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The Modified Form of the Gutenberg-Richter Law in Solar Flare Complex Network : Approach of Genetic Algorithm on the Thresholded Power-Law Behavior

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Abstract. The hybrid model approach is adopted to construct the solar flare complex network. The modified form of Gutenberg-Richter law is obtained as the frequency-magnitude distribution of the empirical data. The frequency-magnitude distributions of positive-definite data are sometimes observed to follow a power-law over several orders of size. There are reasons to the deviation of the frequency-magnitude distribution from an ideal power distribution. Among many alternative forms of the power-law function, we found that the threshold power-law is well fitted with solar empirical data at small values. A statistical method based on optimization of the χ^2-test by application of the genetic algorithm has been developed. Here, the analytical details of a method based on genetic algorithm is presented to calculate the parameters of the frequency-Magnitude distribution of the empirical data sets. This method estimates the best parameters of the threshold power-law function as the frequency-magnitude distribution of the empirical data, as well.

Keywords: Solar Flare, Complex Network, The Gutenberg-Richter Law, Genetic Algorithm, Frequency-Magnitude Distribution, Thresholded Power-Law behavior.

1 Introduction

Solar flares are huge explosions in the sun that release energy and light into space. Solar flares have tremendous effect on the weather earth. Occurrence time, location and energy structure of flaring events are almost unpredictable. Therefore, probabilistic methods are used to predict solar flares[1, 2, 3]. The structure, evolution, and topology of solar magnetic fields are influential parameters in the occurrence of solar flares [4, 5]. Solar flares have Spatio-temporal features with complex spectral [6].

Empirical laws have been investigated in solar flare complex network [7, 8]. Observational evidence provides us with the fact that both solar flares and earthquakes follow the same empirical laws [9]. The Gutenberg-Richter law highlights the complexity of solar flare networks [10].

An approach of hybrid model network developed for both the Gutenberg- Richter and Omori laws in earthquakes phenomena [11]. Various forms of the Gutenberg-Richter are proposed in earthquakes network. There are modifications of the Gutenberg-Richter distribution for large seismic moments [12, 13].

In the network approach, the Gutenberg-Richter law is proposed as the frequency-magnitude distribution of the empirical data. This distribution can be described as the cumulative distribution function of flare versus the size of the released energies [14]. The self-similarity property has been obtained in frequency-magnitude power-law distribution of complex network [15, 16, 17].

The study of experimental data in various fields of research such as physics, astronomy, geology, biology, economics and sociology has revealed the presence of power- law behavior in experimental distributions [18, 19, 20, 21, 22, 23]. Power-law behavior, also known as scale-free behavior, means that there is no characteristic size in the system. The scale-free distributions have an inverse dependence of the frequencies on the event sizes. In other words, bigger sizes events have less probability to occur [20].

Power distribution indicates the relationship between the frequency of events and their size. Frequency-Magnitude distribution of solar data often follows the power-law over the whole range of magnitudes [24]. The cumulative distribution function of flare versus the size can also be described by a q-exponential distribution with $q > 1$, which represents a nonextensive nature of solar flare complex network [25, 26, 7].

The maximum likelihood estimation in Bayesian framework and Genetic Algorithm (hereafter GA) would be suitable methods to calculate the parameters of frequency-magnitude distribution of empirical data. These methods estimate the best parameters of the threshold power-law function as the frequency-magnitude distribution of the experimental data [7, 23, 27].

In the present study, a statistical method based on optimization of the χ^2-test by application of the genetic algorithm is developed to study distributions with threshold power-law behavior. The solar data sets and network model are presented in Section 2. The techniques are introduced in Section 3. The results of applying these methods on solar data sets distribution are discussed. The conclusion is performed in Section 5.

