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The phase transition of corrected black hole with $f(R)$ gravity

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Abstract. In this letter, we consider static black hole in $f(R)$ gravity. We take advantage from corrected entropy and temperature and investigate such black hole. Finally, we study the $P-V$ critically and phase transition of corrected black hole with respect to entropy and temperature. Here also, we obtain the heat capacity for the static black hole in $f(R)$ gravity. This calculation helps us to investigate some singularity. We have shown that the corrections exist for any black hole, but they are important for a small black holes and negligible for the large black holes. The advantage of a static black hole in $f(R)$ gravity is its holographic picture, which is a van der Waals fluid. Then, We have shown that in the presence of corrections there is still a van der Waals fluid as a dual picture. The critical values of event horizon radius for phase transitions are shown due to the corrections of entropy and temperature. We have some figures which show the phase transition and $P-V$ critically.

Keywords: The static black hole, Phase transition, Van der Waals behavior, corrected of entropy and temperature.

1 introduction

As we know, the charged black hole and its generalized with (2 + 1) dimension are studied by [1, 2]. The thermal properties of rotating charged hairy black hole also are investigated by [3, 4, 5, 6]. Recently, the object of black hole with $f(R)$ gravity are studied by several paper [7, 8, 9, 10, 11, 12, 13]. On the other hand, the logarithmic corrections of black hole entropy are discussed by Refs. [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26], which is given by,

$$\widehat{S} = S_0 - \frac{\alpha}{2} \ln S_0 T_0^2 \tag{1}$$

where T_0, α and S_0 are Hawking temperature, dimensionless parameter and Bekenstein-Hawking (BH) entropy respectively. Also, we consider $G = c = \frac{h}{2\pi} = 1$. Here, α is very important for the correction of entropy. When $\alpha = 0$, one can reproduce the ordinary thermodynamics. But here we use the following entropy corrected [27, 28],

$$\widehat{S} = S_0 - \frac{\alpha}{2} \ln S_0 \tag{2}$$

Here, it is possible to do correction Hawking temperature as [29, 30],

$$\widehat{T} = T_0(1 + \frac{\alpha}{2S_0}) \tag{3}$$

So, all above information gives us motivation to organized paper as follow. In section 2, we introduce the static black hole near to $f(R)$ gravity. In that case, we obtain the entropy, pressure, heat capacity of black hole and also discuss the first-order of phase transition. In section3 we, calculate the corrected thermodynamic quantities as entropy and temperature and mass. Here, we have some figures. In section 4, we discuss the thermodynamical structure in extended phase space. Also, here we investigate the $P - V$ critically and the phase transition. In the last section we have some conclusion and results.

2 Static black hole in $f(R)$ gravity

We are going to review the static black hole in $f(R)$ gravity and study the some thermodynamical properties and phase transition. So, the general form of the action is given by [31],

$$S = \frac{1}{2k} \int d^4x \sqrt{-g} f(R) + S_{mat}, \tag{4}$$

where S_{mat} is the matter part of the action. The universal form of the metric of the spherically symmetric space time to this action which is given by,

$$ds^2 = f(r)dt^2 - \frac{dr^2}{f(r)} + r^2 \left(d\theta^2 + sin^2\theta d\phi^2 \right), \tag{5}$$

and,

$$f(r) = 1 - \frac{2M}{r} + \beta_1 r - \frac{1}{3}\Lambda r^2 \tag{6}$$

where M, Λ and β_1 are the mass, cosmological constant and real constant of the black hole [32, 33]. As we know, the position of the black hole event horizon is determined with $f(r_+) = 0$. So, the black hole mass is obtain by following,

$$M = \frac{r_+}{2} + \frac{\beta r_+^2}{2} - \frac{\Lambda r_+^3}{6}, \tag{7}$$

where r_+ is the outer horizon radius. We draw the behavior of physical mass in terms of horizon radius in Fig. (1.a). Here, the mass of this black hole reach to zero at three point, $r_+ = 0$, $r_+ = 1$, and $r_+ = 4.6$, when β_1 is negative. Also in Fig. (1.a), we see the mass physical were shifted by the different values of β_1. Also, the entropy of this black hole is,

