Iranian Journal of Astronomy and Astrophysics
Vol. 6, No. 1, Spring 2019

DOI:10.22128/ijaa.2019.335.1051

Iranian Journal of Astronomy and Astrophysics

The Braneworld Stability and Large-scale Correction in Graphene Like Background

Zahra Zali[1] · Jafar Sadeghi[2] · Behnam Pourhassan[3]

[1] Department of Physics, Islamic Azad University, Ayatollah Amoli Branch, P .O .Box 678, Babolsar, Amol, Iran;
email: st.z.zali@iauamol.ac.ir

[2] Department of Physics, University of Mazandaran, P .O .Box 47416-95447, Babolsar, Iran;
email: pouriya@ipm.ir

[3] School of Physics, Damghan University, Damghan, 3671641167, Iran;
email: b.pourhassan@du.ac.ir

Abstract. In this work, we consider a graphene like background in braneworld scenario which is one of the interesting models in particle physics and cosmology. Indeed, this paper is an application of holography in condensed matter physics. We use the geometric form of the potential which helps us to obtain field equations and solve it to obtain the energy spectrum. In that case, we calculate superpotential and energy density of this model. Thermodynamical study of this model suggests that the entropy should be modified due to the large-scale corrections. An important point is the stability of this model in braneworld scenario. We study model stability by two separated methods. We calculate the heat capacity, Gibbs and Helmholtz free energy to analyze thermodynamics stability; then, we consider small perturbations in graphene like metric background to investigate gravitational stability. We find that the model is completely stable unless in very low temperatures where the graphene structure may break. Hence, we confirm that our braneworld model is thermodynamically stable and also small perturbations yields to an oscillation around the equilibrium point.

Keywords: Braneworld; Perturbations; Graphene; Thermodynamics.

Holographic principles connected various branches of physics. One of the recent and interesting one is the connection between condensed matter and high energy physics [2, 3]. An interesting phenomena which can study by holographic principles is superconductivity, especially in high temperatures [4, 5, 6, 7]. Another interesting issue is the construction of graphene like behaviors by using a quantum field in a curved space-time [8, 9, 10]. The fact is that graphene behaves as a quantum relativistic-like Dirac massless field which is constructed by two-dimensional sheet of carbon atoms. Importance of graphene, as two-dimensional object, is for example describing unusual quantum Hall effect which is a fundamental effect to design an electronic device [11, 12, 13, 14].

An important phenomenon is an electron motion in graphene space, which is relativistic motion, hence it is useful model to test relativistic quantum theory. In that case, an electron moving in graphene interpreted as a massless charged fermion in two dimensional space. The fact is that all massless elementary particles are electrically neutral (like photon). Hence, graphene can relate condensed matter and high energy limits of elementary particle physics. As curvature influences the dynamics of quantum particles, it can be used to manipulation of the electronic properties of two-dimensional materials [15] which is testable process.

An important backgrounds to construct graphene-like behavior is BTZ black hole which is related to three dimensional anti de sitter space-time [16, 17, 18], where the light velocity replaced by the Fermi velocity. These tree dimensional backgrounds can holographically dual

of a two-dimensional graphene. We should note that the holographic dual of a graphene is a curvature background which is a good reason to think about geometric potential. In another word, the curvature effects may be induced in a deformed sheet of graphene.
Another important backgrounds which can be considered as holographic dual of graphene may be found in braneworld scenario of cosmology which deals with a curved space-time [19]. Dimensional reduction process can reduce five-dimensional metric of braneworld model to a two-dimensional one including an extra dimension, which can be considered as graphene like metric. The graphene like behavior in the context of braneworld scenario already studied by the Refs. [20, 21]. Recently, we considered the motion of particle in curved graphene like metric background [22] and obtained the geometric potential, wave function and energy spectrum. We also briefly discussed about the thermodynamics and found that the entropy modified due to the large scale correction. Such correction appears as logarithmic term as well as quantum correction. Now, we would like to study the stability of this model by two separate ways. First of all, we study thermodynamical stability, then study gravitational stability.
This paper is organized as follows. In the section 2, we introduce our model which is holographically dual of the graphene (adaptive model of graphene). In the section 3, we study the thermodynamics and study thermal stability. In the section 4, we consider the small perturbation of the background metric to study the gravitational stability. Finally, in the section 5, we give conclusion and summary of results.

