 - \Big(\frac{m_j c^2}{q_j \varphi_d}\Big)^2}. \tag{5}$$

The equation 3 can be calculated using the Gaussian integrals, integration by parts, Gamma integrals[40] and combination methods. Since electrons are much lighter than ions, initially the electron current is significantly less than ion current, and dust particles cause a negative charge. In the following, the electron current decreases to the grain. Ultimately, both the electron and ion currents are equal, which is a very interesting situation for our consideration. But a repulsive potential case $q_j \varphi_d > 0$) is as follows:

$$I_e = r_d^2 n_e e^{-\alpha_e u_e} \Big[G_1 u_e^{-\frac{1}{2}} + G_2 u_e^{-1} + G_3 u_e^{-\frac{3}{2}} + G_4 u_e^{-\frac{5}{2}} + G_5 u_e^{-\frac{7}{2}}\Big], \tag{6}$$

where G_1, G_2, G_3, G_4 and G_5 are, respectively, given as

$$G_1 = \frac{4}{3} A_e c^4 \Big(\frac{4}{5} - \alpha_e - \frac{2}{15}\alpha_e^2 + \frac{4}{15}\alpha_e^3\Big), \tag{7}$$

$$G_2 = \frac{2\sqrt{\pi}}{3} A_e c^4 \alpha_e^{\frac{1}{2}} \Big(1 - \frac{4}{15}\alpha_e^2\Big), \tag{8}$$

$$G_3 = -\frac{2}{5} A_e c^4 \Big(1 - \frac{2}{5}\alpha_e - \frac{4}{15}\alpha_e^2\Big), \tag{9}$$

$$G_4 = -\frac{2}{5} A_e c^4 \Big(1 + \frac{2}{3}\alpha_e\Big), \tag{10}$$

$$G_5 = \frac{2}{3} A_e c^4, \tag{11}$$

$$A_e = -4\pi e \Big(\frac{m_e}{2\pi k_B T_e}\Big)^{3/2}. \tag{12}$$

Equation 6 is calculated for the electron current on the negative dust particle using Equation 3. In calculating this equation and Equations 6-12, Gaussian and Gamma integrals have been used. As can be seen from Equation 6, this equation shows the nonlinear relationship between the electron current Ie and the normalized potential energy of the dust grain ue. As can be seen from this equation, the electron current is inversely related to the normalized potential energy of the dust grain, due to the repulsion between the electron current and the negative charge of the dust grain. Also, when potential is attractive ($q_j\varphi_d < 0$), the ion current is as

$$I_i = r_d^2 n_i [F_1 - F_1 u_i + F_2 u_i^2], \tag{13}$$

where F_1 and F_2 are, respectively, given as

$$F_1 = A_i c^4 S_1, \tag{14}$$

$$F_2 = -A_i c^4 (S_1 + S_2), \tag{15}$$

and in these

$$A_i = 4\pi^2 e\Big(\frac{m_i}{2\pi k_B T_i}\Big)^{3/2}, \tag{16}$$

$$S_1 = e^{-\alpha_i}\Big(\frac{4}{15} + \frac{8}{5}\alpha_i - \frac{8}{45}\alpha_i^2 - \frac{16}{45}\alpha_i^3\Big) - \frac{2}{3}\Big(\sqrt{\pi} - \frac{4}{3}\Big(\alpha_i^{\frac{1}{2}}\Big(1 - \frac{4}{15}\alpha_i^2\Big), \tag{17}$$

$$S_2 = -e^{-\alpha_i}\Big(\frac{2}{3} + \frac{4}{3}\alpha_i\Big) + \frac{2}{3}\alpha_i^{\frac{1}{2}}\Big(\sqrt{\pi} - \frac{4}{3}\Big). \tag{18}$$

