Iranian Journal of Astronomy and Astrophysics
Vol. 6, No. 2, Autumn 2019

DOI:10.22128/ijaa.2019.376.1073

Iranian Journal of Astronomy and Astrophysics

Cherenkov Radiation in Uniaxial and Biaxial Anisotropic Media

Marjan Jafari
Department of Physics, Faculty of Science, Imam Khomeini International University, 34148 - 96818, Qazvin, Iran
email: Jafary.marjan@gmail.com, m.jafari@sci.ikiu.ac.ir

Abstract. A quantum mechanic approach is presented for Cherenkov radiation of a moving atom in uniaxial and biaxial anisotropic medium. A moving particle in a medium emits Cherenkov radiation when its speed is larger than the phase velocity of light in the medium. The electromagnetic field is quantized in the anisotropic media, using maxwell and related constitute equations, phenomenologically. The Cherenkov radiation for a moving charged particle in the anisotropic medium with arbitrary real permittivity tensor is obtained. The dielectric structure can be changed the Cherenkov radiation. For uniaxial media, we give a close form of the Cherenkov radiation for arbitrary dipole alignment. In this manner, the Cherenkove radiation of the biaxial media is calculated.. It is shown that Cherenkov radiation in an isotropic medium can be obtained from the form of an anisotropic medium.

Keywords: Biaxial media, Cherenkov Radiation, Uniaxil medai.

1 Introduction

Cherenkov radiation is one of the problems in light- matter interaction that can be opened new ideas in quantum electrodynamic. A moving particle in a medium emits Cherenkov radiation when its speed be larger than the phase velocity of light in the medium [1]. The Cherenkov radiation is the radiation with a continuous spectrum and specific angular distribution. The Cherenkov radiation was observed experimentally by Cherenkov, and the first formulation of this radiation was presented by Ginzburg and frank [2].
The Cherenkov effect has been used in different systems since the first discovery and leads to discoveries in high energy particle physics, cosmic ray physics, astrophysics, nonlinear phase-matched system [3, 4] to biomedical imaging[5], etc [6, 7, 8]. Discoveries in the Cherenkov effect, such as backward Cherenkov radiation [9], is observed in metamaterials [10], photonic crystals [11] and graphene [12]. This new wide range of applications is attracted by more scientists to study the theory of Cherenkov radiation [13, 14].
A neutral moving particle with constant velocity emits because of the fluctuating dipol moment. The energy of radiation in the neutral particle is determined by a change in the kinetic energy of the particle, and this comes from p-polarized electromagnetic waves. It has to mention that two types of radiation are possible if the particle has an internal degree as freedom. In particular, an explicit expression for the Cherenkov radiation in general anisotropic media has not been presented, while for isotropic media, this is a well studied problem.
The anisotropic media cannot be described by a scalar permittivity and permeabilitty and they have different responses to the external electric and magnetic fields according to the direction. Therefore, the permittivity and the permeabilitty have tensorial forms in the

anisotropic media. Anisotropic media are in one of two forms, uniaxial or biaxial, depending on whether they have a single optical axis or more than one.
Metamaterials and photonic crystals are received more attention because of their optical properties. These materials have negative permittivity and permeability. The studies show that the properties of negative refraction can occure in some uniaxially anisotropic media [15]. The optical properties of uniaxially anisotropic media are essential for many applications, such as liquid crystals. Other kind of media is biaxially anisotropic media that behave differently than isotropic and anisotropic media in interaction with light. The biaxial materials are twofold, one natural media with biaxial properties and another handmade biaxial materials like left-hand materials. Because of the compelet application of these types of optical materials, there is an interest in studying the light radiation in anisotropic media.
In this paper, we consider a particle of mass m and charge e moving in an anisotropic medium in which the media is charactrized with arbitrary real permittivity tensor. We quantize the electromagnetic field inside the medium and use this to obtain the Cherenkov radiation. For uniaxial media, we give a close form of the Cherenkov radiation for arbitrary dipole alignment. In this manner, the Cherenkove radiation of the biaxial media is calculated.
The paper is organized as follows: In section II, we present the phenomenological quantization of the electromagnetic field in the presence of an anisotropic media. In section III, we use this to drive the Cherenkov radiation for an moving charged particle in the anisotropic medium with arbitrary real permittivity tensor. In Section IV, the close form of the Cherenkov radiation is obtained for the uniaxial medium and the biaxial media. A conclusion is presented in section V.