$$S = \pi r_+^2. \tag{8}$$

Here, the negative cosmological constant could interpreted as the positive thermodynamic pressure,

$$P = -\frac{\Lambda}{8\pi} = \frac{3}{8\pi\ell^2}. \tag{9}$$

By using equations (6) and (9), the Hawking temperature is given by,

$$T = \frac{1}{4\pi}\frac{df(r)}{dr}|_{r=r_+} = \frac{1}{4\pi r_+}\left(1 + 2\beta_1 r_+ + 8\pi P r_+^2\right). \tag{10}$$

Now, we can drive the equation of state as, $P = P(T, r_+)$,

$$P = \frac{T}{2r_+} - \frac{1}{8\pi r_+^2} - \frac{\beta_1}{4\pi r_+}. \tag{11}$$

The heat capacity obtain as following [34, 35],

$$C_P = T\left(\frac{\partial S}{\partial T}\right)_P = \frac{\left(\frac{\partial M}{\partial S}\right)}{\left(\frac{\partial^2 M}{\partial S^2}\right)}. \tag{12}$$

The equation (12) leads us to obtain the following heat capacity,

$$C_P = 2\pi r_+^2\left(\frac{1 + 2\beta_1 r_+ + 8\pi P r_+^2}{8\pi r_+^2 - 1}\right). \tag{13}$$

As we know, if $C > 0$, the black hole is stable, and if $C < 0$, the black hole is an unstable. As regards the change of sign in heat capacity is representing the phase transition between unstable / stable states. So, $C = 0$ corresponds to the phase transition and it leads to following relation,

$$1 + 2\beta_1 r_+ + \frac{3}{\ell^2}r_+^2 = 0, \qquad 2\pi r_+^2 = 0. \tag{14}$$

If $\frac{3}{\ell^2\beta_1^2} \ll 1$, in that case, we can say that the type one of phase transition is occurred by following points,

$$r_{+,1} = 0, \qquad r_{+,2} = \frac{-2\beta_1\ell^2}{3}. \tag{15}$$

In that case, the values β_1 should be negative. On the other hand, the type two of phase transition is propered by divergency of heat capacity which is given by,

$$r_+ = \pm\sqrt{\frac{1}{8\pi}}. \tag{16}$$

In the next section, we will discuses the corrected thermodynamical quantities in extended phase space and obtain the critical points and the stability of the static black hole in $f(R)$ gravity.

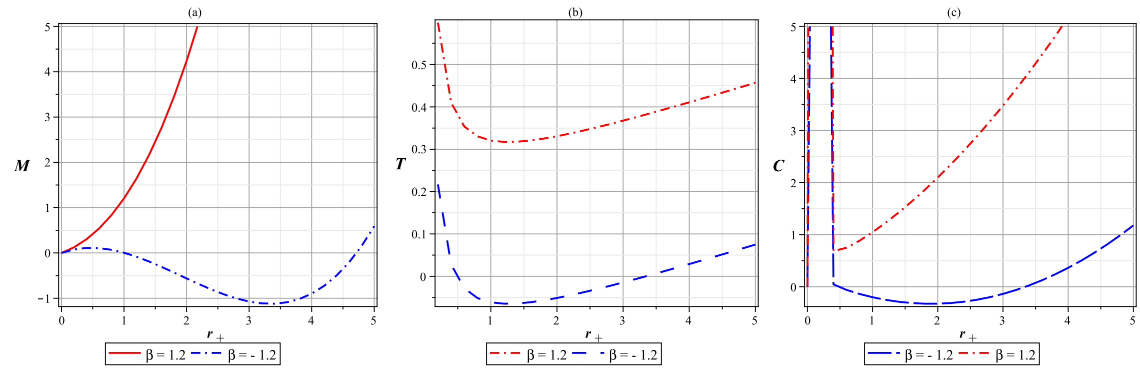

Figure 1: The static black hole in $f(R)$ gravity for $P = 0.025$ and all values of β_1 ; (a) mass variation, (b) temperature variation, (c) specific heat variation.