2 Gutenberg-Richter Law

Gutenberg-Richter law is an empirical law that describes the relationship between magnitude and the number of events with size larger than a threshold value. It is expressed by the following relation

$$log_{10}(N_{>M}) = a - bM \tag{1}$$

The law was first introduced for seismic phenomena by Charles Francis and Richter-Benno Gutenberg in 1956, [10]. Gutenberg-Richter law expresses an exponential distribution for the size of events. This law actually states the statistics of solar events and earthquake as well. Gutenberg-Richter law expresses self-similarity in solar flare complex network.

This law is well explained by using the threshold power-law distribution model as well as the application of nonextensive statistical mechanics. A nonextensive modification of the Gutenberg-Richter law for seismic data was obtained in Fig. 2, [16]. They retrieved a modified version of Gutenberg-Richter with establishing Cumulative distribution of earthquakes versus magnitude for empirical seismic data of different regions. There are good Coincidence between the modified GR law (green solid line) and the original GR law (red dashed line) for magnitudes greater than 2.5, See Fig. 2.

3 Flare Data and Model

We used the information of 14395 solar flares data between 2006 January 1 to 2016 July 21.

Occurence time, event type, NOAA number, position of the sun surface, peak energy and total energy of each recorded flaring events are included in the archive. Scattering of the of 14395 solar flares data between 2006 January 1 to 2016 July 21 over the solar latitudes is shown in Fig. 1. The flare information is extracted from LMSAL web site: (`http://www.lmsal.com/solarsoft/latest-events-archive.html`). There is long-range correlation in the solar data time series [28, 29, 30, 31, 32, 33].

We introduce a hybrid model based on the Abe-Suzuki model (hereafter AS) and satisfying the visibility graph condition to construct the solar flare network [11, 28, 7]. According to hybrid model, the Abe -Suzuki approach is adopted for modeling the flare networks based on the spatial location and event time of solar flares [34, 35, 36]. The visibility graph is employed to the occurrence time and size of the flaring events [37, 38].

4 Approach of the χ^2 Function and Genetic Algorithm on Determination of Thresholded Power-Law Parameters

The ideal power-law distribution for the frequency (probability) of event sizes S is:

$$P(S) \propto S^{-\gamma}, \tag{2}$$

where γ is the power-law index. The threshold power-law distribution for a scale-free data set is described as

$$P(S)dS = P_0(S + S_0)^{-\gamma}dS, \tag{3}$$

where S_0 and P_0 are the threshold and normalization constants, respectively. The constant P_0 is given by:

$$P_0 = (\gamma - 1)[(S_1 + S_0)^{-\gamma+1} - (S_2 + S_0)^{-\gamma+1}]^{-1}. \tag{4}$$

The index of the power-law, which is often estimated by binning the data and fitting threshold power-law models to the frequency-magnitude distribution, can give an insight into the intrinsic complex and stochastic nature of the subject system [39].

There are reasons to the deviation of the frequency-magnitude distribution from an ideal power distribution.There are deviations from power-law in seismic moment distribution [40]. There are factors such as limited size effect and background contamination, which causes deviation from an ideal power-law distribution. Therefore, different forms of power-law function are proposed to prepared a better fit on empirical data [41, 42, 23]. The threshold power-law is well fitted with solar experimental data [24].

Various techniques have been obtained to infer the power-law index of power-law distributed data, and the range over which the power-law behavior applies. There are the following techniques for this purpose: The method of moments, graphical methods, linear least-square, and maximum likelihood estimation and genetic algorithm.

The method of moments approximately estimates the parameters of a given distribution based on simple assumptions e.g., linear relationships between the parameters [43]. It has been shown that this method can fail to accurately determine parameters for some estimation problems [44].

With graphical methods, the frequency-magnitude distribution of empirical data is constructed by binning the data range into smaller intervals and measuring the number of events in each interval (frequency). Then, the frequency-magnitude distribution is determined by minimizing the differences between the observational values and those of a model.