2 Quantum electrodynamic

An anisotropic medium in the presence of the electromagnetic field is considered. The electromagnetic response of the medium is characterized by electric permittivity and magnetic permeability. By assuming that the permeability of the medium is unit, the anisotropic dielectric medium is described by a permittivity matrix $\bar{\bar{\varepsilon}}$. In the following, the system is considered in a coordinate in which $\bar{\bar{\varepsilon}}$ is diagonal. So

$$\bar{\bar{\varepsilon}} = \begin{pmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix} \tag{1}$$

We use the macroscopic maxwell equations and constitution relations to describe the propagation of the electromagnetic waves in the medium. The maxwell's equations are

$$\nabla.\mathbf{B}(r,t) = 0, \tag{2}$$

$$\nabla.\mathbf{D}(r,t) = 0 \tag{3}$$

$$\nabla \times \mathbf{E}(r,t) = -\frac{\partial \mathbf{B}(r,t)}{\partial t} \tag{4}$$

$$\nabla \times \mathbf{H}(r,t) = \frac{\partial \mathbf{D}(r,t)}{\partial t} \tag{5}$$

E is the electric field, D is the displacement field, B is the magnetic flux density and H is the magnetic field. The constitution relations are

$$\mathbf{D}(r,t) = \varepsilon_0 \bar{\bar{\varepsilon}} \mathbf{E}(r,t) + \mathbf{P}_N(r,t) \tag{6}$$

$$\mathbf{B}(r,t) = \mu_0 \mathbf{H}(r,t) \tag{7}$$

$\hat{P}_N$ is the noise polarization operator associated with electric loss due to material absoption and μ_0 is the permeability of free space. The wave equation for the electric field is obtained by the maxwell equations,as follows

$$\nabla \times \nabla \times \mathbf{E}(r,t) + \mu_0 \bar{\bar{\varepsilon}} \frac{\partial^2 \mathbf{E}(r,t)}{\partial t^2} = \frac{\partial^2 \mathbf{P}_N(r,t)}{\partial t^2} \tag{8}$$

Here, the electric field is defined in terms of the vector potential as $\frac{\partial}{\partial t}\mathbf{A}(r,t)$. So, the vector potential equation is

$$\nabla \times \nabla \times \mathbf{A}(r,\omega) - \omega_{k\lambda}^2 \bar{\bar{\varepsilon}} \mathbf{A}(r,\omega) = \mu_0 \omega_{k\lambda}^2 \mathbf{P}_N(r,\omega) \tag{9}$$

where $\mathbf{A}(r,\omega) = \int dt \mathbf{A}(r,t) e^{i\omega_{k\lambda} t}$. We quantize the system by introducing the canonical bosonic creation and annihilation operators $\hat{a}_{k\lambda}^\dagger$ and $\hat{a}_{k\lambda}$ that satisfy the commutation relation as follows

$$[\hat{a}_{k\lambda}, \hat{a}_{k'\lambda'}^\dagger] = \delta(k - k')\delta_{\lambda\lambda'} \tag{10}$$

So, the noise polarization can be described in terms of the bosonic operators as

$$\hat{\mathbf{P}}_N(r,\omega) \to \sqrt{\frac{\hbar\omega_{k\lambda}}{2(2\pi)^3 \mathbf{e}_{k\lambda}.\bar{\bar{\varepsilon}}\mathbf{e}_{k\lambda}}} \hat{a}_{k\lambda}^\dagger, \tag{11}$$