Equations 13-18 are calculated by using Gaussian and Gamma integrals for the ion current on the negative dust particle. These equations shown that because of the coulomb attractive, the ion current Ii is directly related to the energy potential of the dust grain ue. Assuming that system is neutral from an electrical view point, the quasi-neutrality condition is as follows

$$\frac{n_e}{n_i} = 1 - Z_d\frac{n_d}{n_i} \tag{19}$$

where Z_d and nd are the number of the dust grain charges and the dust density, respectively. In $Z_d n_d/n_i \ll 1$, the dust grains will be assumed isolated, and in ratio comparable to unity, it will be assumed non-isolated. By increasing of the number of dust grains chargers, it is seeds that have a great deal of appetite for electrons, but the number of electrons in each dust grain is small dust. The ratio $Z_d n_d/n_i \ll 1$ is a very important factor for studying the behavior of dusty plasma, and according to literature the parameter is defined as follow

$$P = -\frac{m_e c^2 r_d n_d}{e^2 n_i}. \tag{20}$$

Finally, for evaluating the charging process of dust grain and the dust grain electrical potential, we use equation 13 for electron and equation 6 for ion and $I_i + I_e = 0$ in stady state. Therofore we obtain the relationship between ion density (n_i) and electron density (n_e). Then, by replacing the obtained relation in the quasi-neutral condition (equation 19) and using the P parameter (equation 20), we can write

$$\Big[\frac{F_1(1-u_i) + F_2 u_i^2}{G_1 + G_2 u_e^{-\frac{1}{2}} + G_3 u_e^{-1} + G_4 u_e^{-2} + G_5 u_e^{-3}}\Big] u_e^{\frac{1}{2}} e^{\alpha_e u_e} = 1 + P u_e, \tag{21}$$

where

$$u_i = -\Big(\frac{m_e}{m_i}\Big)u_e. \tag{22}$$

In a non-relativistic regime, the equation 3 by Shukla and Mamun [39] is calculated as

$$\Big(\frac{T_i m_e}{T_e m_i}\Big)^{\frac{1}{2}} \exp\Big(-\frac{e\phi_d}{k_B T_e}\Big)\Big(1 - \frac{e\phi_d}{k_B T_i}\Big) = 1 + P^* \frac{e\phi_d}{k_B T_i}, \tag{23}$$

where

$$P^* = \frac{r_d n_d k_B T_i}{e^2 n_i}. \tag{24}$$

Comparing equations 21 and 23 show the difference between the relativistic and the non-relativistic regimes, respectively. The equation 21 has no analytical solutions and is very nonlinear. Thus, by using the numerical methods, the dust grain electric potential can be obtained in terms of the P parameter. We use a logarithmic form in studying of the dust grains electrical potential behavior to make the dust charging process more clearly.