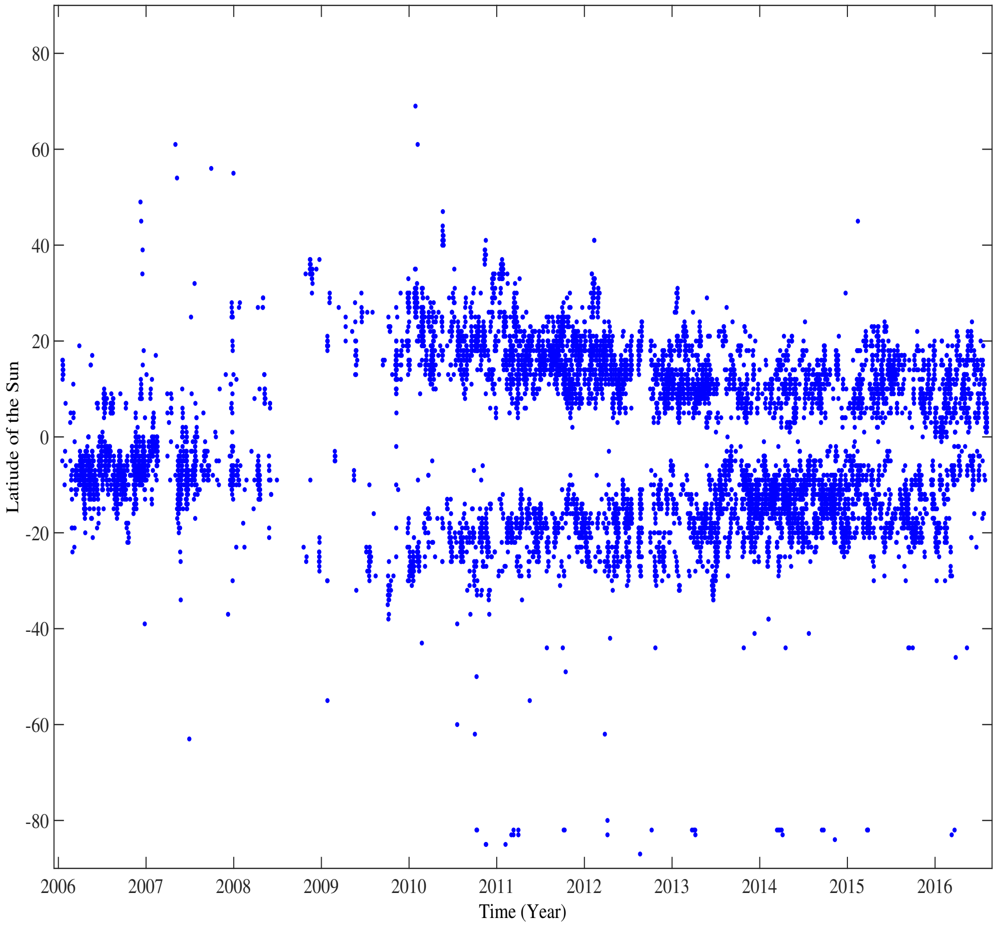

Figure 1: A portrait of scattering of the of 14395 solar flares data between 2006 January 1 to 2016 July 21 over the solar latitudes [28].

Least squares fits are not fullfiled by power-law distributions of empirical data. Therefore, they are not suitable to fit parameters of power-law distribution.

Maximum likelihood estimation (hereafter MLE) seeks the model parameters based on some prior information, including knowledge of the adopted model, and the distributions assigned to parameters as their lower and upper limits [39, 22, 23]. χ^2-test, is another class of graphical methods. χ^2-test is established in which the optimization of the χ^2-function is achieved by the application of genetic algorithm (hereafter GA).

In this paper, genetic algorithm (GA) has been presented as a trustworthy alternative to graphical methods. These estimators have asymptotically property. They apply to wide class of power-law data distributions.