Where $\mathbf{e}_{k\lambda} = \mathbf{E}_{k\lambda}/|\mathbf{E}_{k\lambda}|$ are eigenvectors of the electric field. So, the vector potential operator can be written as follows

$$\begin{aligned}\hat{\mathbf{A}}(r,t) \quad &= \int d^3\mathbf{k} \sum_\lambda \mathbf{e}_{k\lambda} \sqrt{\frac{\hbar\omega_{k\lambda}}{2(2\pi)^3 \mathbf{e}_{k\lambda}.\bar{\bar{\varepsilon}}\mathbf{e}_{k\lambda}}} \frac{1}{\omega_{k\lambda}} \\ &\times (\hat{a}_{k\lambda} e^{i(k.r - \omega_{k\lambda} t)} + \hat{a}_{k\lambda}^\dagger e^{i(k.r - \omega_{k\lambda} t)}),\end{aligned} \tag{12}$$

and the electric field is

$$\begin{aligned}\hat{\mathbf{E}}(r,t) \quad &= \int d^3\mathbf{k} \sum_\lambda \mathbf{e}_{k\lambda} \sqrt{\frac{\hbar\omega_{k\lambda}}{2(2\pi)^3 \mathbf{e}_{k\lambda}.\bar{\bar{\varepsilon}}\mathbf{e}_{k\lambda}}} \\ &\times (\hat{a}_{k\lambda} e^{i(k.r - \omega_{k\lambda} t)} + \hat{a}_{k\lambda}^\dagger e^{i(k.r - \omega_{k\lambda} t)}),\end{aligned} \tag{13}$$

the magnetic field can be obtained easily . It can be seen that in an anisotropic medium, frequency is dependent on the polarization and the diraction of k. It can be showen that the Hamiltonian of the system consists of the medium and electromagnetic field, which is diagonal in terms of ladderar operators [16].

3 Cherenkov radiation

In this section, the radiation intensity of the moving particle is calculated. Consider a charged particle moving at velocity V in the presence of the anisotropic dielectric media. The Hamiltonian of the free charged particle, the total field (an interacting system consists of an electromagnetic field and anisotropic dielectric medium is named total field) and interaction of the charge particle and total field, is as follows

$$H = H_p + H_f + H_{int}, \tag{14}$$

where

$$H_p = \frac{\hat{\mathbf{p}}^2}{2m}, \tag{15}$$

$$H_f = \frac{1}{2}\int d^3r[\hat{\mathbf{E}}(r,t).\hat{\mathbf{D}}(r,t) + \hat{\mathbf{H}}(r,t).\hat{\mathbf{B}}(r,t)] \tag{16}$$

$$= \int d^3k \sum_{\lambda} \hbar\omega_{k\lambda}(\hat{a}^{\dagger}_{k\lambda}\hat{a}_{k\lambda} + \frac{1}{2}),$$

$$H_{int} = -\frac{q}{m}\hat{\mathbf{P}}.\hat{\mathbf{A}}(r,t), \tag{17}$$

where $\hat{\mathbf{P}}$ is the momentum operator of the moving particle. The Hamiltonian of the interaction is written in the dipol approximation. The eigenstate of the momentum of the particle, before the emission of the photon is $|q\rangle$, and after the emission is $|q-k\rangle$. $|\{0\}\rangle$ is the vacuum state of the total field and $|\{1_{k\lambda}\}\rangle$ is the excited state of the total field with $\hat{a}^{\dagger}_{k\lambda}|\{0\}\rangle = |\{1_{k\lambda}\}\rangle$. The eigenstate of the combined system is the direct producte of the eigenstates of the particle and field. The frequencies of the particle before and after emission are respectively $\omega_A = \frac{|E_q|}{\hbar}$ and $\omega_B = \frac{|E_{q-k}|}{\hbar}$ where $\tilde{\omega}_A = \omega_B - \omega_A$.
In this step, the transient probability amplitude of the system from the initial to the final state is obtained.