1 Braneworld scenario

In order to investigate the particle motion in a graphene like background, we have to consider the graphene in a curved space with Lobachevsky geometry [23, 24], which is given by

$$ds^2 = \frac{r_0^2}{\bar{y}^2}\left(d\bar{x}^2 + d\bar{y}^2\right), \tag{1}$$

where $\bar{x}$ and $\bar{y}$ are the coordinates of the Lobachevsky geometry in the upper half - plane ($\bar{y} > 0$) [23, 24]. Adding time part one can obtain the full background of graphene curve as

$$ds^2_{graphen} = \frac{r_0^2}{\bar{y}^2}\left[\frac{\bar{y}^2}{r_0^2}dt^2 - d\bar{x}^2 - d\bar{y}^2\right] \tag{2}$$

In order to investigate the motion of particle in such background, we choose the following change of variable [9]

$$\bar{x} = e^{\frac{v_f v}{r_0}} \tanh\frac{u}{r_0}, \qquad \bar{y} = e^{\frac{v_f v}{r_0}} \frac{1}{\cosh\frac{u}{r_0}}, \tag{3}$$

where u and v are new coordinates, and the light speed replaced by Fermi velocity (v_f) [10]. In that case, the background metric in terms of measurable objects u and v can be written as

$$ds^2 = du^2 + e^{2A}(\frac{u}{u_0})dv^2, \tag{4}$$

where u may denotes the extra dimension and

$$A = \frac{1}{2}\ln v_f^2 \cosh^2(\frac{u}{u_0}), \tag{5}$$

is warp function which is related to the space curvature. Also, $v_f \approx 0.003c$ is the Fermi velocity. It is indeed experimental verification that graphene is described by a massless

Dirac equation in three dimensions, and hence the effective field theory of a flat sheet of graphene is the Dirac equation in (2+1) dimensional space-time. And the fact is that, in the mentioned effective field theory, the velocity of light being replaced by Fermi velocity [10]. This picture is indeed the main difference between the three dimensional relativistic Dirac equation and described effective field theory of graphene. In that case, the corresponding action in unit of $4\pi G = 1$ may give by,

$$\mathcal{S} = \int dvdu\sqrt{-g}(R + \mathcal{L}), \tag{6}$$

where matter lagrangian density given by,

$$\mathcal{L} = \frac{1}{2}g_{ab}\partial^a\phi\partial^b\phi - V(\phi), \tag{7}$$

where indices $a, b = u, v$ and ϕ is scalar field. It is also possible to consider more than one scalar field which is not of our interest.
Einstein's equation gives us the following relation for the superpotential [25],

$$W = -\frac{3}{2}A' = -\frac{3}{2u_0}\left(\frac{\sinh(\frac{u}{u_0})}{\cosh(\frac{u}{u_0})}\right), \tag{8}$$

where prime denotes derivative with respect to u. Then, we can use superpotential to calculate scalar field as

$$\phi = \frac{\sqrt{6}}{2u_0}\int du\sqrt{\frac{\sinh^2(\frac{u}{u_0})}{\cosh^2(\frac{u}{u_0})} - 1}, \tag{9}$$

where we used the following equation [25]

$$\phi' = -\frac{3}{2}\frac{\partial W}{\partial \phi}. \tag{10}$$

Then, we can obtain potential via the following relation [25],

$$V = -\frac{4}{3}W^2 + \frac{9}{8}(\frac{\partial W}{\partial \phi})^2. \tag{11}$$

which yields

$$V = -\frac{3}{4u_0^2}\left(1 + 3\frac{\sinh^2(\frac{u}{u_0})}{\cosh^2(\frac{u}{u_0})}\right). \tag{12}$$

Finally, by using the equations (7), (10) and (12) we can obtain Hamiltonian density as

$$\mathcal{H} = -\frac{3v_f^2}{8u_0}\frac{3\cosh^2(\frac{u}{u_0}) + \sinh^2(\frac{u}{u_0})}{\cosh(\frac{u}{u_0})}. \tag{13}$$

In the Fig. 1 we can see behavior of the energy density for various values of u_0.