3 Results and discussion

In the previous section, the nonlinear equation of the dust grain electrical potential in the relativistic plasma is obtained. This equation shows that the electrical potential of dust grain in the relativistic state is dependent to the characteristics of the dusty plasma such as the ion, electron and dust density, the dust particle size, the ion and electron temperature, and gas of plasma. In this section, we slove the nonlinear equation 21 by numerical methode and compare it to the non-relativistic regime. In Figure 1 $(a) - (c)$, the effect of ion-to-electron temperature ratio in the relativistic regime on the electrical potential of dust grain for three gases of plasma is plotted as a function of log P. Plasmas are considered as hydrogen, helium, and oxygen plasma gases as shown in Figures 1 $(a) - (c)$, respectively. These curves are plotted for the different temperature ratios of ion-to-electron $T_i/T_e = 0.25$ (solid line), $T_i/T_e = 0.5$ (dashed line), $T_i/T_e = 0.75$ (dotted line) and $T_i/T_e = 1$ (dash dotted line) in relativistic regime. If the density of ion and dust grain radius does not chang, Figure 1 shows the effects of density of dust particle on the its electrical potential φ_d. It is clear from Figure 1 that with increasing the dust density (the means decreasing the average distance between dust grains), in the relativistic state the ue and the average of charge of dust grain ($Z_d = r_d u_e m_e c^2/e^2$) decreases. In other words, increasing in the dust density causes decreasing of electron and ion incident currents on dust grains, and thus, the decrease in the charge of any dust grains. This Figure shows that a critical region is seen for the dust electrical potential when it is very close to zero. In other words, due to the increase in the dust density, grains of dust are a component of conventional multi-ionic plasma. Figure 1 can be analyzed from a different point of view. By assuming the density of ion and the dust grains radius be constant, the effect of ion density and also under condition of quasi-neutrality, the electron density on the electrical potential of dust grain φ_d and the dust charge Z_d can be obtained from Figure 1. This shows that as the ion density ni increases, the value of ue increases in relativistic plasma and also, dust charge Z_d increases. This is quite obvious because, by increasing the density of ions or electrons, the ion and electron currents on the dust grains increase and as a result of the charge of the dust grain and thus the dust grain electrical potential increases. Moreover, it is indicated in Figure 1 when the temperature of ion increases, the electrical potential of dust grain increase, too. As a result a temperature ratio of ion-to-electron smaller caused a smaller dust electrical potential, also. Its physical reason is that by raising the ion temperature, the ion velocity in relativistic state increases as a result of the equation 1, the collision cross-section, and thus the current of ions in the charging process of dust grains decrease. In steady state assumption, the decrease of ion current causes increasing of the electron current, thus the dust grain charge and its electrical potential increase. In Ref [30], we calculate the charging process of the dust grain by the relativistic Tsallis distribution function. The statistical of Tsallis is based on entropy of nonextensive that is suitable for describing systems out of equilibrium. The Tsallis distribution can be seen in physical phenomena such as long-range interactions, space plasmas and gravitational systems. In this distribution, the q parameter is degree of nonextensive that is in the limit $q = 1$ and the Taslis distribution function tends to the Maxwell distribution function. In Ref [30], it is indicated that the low ratio of dust grain to ion densities or P_s small, are not in the amplitude of the electrical potential of dust grain. By comparing Figure 1 and the results of Ref [30] it is indicated that in low ratio of dust grain to ion densities, the relativistic Maxwellian distribution shows the charging process of the dust grain but the Tsallis distribution in relativistic regime cannot show the dust grain charging process. In addition, we also find that in the oxygen plasma, the electrical potential of the dust grain is smaller than the helium and hydrogen plasmas. Clearly, ions in hydrogen plasma are lighter than helium plasma, and helium plasma is lighter than plasma oxygen and indeed, heavier ions may lead to a smaller dust electrical potential. The reason is that increasing the mass of ion, ue decreases in the relativistic plasma according to equation 2. The dust electrical potential for hydrogen plasma is about two times bigger than helium plasma and sixteen times larger than the oxygen plasma which is well matched to the mass proportions of these gases. Figures 2 $(a) - (c)$ show the variations of the dust grain electrical potential in terms of $\log P$ for ion-to-electron temperature different ratios that are larger than unity, in three kinds of hydrogen, helium and oxygen plasmas gases,

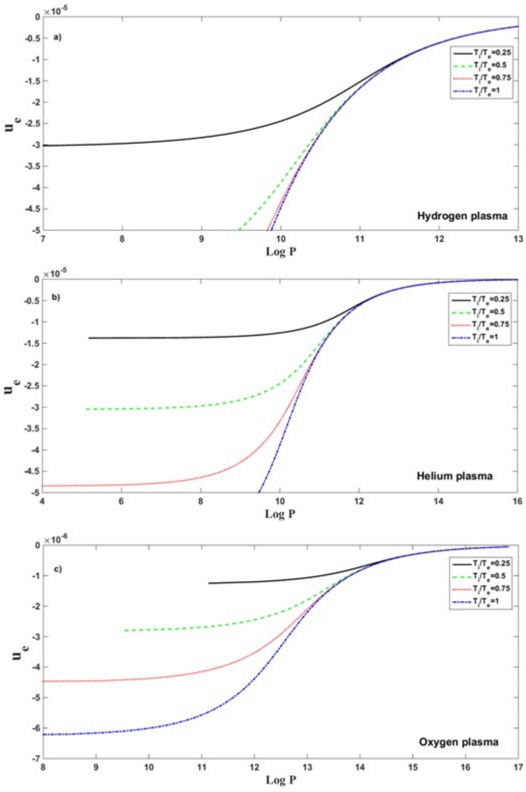

Figure 1: The variations of the electrical potential of the dust grain in terms of $\log P$ with temperatures changes ratios (ion-to-electron) less than unity in relativistic state (a) hydrogen plasma and (b) helium plasma and (c) oxygen plasma that $T_i/T_e = 0.25$ (solid line), $T_i/T_e = 0.5$ (dashed line), $T_i/T_e = 0.75$ (dotted line) and $T_i/T_e = 1$ (dash dotted line).