$$\Gamma_{q\to q-k} = \frac{2\pi}{\hbar}\left|\langle 1_k|\langle q-k| H_{int}|q\rangle|0\rangle\right|^2 \delta(\omega_{k\lambda} - \tilde{\omega}_A) \tag{18}$$

The Dirac δ function displays the conservation of energy. To calculate the Cherenkov radiation intensity of the moving atom in an anisotropic dielectric medium, multiply Transient probability amplitude by $\hbar\omega$ and integrate over k and ω, as follows

$$\frac{dW}{dt} = \int d^3k \int d\omega\hbar\omega\Gamma_{q\to q-k} \tag{19}$$

by substituting (17) in (18) and using (19), we have

$$\frac{dW}{dt} = \frac{\hbar}{8\pi^2}\int d^3k \sum_{\lambda} \frac{\left|\hat{\mathbf{P}}.\mathbf{e}_{k\lambda}\right|^2}{\mathbf{e}_{k\lambda}.\bar{\bar{\varepsilon}}\mathbf{e}_{k\lambda}}\delta(\omega_{k\lambda} - \tilde{\omega}_A) \tag{20}$$

Where the momentum of the particle is $\mathbf{p} = (p_1, p_2cos\phi, p_2sin\phi)$.

4 Examples

4.1 Uniaxial media

An essential class of the anisotropic medium is the uniaxial media where the permitivity of the medium in two directions is the same. Uniaxial anisotropic media have a single optic axis that defines a unique direction in which light of all polarizations propagates with the same refractive index. For all other directions, waves analyze into two orthogonally polarized elements with different refractive indices. One element is the ordinary wave because it propagates in the usual way. Its refractive index is independent of the direction of the wave vector k, and energy flows in the same direction as k. The other element is the extraordinary wave. Its refractive index varies with the angle between k and the optic axis [17].

$$\bar{\bar{\varepsilon}} = diag(\varepsilon_1, \varepsilon_2, \varepsilon_2) \tag{21}$$

The eigenvectors of the electric field in uniaxial media are as follows [18]

$$\mathbf{e}_{ko} = \begin{pmatrix} 0 \\ -k_3 \\ k_2 \end{pmatrix} \tag{22}$$

and

$$\mathbf{e}_{ke} = \begin{pmatrix} -\varepsilon_2(k_2^2 + k_3^2) \\ \varepsilon_1 k_1 k_2 \\ \varepsilon_1 k_1 k_3 \end{pmatrix} \tag{23}$$

with corresponding frequencies

$$\omega_{ko} = \frac{ck}{n_o} = \frac{k}{\mu_0\varepsilon_2}, , \quad \omega_{ke} = \frac{ck}{n_e} = \sqrt{\frac{\mathbf{k}.\bar{\bar{\varepsilon}}\mathbf{k}}{\mu_0\varepsilon_1\varepsilon_2 k^2}}k, \tag{24}$$

where n_o and n_e are the ordinary and extraordinary refractive indices [17]. The media with $n_o < n_e$ are named positive uniaxial and with $n_o > n_e$ are named negative uniaxial. It has to mention the $n_e(\theta) = \varepsilon_0^{-1/2}(cos^2\theta/\varepsilon_2 + sin^2\theta/\varepsilon_1)^{-1/2}$ is the refractive index of the extraordinary wave varies with θ. θ is angle $\mathbf{k}$ and x-direction. n_e is an extremum of $n_e(\theta)$. To obtain the Cherenkov radiation, we choose $\phi = 0$ for momentum orientation without losing the generality. By substituting $dk = n_\lambda d\omega_{k\lambda}/c$, in sperical coordinates $\mathbf{k} = k(cos\theta, sin\theta cos\phi, sin\theta sin\phi)$, we have

$$\frac{dW}{dt} = \frac{\tilde{\omega}_A^2}{8\hbar\pi^2}\int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sum_\lambda (\frac{n_\lambda}{c})^3 \frac{|\mathbf{P}.\mathbf{e}_{k\lambda}|^2}{\mathbf{e}_{k\lambda}.\bar{\bar{\varepsilon}}\mathbf{e}_{k\lambda}} sin\theta \tag{25}$$