It is clear that the Hamiltonian density is completely negative which may sing of stability of the model. We investigate this in details by two separated methods which described in the following sections.

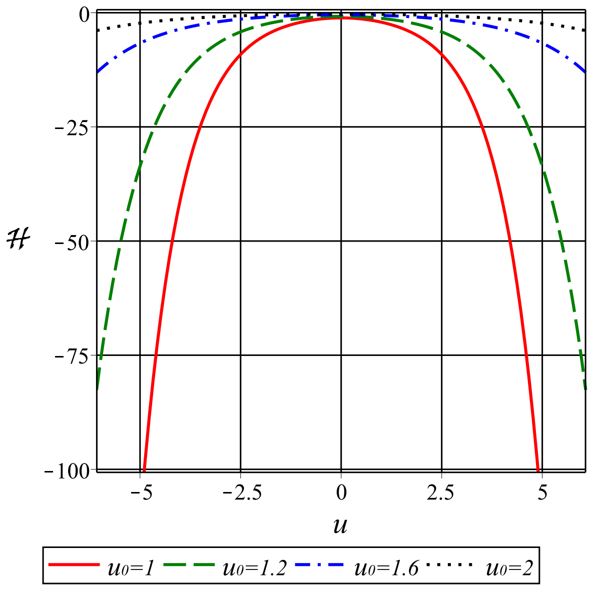

Figure 1: Hamiltonian density in terms of extra dimension in unit of the Fermi velocity.

2 Thermodynamical stability

We have found that the entropy should correct by a logarithmic term as follow [22]

$$S = \frac{k_B}{2}\left[1 + \ln\frac{\pi k_B}{4\gamma} + \ln T\right], \tag{14}$$

where $\beta = \frac{1}{k_B T}$ is inverse of the temperature and $\gamma = \frac{\hbar^2}{2Mu_0^2}$ coming from the Schrödinger equation. In that case, the partition function obtained as

$$Z = \frac{1}{2}\sqrt{\frac{\pi}{\beta\gamma}}e^{-\beta(E_0+\gamma\lambda^2)}, \tag{15}$$

where λ is a real parameter with the condition $\lambda > -1$. Hence, the internal energy given by

$$U = -\frac{\partial}{\partial\beta}\ln Z = E_0 + \gamma\lambda^2 + \frac{1}{2}k_B T. \tag{16}$$

Hence, the Helmholtz free energy is given by

$$F = E_0 + \gamma\lambda^2 - k_B T\left(\ln\frac{1}{2}\sqrt{\frac{\pi k_B T}{\gamma}}\right). \tag{17}$$

and the specific heat is calculated as,

$$C_P = C_V = \frac{k_B}{2}. \tag{18}$$

The positive value of specific heat is a sign that the model is thermodynamically stable, however there is a problem at low temperature (near net zero), which illustrated by analyzing

Helmholtz free energy. In the Fig. 2, we can see typical behavior of the Helmholtz free energy which is similar to the Ref. [10] in agreement with the general behavior of the adaptive model of graphene [26]. In the Fig. 2 we see a maximum at low temperature which is sign of an instability at very low temperature which graphene structure may broke. The same result will obtain for the Gibbs free energy which is given by

$$G = F + PV. \tag{19}$$

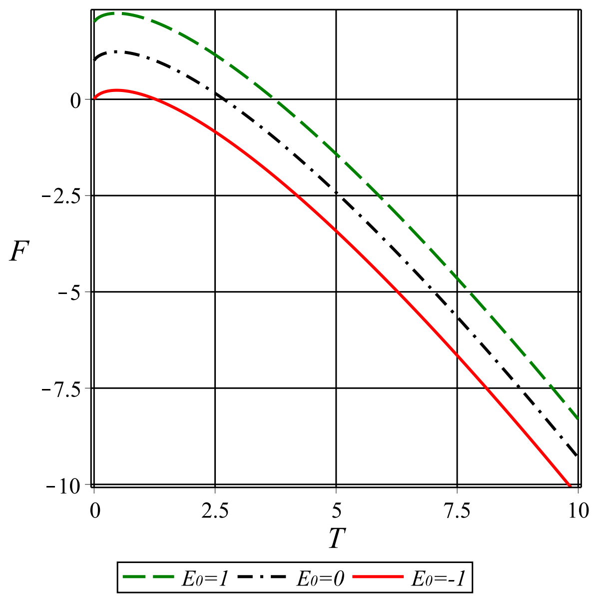

Figure 2: Helmholtz free energy in terms of temperature for $\gamma = \lambda = 1$ in unit of k_B.