respectively. In these Figures, the scales are the same for easier comparison and curves, also are plotted for the different temperature ratios of ion-to-electron $T_i/T_e = 1.25$ (solid line), $T_i/T_e = 1.5$ (dashed line), $T_i/T_e = 1.75$(dotted line) and $T_i/T_e = 2$ (dash dotted line). Figure 2 indicates that by rising temperature of ion, the ions are closer to the ultra-relativistic range, and thus the dust electrical potential increases. In fact, not only the dust grain electrical potential increases with ion temperature, but also the difference between the dust electrical potential of oxygen, helium and hydrogen plasmas becomes more and more by increasing the ion temperature. In other words, it is clear that in a light plasma gas such as hydrogen, the change in the ion temperature causes low changes in the dust grain electrical potential changes than a heavy plasma gas such as oxygen in the relativistic regime. So that the higher ion temperature in a heavy plasma gas causes an increase in the electrical potential of the dust grain, and also the slope of the electrical potential changes increases, and the critical transition range to zero potential (and ordinary plasma from dusty plasma) becomes much smaller. The reason is the relationship between u_e and β in the cross-sectional equations 1 and 2. Changes of the ion temperature against ion mass for hydrogen plasma is not very important, but in heavy oxygen plasma, the role of ion temperature changes is dominate on its mass. On the other hand, it is indicated from the comparison of Figures 1 and 2 that in light plasmas such as hydrogen and helium plasmas when the temperature of the ion approaches the electron temperature and plasma is isothermal approximately, the difference between the electron temperature and the ion shows smaller variations on the electrical potential of the dust grain. The impact of plasma gas on the electrical potential of the dust grain for ion-to-electron temperature different ratios; equal to one, smaller than unity and bigger than one is plotted as a function of $\log P$ in Figures 3 $(a)-(c)$, respectively. The Figure is presented for plasma as hydrogen plasma (solid line), helium plasma (dashed line), carbon plasma (dash dotted line) and oxygen plasma (dotted line). It is indicated that by increasing of ion mass, the electrical potential of the dust grain decreases. In another way, by raising the ion mass, the effects of relativistic are decreased then an increase in the relativistic effects causes reduction in the electrical potential of the dust grain. In addition, from Figures 3 (a) and (c) compared to (b) it has been shown that, when in a isothermal plasma state, the determinate of the change in the ion temperature on the electric potential of the dust grain is difficult and in this situation, the role of mass is more colorful than the ion temperature especially in the lighter hydrogen and helium plasmas. Comparison of Figures 1 and 3 shows that with increase the mass of the plasma ions, the critical transition region to zero potential (from the dusty plasma to the normal plasma) occurs in larger amounts of P, i.e. under conditions that the ion density and the radius of dust grain are constant, a larger dust density is possible and accessible. In Figure 4, comparing between the electrical potential of dust grain in the relativistic and the non-relativistic plasma regimes is presented. In this Figure, plasma gas is assumed as oxygen plasma and the electron and the ion are isothermal. For a better comparison, the electrical potential of dust grain is plotted on a scale of 10^6 for the relativistic regime. Also, the curves are plotted for the relativistic plasma regime (solid line) and the non-relativistic plasma regime (dashed line) as a function of log P. The difference between the values of ue is due to the difference in the dust grain electrical potential normalized in the relativistic and the non-relativistic plasmas. The relationship of ue in the relativistic regime is obtained from equation 2, and in the non-relativistic regime it is obtained from $e\varphi_d/k_B T_i$ [39] and, the ratio of the dust electrical potential in the vertical axis for the relativistic to the non-relativistic regimes, is in order of 10^{-6}. As the Figure shows, in the relativistic regime, the electrical potential of the dust grain increases. It should be noted that the dependence of P is different in the relativistic and non-relativistic regimes according equations 20 and 24 and their the logarithmic ratio of the regime of relativistic to non-relativistic for P with the ion temperature of order $5eV$ is