The Cherenkov radiation of the ordinary refractive indices for $\omega_{k0} = \frac{ck}{n_0} = \frac{k}{\sqrt{\mu_0\varepsilon_2}}$ is as follows

$$\frac{dW_o}{dt} = \frac{p_2^2\tilde{\omega}_A^2}{8\pi^2}\int_0^{2\pi} d\phi \int_0^{\phi} d\theta \frac{sin^3\theta}{\varepsilon_2}(\sqrt{\mu_0\varepsilon_2})^3 \tag{26}$$

where the p_1 does not contribute in this part of the Cherenkov radiation.

$$\frac{dW_o}{dt} = \frac{p_2^2\tilde{\omega}_A^2\mu_0^{3/2}\varepsilon_0^{1/2}}{4\pi}\sqrt{\varepsilon_2} \tag{27}$$

In the extraordinary waves, all parts of the momentum are contributed; so, for ω_{ke} we have

$$\begin{aligned} \frac{dW_e}{dt} &= \frac{\tilde{\omega}_A^2}{8\pi^2}\int_0^{2\pi} d\phi \int_0^{\pi} d\theta \frac{(\mu_0\varepsilon_1\varepsilon_2)^{3/2} sin\theta}{\varepsilon_1\varepsilon_2(\varepsilon_2 sin^2\theta + \varepsilon_1 cos^2\theta)^{5/2}} \\ &\quad \times [p_2^2\varepsilon_1^2 cos^2\theta cos^2\phi + p_1^2\varepsilon_2^2 sin^2\theta] \\ &= \frac{\tilde{\omega}_A^2}{3\pi}\mu_0^{3/2}(\frac{p_2^2\varepsilon_1 + 4p_1^2\varepsilon_2}{4\sqrt{\varepsilon_2}}) \end{aligned} \tag{28}$$

So the total Cherenkov radiation is as follows

$$\frac{dW}{dt} = \frac{dW_o}{dt} + \frac{dW_e}{dt} = \frac{\tilde{\omega}_A^2\mu_0^{3/2}}{3\pi}(\frac{\varepsilon_1 + 3\varepsilon_2}{4\sqrt{\varepsilon_2}}p_2^2 + \sqrt{\varepsilon_2}p_1^2) \tag{29}$$

According to the above relation, for a particle with momentum parallel to the ε_1 axis, the Cherenkov radiation is similar to the Cherenkov radiation in isotropic media. Also, in the case that $\varepsilon_1 = \varepsilon_2$, the relation (29) reduces to the cherenkove radiation isotropic media.

4.2 Biaxial media

The propagation of light in biaxial media is more complicated than other media. In biaxial media all three elements of the dielectric tensor, that presented by diagonal matrix, are different [19].

$$\bar{\bar{\varepsilon}} = \begin{pmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_2 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix} \tag{30}$$

The eigenvectors are

$$\mathbf{e}_{k\pm} = \begin{pmatrix} k_1/(\varepsilon_1 - \varepsilon_k) \\ k_2/(\varepsilon_2 - \varepsilon_k) \\ k_3/(\varepsilon_3 - \varepsilon_k) \end{pmatrix} \tag{31}$$

and angular frequencies are

$$\omega_{k\pm} = \frac{ck}{n_\pm} = \frac{k}{\sqrt{\mu_0 \varepsilon_{k\pm}}} \tag{32}$$

with

$$\varepsilon_{k\pm} = \frac{2\varepsilon_1\varepsilon_2\varepsilon_3}{t_k \pm s_k}, \tag{33}$$

$$t_k = \frac{1}{k^2}\mathbf{k}.\bar{\varepsilon}(Tr\bar{\bar{\varepsilon}}\mathbf{I} - \bar{\bar{\varepsilon}})\mathbf{k}, \quad s_k = \sqrt{t_k^2 - \frac{4}{k^2}\varepsilon_1\varepsilon_2\varepsilon_3\mathbf{k}.\bar{\varepsilon}\mathbf{k}} \tag{34}$$