We used the genetic algorithm to minimize the chi-square function to obtain the power-law exponent and threshold parameter [24].
The chi-square function is introduced by:

$$\chi^2 = \frac{1}{n_{bin} - 3} \sum_{i=1}^{n_{bin}} \frac{[P_{model}(S_i) - P_d(S_i)]^2}{\sigma_d^2}, \tag{5}$$

where n_{bin} is the number of bins, p_{model} is the theoretical PDF Eq. (3). The distribution of each bin for observational data obtained by p_d. Uncertainty of the observational data is expressed by σ_d as well. Using the genetic algorithm, we minimize chi-square function with regard to the K_0 and γ parameters in order to obtain best threshold fit. Method of genetic algorithm (GA) is applied to probability distribution function (PDFs) model.

GA is a probabilistic search algorithm which evaluates the optimal value of a function through an evolutionary process [45, 46, 47]. This algorithm iteratively revolutionizes a set of populations (mathematical objects) into a new population with regard to a relevant fitness value. This procedure is governed by the Darwinian principle of natural selection and uses three main types of genetic operators at each iteration: The selection method, the crossover method, and the mutation method.

Therefore, providing two initial sets of data as the lower and upper limits of the parameters (namely the search/genome space), the GA successively generates populations of points to obtain the high-quality solution to the subject optimization problem. This succeeding process ceases if the average relative change in the best fitness function value over several generations is less than or equal to a preset tolerance, or the arbitrary maximum number of generations is exceeded.

In order to calculate the uncertainty of each parameter (power-law index and threshold), the χ^2-test is repeatedly performed on re-sampled distributions using the bootstrapping technique [48]. The average of obtained values of the samples are considered as the value of the parameters.

5 Results

We develop hybrid model including the Abe-Suzuki and visibility graph condition for construct the solar flare network. A visibility graph condition applied to the occurrence time and magnitude of the events.

Gutenberg-Richter law is well explained by applying probability distribution function (PDFs model) versus the magnitude of the released energies by establishing hybrid model, See Fig. 3 and Fig. 4. This is performed for different resolution. This law is well explained by using the threshold power-law distribution model as well as the application of nonextensive statistical mechanics.

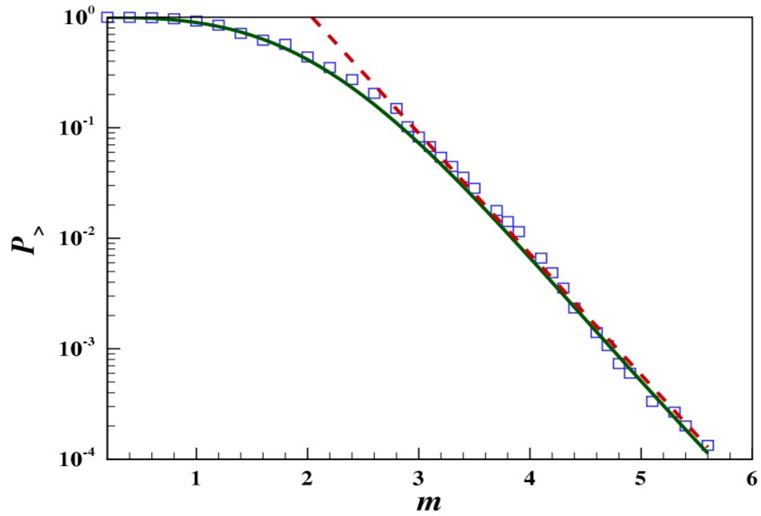

Figure 2: Cumulative distribution versus magnitude as frequency-magnitude distribution for seismic data. There are good Coincidence between the modified GR law (green solid line) and the original GR law (red dashed line) for magnitudes greater than 2.5, [16].

Figures 3-4 illustrates how the thresholded power-law model characterizes the studied data sets. As seen in the figures, the relevant PDFs alongside the model derived from χ^2-test with GA are displayed.Genetic algorithm (GA) is applied to minimize the χ^2-function of Equation (3) in order to find the parameters describing the frequency-size distribution (Red dashed line in Fig. 3 and Fig. 4). The best values of parameters of the threshold power-law distribution model (threshold constant and exponent degree) were calculated by the genetic algorithm.