Hence, in order to conclude about thermodynamical stability, we should use the specific heat together thermodynamics function.

3 Perturbations

To study the gravitational stability of the model, we consider small perturbation in the metric background (4). In that case, we can write the following line element

$$ds^2 = du^2 + (1 + \epsilon h_{\mu\nu})v_f^2 \cosh^2(\frac{u}{u_0})dv^2, \tag{20}$$

where ϵ is an infinitesimal parameter and metric fluctuation $h_{\mu\nu}$ is depend on u and v. By using a gauge transformation, we yield to the following relation,

$$\left(\left(1 - \frac{1}{v_f^2 \cosh^2(\frac{u}{u_0})}\right)\frac{\partial^2}{\partial u^2} - \frac{1}{v_f^2 \cosh^2(\frac{u}{u_0})}\frac{\partial^2}{\partial v^2} + \frac{4}{u_0}\frac{\sinh(\frac{u}{u_0})}{\cosh(\frac{u}{u_0})}\frac{\partial}{\partial u}\right)h_{\mu\nu} = 0. \tag{21}$$

By using the following change of variable

$$z = \frac{2u_0}{v_f}\tan^{-1}(e^{\frac{u}{u_0}}), \tag{22}$$

or its reversion,

$$u(z) = u_0 \ln(\tan(X)), \tag{23}$$

where $X = \frac{v_f z}{2u_0}$, one can obtain the following equation [25]

$$\left(-\frac{d^2}{dz^2} + U(z)\right) H_{\mu\nu} = m^2 H_{\mu\nu}, \tag{24}$$

where m^2 is eigenvalue of energy and we obtain

$$H_{\mu\nu} = e^{-imv} v_f \cosh(\frac{u(z)}{u_0}) h_{\mu\nu}, \tag{25}$$

hence, find the stability potential as

$$U(z) = \frac{3v_f^2}{16u_0^2} \frac{5 + 5\tan^4(X) - 2\tan^2(X)}{\tan^2(X)}. \tag{26}$$

In the plots of the Fig. 3, we can see that there is a minimum for the potential which sound stability. Also, we can see that potential is periodic in terms of z. In the Fig. 3 (a) we draw $U(z)$ and see that potential repeated periodic. In the Fig. 3 (b) we draw U in terms of extra dimension u and find that there is a minimum at the origin. Hence, we can see that the model is stable and small perturbations yields to an oscillation around the equilibrium point $z_0 = \frac{n\pi u_0}{2v_f}$ with $n = 1, 2, 3, \cdots$.

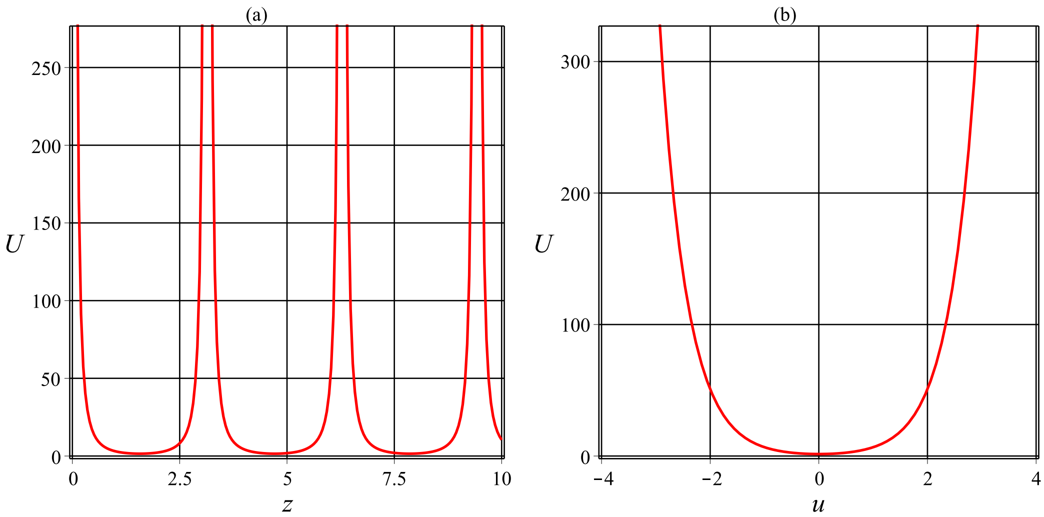

Figure 3: Stability potential in terms of (a) z and (b) u in unit of u_0 and v_f.