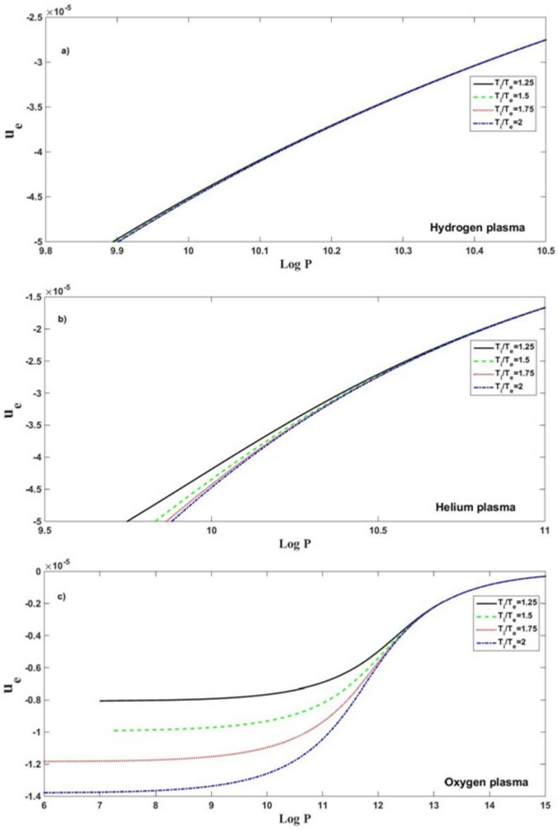

Figure 2: The variations of the electrical potential of the dust grain in terms of $\log P$ with temperatures changes ratios of ion-to-electron larger than unity in relativistic r$T_i/T_e = 1.5$ (dashed line), $T_i/T_e = 1.75$ (dotted line) and $T_i/T_e = 2$ (dash dotted line).

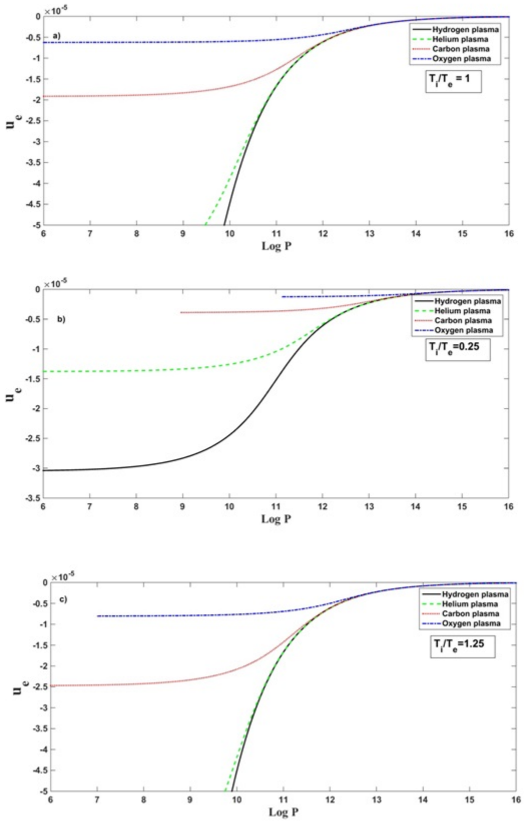

Figure 3: The relationship between dust electrical potential and log P for different kinds of plasma gases in temperature ratios (a) $T_i/T_e = 1$, (b) $T_i/T_e = 0.25$ and (c) $T_i/T_e = 1.25$in relativistic state that hydrogen plasma (solid line), helium plasma (dashed line), carbon plasma (dash dotted line) and oxygen plasma (dotted line).