Performing the χ^2-test the power-law index and the threshold obtained are $\gamma = 3.91$ and $S_0 = 6.74 \times 10^{-5}$, $\gamma = 3.33$ and $S_0 = 4.52 \times 10^{-5}$, $\gamma = 2.75$ and $S_0 = 2.21 \times 10^{-5}$, $\gamma = 2.81$ and $S_0 = 3.08 \times 10^{-5}$ for n=44, n=62, n=74, and n=88, respectively.

Usually, due to the weakness of observation tools and data monitoring, we are unable to record the location of small events. This will distort the distribution of real experimental data from a power-law. Threshold power distribution would be a relevant candidate to fit the data distribution.

As shown in the figures 3-4, the frequency-magnitude distribution of solar events for magnitude greater than 10^{-5} have power-law behavior. This states the relation between occurrence probability of solar events and magnitude greater than a threshold value. As the magnitude of solar events increases, their frequency decreases. Consequently, the relationship between the magnitude and frequency of solar events is inversely related. The logarithmic relationship between the magnitude and frequency of solar events is as the power-law.

In summary, a statistical method based on optimization of the χ^2-test by application of the genetic algorithm (GA) is developed. Threshold power-law distribution model is used to fit the solar empirical data in the Fig. 3 and Fig. 4.

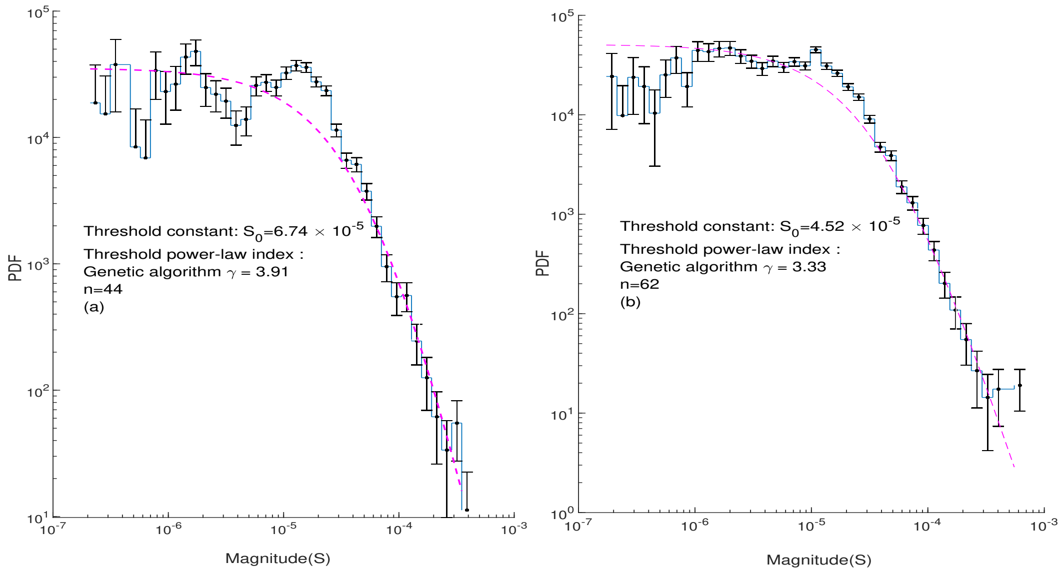

Figure 3: PDFs versus the magnitude of the released energies as the frequency-magnitude distribution with a thresholded power-law. Performing the χ^2-test with GA, the parameters obtained are $\gamma = 3.91$ and $S_0 = 6.74 \times 10^{-5}$, $\gamma = 3.33$ and $S_0 = 4.52 \times 10^{-5}$ for n=44 and n=62, respectively. The threshold power-law index and the threshold constant are calculated using the χ^2-test with GA (red dashed line).