4 Conclusion

In this paper, we considered a braneworld scenario and used dimensional reduction to reduce the corresponding metric background to two-dimensional space including one extra dimension. Then, we interpreted it as dual background of the graphene adaptive model. We obtained scalar field and potentials in terms of extra dimension and wrote Hamiltonian density. We found that energy is completely negative which is the sign of model stability. In order to confirm this fact, we studied thermodynamics and calculated heat capacity, Helmholtz and Gibbs free energies by using partition function. We found that heat capacity is completely positive which means that the model may thermodynamically stable. However, we found that the model may unstable at very low temperatures corresponding to maximum value of the free energy. Finally, we considered small perturbation in the metric background and obtained stability potential. We show that the potential has a minimum corresponding to equilibrium point and found oscillation behavior around this equilibrium point. Hence, we found that our model is stable at usual temperature.

References

[1] Peng, R. B. 1999, ApJ, 194, 802

[2] Hartnoll, S. A. 2009, Class. Quant. Grav., 26, 224002

[3] Benini, F. 2012, Fortschr. Phys., 60, 810

[4] Hartnoll, S. A., Herzog, C. P., & Horowitz, G. T. 2008, JHEP, 0812, 015

[5] Horowitz, G. T. 2011, Lect. Notes Phys., 828, 313.

[6] Cai, R-G., Li, L., Li, L-F., & Yang, R-Q. 2015, Sci. China Phys. Mech. Astron., 58, 060401

[7] Pourhassan, B., & Bagheri-Mohagheghi, M. M. 2017, Eur. Phys. J. C, 77, 759

[8] Cvetic, M., & Gibbons, G. W. 2012, Ann. Phys., 327, 2617

[9] Iorio, A., & Lambiase, G. 2014, Phys. Rev. D, 90, 025006

[10] Pourhassan, B., Faizal, M., & Ketabi, S. A. 2018, Int. J. Mod. Phys. D, 27, 1850118

[11] Cea, P. 2012, Mod. Phys. Lett. B, 26, 1250084

[12] Roy, B., Hu, Z-X., & Yang, K. 2013, Phys. Rev. B, 87, 121408(R)

[13] Yonaga, K., Shibata, N. 2018, J. Phys. Soc. Jpn., 87, 034708

[14] Kiryu, T., & Koshino, M. 2019, Phys. Rev. B, 99, 085443

[15] Atanasov, V. & Saxena, A. 2010, Phys. Rev. B, 81, 205409

[16] Banados, M., Teitelboim, C., & Zanelli, J. 1992, Phys. Rev. Lett., 69, 1849

[17] Banados, M., Henneaux, M., Teitelboim, C., & Zanelli, J. 1993, Phys. Rev. D, 48, 1506

[18] Carlip, S. 1995, Class. Quantum Grav., 12, 2853

[19] Randall, L. & Sundrum, R. 1999, Phys. Rev. Lett., 83, 4690

[20] Sarrazin, M., & Petit, F. 2014, Eur. Phys. J. B, 87, 26

[21] Bellucci, S., Saharian, A. A., Simonyan, D. H., & Vardanyan, V. V. 2018, Phys. Rev. D, 98, 085020

[22] Zali, Z., Sadeghi, J., & Pourhassan, B. submitted to PLA

[23] Spivak, M. 1999, Publish or Perish Inc. (Houston) 1999

[24] Eisenhart, L. P. 1909, Princeton Univ. Press (Princeton) 1909

[25] DeWolfe, O., Freedman, D. Z., Gubser, S. S. & Karch, A. 2000, Phys. Rev. D, 62, 046008

[26] Xia, M., Song, Y. & Zhang, S. 2011, Phys. Lett. A, 375, 3726