With these relations, the integration over $\mathbf{k}$ can be performed similarly to the uniaxial media. So,

$$\frac{dW}{dt} = \frac{\tilde{\omega}_A^2}{8\pi^2}\int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sum_{\lambda\pm} \frac{|\mathbf{p}.\mathbf{e}_{k\lambda}|^2}{\mathbf{e}_{k\lambda} - \bar{\bar{\varepsilon}}\mathbf{e}_{k\lambda}}(\mu_0\varepsilon_k)^{3/2} sin\theta. \tag{35}$$

5 Conclusion

We drive the quantized electromagnetic field operators for a system consist of a moving atom in an anisotropic media to calculate the Cherenkove radiation in uniaxial and biaxial media. It is obvious that the anisotropic nature of the medium does not change Cherenkov angle.

References

[1] Cherenkov, P. A. 1934, C. R. Ac. Sci. U.S.S.R., 8, 451.

[2] Tamm, I., and Frank, I. 1937, C. R. Ac. Sci. U.S.S.R., 14, 109.

[3] Sheng, Y., Wang, W., Shiloh, R., Roppo, V., Kong, Y., Arie, A., Krolikowski, W. 2011 , Appl. Phys. Lett., 98, 241114.

[4] D'Amico, C., Houard, A., Franco, M., Prade, B., Mysyrowicz, A., Couairon, A., and Tikhonchuk, A. T. 2007, Phys. Rev. Lett. 98, 235002.

[5] Shaffer, T. A., Pratt, E. C., Grimm, J. 2017, Nat. Nanotechnol., 12, 106.

[6] Bogdanov, O., Fiks, E., and Pivovarov Y. 2012, J. Exp. Theor. Phys., 115, 392.

[7] Volotka, A. V., Glazov, D. A., Plunien, G., and Shabaev, V. M. 2013, Ann. Phys, 525, 636.

[8] Calajó, G., Rabl, P. 2017, Phys. Rev., A 95, 043824.

[9] Veselago, V. G. 1968, Sov. Phys. Usp. 10, 509.

[10] Liu, F., Xiao, L., Ye. Y, Wang, M., Cui, K., Feng, X., and Zhang, W. 2017, Nat. Photonics, 11, 289.

[11] Luo, C., Ibanescu, M., Johnson, S. G., Joannopoulos, J. D. 2003, Science, 299, 368.

[12] Xi, S., Chen, H., Jiang, T., Ran, L., Huangfu, J., Wu, B. I., Kong, J. A., Chen, M. 2009, Phys. Rev. Lett., 103, 194801.

[13] Kheirandish, F., Amooghorban, E. 2010, Phys. Rev. A, 82.4, 042901.

[14] Roques Carmes, C., Rivera, N., Joannopoulos, J. D., Soljacic, M., Kaminer, I. 2018, Phys, Rev. X, 8(4), 041013.

[15] Yao, J., Liu, Z. W., Liu, Y. M., Wang, Y., Sun, C., Bartal, G., Stacy, A. M., Zhang, X. 2008, Science, 321, 930.

[16] Suttorp, L. G., Van Wonderen, A. G. 2004, EPL, 67.5 ,766.

[17] New, G. H. C. 2013, European Journal of Physics, 34, 1263.

[18] Messinger, A., Westerberg, N., Barnett, S. M. 2020, arXiv preprint arXiv:2004.11107.

[19] Braat, J., Torok, P. 2019, Imaging Optics, Cambridge Univ. Press, Cambridge.