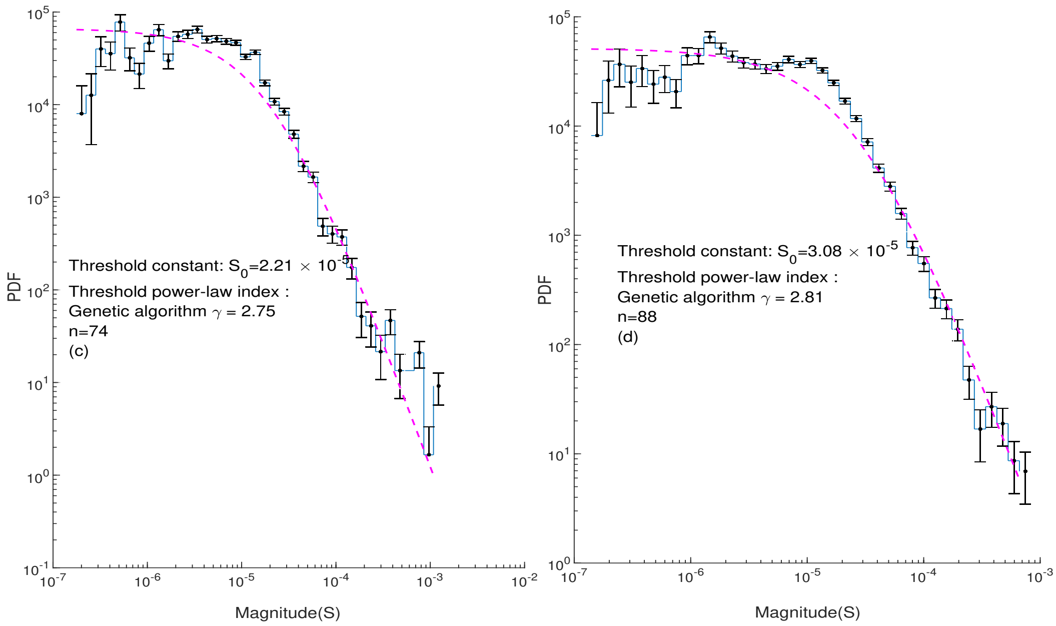

Figure 4: PDFs versus the magnitude of the released energies as the frequency-magnitude distribution with a thresholded power-law. Applying the χ^2-test with GA, the parameters obtained are $\gamma = 2.75$ and $S_0 = 2.21 \times 10^{-5}$, $\gamma = 2.81$ and $S_0 = 3.08 \times 10^{-5}$ for n=74 and n=88, respectively. The threshold power-law index and the threshold constant are calculated using the χ^2-test with GA (red dashed line).

6 Conclusion

In the study, using hybrid model of solar flare complex network, a modified form of the Gutenberg-Richter magnitude-frequency relation for flaring events was obtained. A statistical method was developed to help users model scale-free simulated and empirical distributions. Due to the hardware limitations of solar monitoring tools and related computing tools, it is not possible to record position, time, and energy small flare in the timeseris data, accurately. Therefore, small flares (for example: flare type A) with registered position are very rare in the timeseries. As a result, the thresholded power-law model can gives better interpretation of the studied distributions compared with a simple power-law. A graphical approach based on minimization of the χ^2-function using the GA was presented. The χ^2-test also provided valid estimations for the model parameters. This technique estimated best parameters describing data sets with the thresholded power-law behavior. The established method was examined over the solar flare data empirical distributions which depart from the ideal power-law behavior. Finally, we conclude that the established technique is suitable to investigate distributions suffering from truncations due to the sampling effects at small sizes, background contamination, etc. with the thresholded power-law behavior. Furthermore, the developed routines are easy to use and capable of efficiently analyzing distributions departing from the ideal power-law behavior.

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