approximately equal to 4. Also, in the non-relativistic regime, in addition to the density of the dust grain and ion, as well as the size of the radius of the dust grain, the ion temperature also plays a role in the amount of P, while in the relativistic state; the ion temperature does not exist in the P equation. According to the Figure 4, the comparison of the relativistic and the non-relativistic effects shows that the critical transition region is taken to the zero potential in the relativistic regime to larger values of P. In fact, the possible values for P are shifted to the larger amounts. In other words, when the ion temperature, Provided that the density of ion and dust grain size do not change, it is available larger values for the dust density in relativistic conditions and this value is much larger with respect to the difference in order 4 in the relationship between $P's$ in relativistic and non-relativistic regimes that is above mentioned. Also, comparing two regimes shows that in the relativistic regime, the slope of the transition region to zero potential (as well as reaching the ordinary plasma state from dusty plasma) is much more severe than the non-relativistic state.

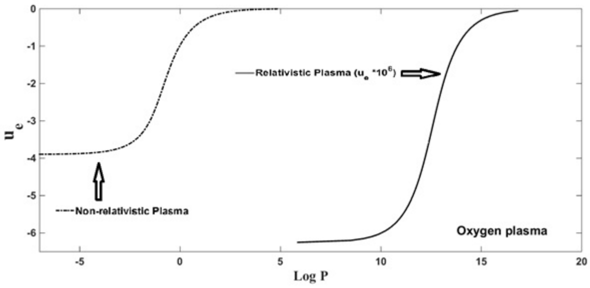

Figure 4: The relationship between dust electrical potential and log P for relativistic plasma regime (solid line) and non-relativistic plasma regime (dashed line), kind of plasma gas is oxygen plasma and the electrons and ions temperature is isothermal.

4 Summary and Conclusion

In this work, the dust charging process via the currents carried by electron and ion in the relativistic regime were calculated by using a kinetic model and the cross section of relativistic in the OLM theory. In particular, we employed a Maxwellian distribution of relativistic for ion and electron currents. Because of ions are much heavier than electrons, the current of ion to the dust grain is initially much smaller than the electron current and the dust grain is negative for the first time. In the next few moments, this leads to a larger ion current and a smaller electron current and ultimately, both currents are equal; from this moment in the literature, the behavior of the system is being investigated. Thus, we performed the calculations for finding the electrical potential of dust grain in charging process of dust grains in the relativistic regime. Then, we investigated the impact of Effective parameters on the electrical potential of dust grain by the numerical method. We showed that, by increasing the dust density, the dust grain electrical potential and the average of dust grain charge were decreased. Also, due to the increase in the dust grain density, the dust particle was as one of the components of ordinary plasma that have behavior of collective. Moreover, it was found that by increasing the ion density, the dust grain electrical potential and the dust charge were increased. It was indicated that, by increasing the ion temperature, the electrical potential of dust grain will be bigger and we found that the electrical potential of the dust grain in the oxygen plasma was smaller than the helium and hydrogen plasmas.

It was shown that, when ions were closer to the ultra-relativistic range, the dust electrical potential increased and the difference between the dust grain electrical potential in oxygen, helium and hydrogen plasmas became more and more. In addition to, it was obtained that in a light plasma gas such as hydrogen; the change in the ion temperature causes low changes in the dust grain electrical potential comparing a heavy plasma gas such as oxygen in the relativistic regime. It was shown when ions and electrons are isothermal approximately, the difference between the electron and ion temperature showed low variations on the electrical potential of dust grain in light plasmas such as hydrogen and helium. It was indicated that the role of mass was more colorful than the ion temperature especially in lighter hydrogen and helium plasmas. Furthermore, it was found that, the critical transition region and the possible values for P were shifted to the larger amounts and larger values for the dust density in the relativistic conditions were available. Finally, it was obtained that in the relativistic state, the amount of electrical potential of the dust grain increased and the slope of the transition region to zero was much more severe than the non-relativistic state.

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