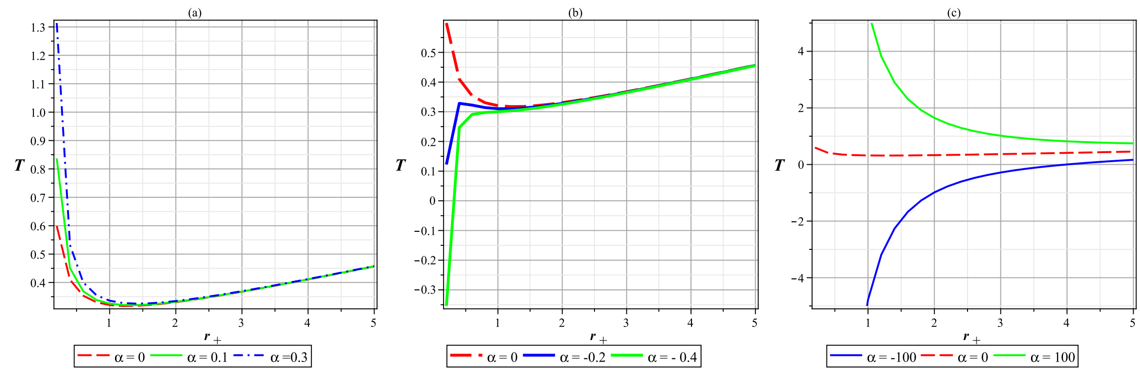

Figure 2: The corrected temperature variation in terms r_+ for; (a) α is positive and small, (b) α is negative and small, (c) α is large.

3 Corrected thermodynamics for static black hole in $f(R)$ gravity

In this section, we obtain the first-order corrected entropy and temperature of above mentioned black hole. By using the correction of (1) and (3) in equations (8) and (10), one can obtain the following [36],

$$\widehat{S} = \pi r_+^2 - \frac{\alpha}{2} \ln\left(\pi r_+^2\right) \tag{17}$$

and,

$$\widehat{T} = \frac{2\pi r_+^2 + \alpha}{8\pi^2 r_+^3}\left(1 + 2\beta_1 r_+ + 8\pi P r_+^2\right), \tag{18}$$

Here we draw Fig. 2, the temperature in terms of horizon radius the corresponding static black hole and see the effect of corrections when the horizon radius is small. Also, we see in figure (2.a) for the case of small α with positive correction, the corrected temperature is same with uncorrected temperature when the horizon radius is large. But the temperature values at minimum points increases when the α parameter increases. Also, Fig. (2.b) shows the values of small α with negative correction. The corrected temperature is contrast

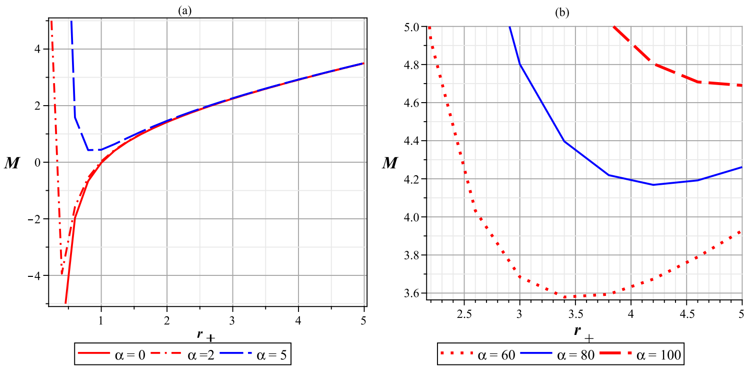

Figure 3: The corrected mass variation in terms r_+ for; (a) α is small, (b) α is large.

with uncorrected temperature when the horizon radius is very small. In this range, the temperature is shifted to negative values which sounds instability and may interpreted as a non-physical state. Finally, by drawing the corrected temperature of such black hole in the case of large α, in Fig. (2.c), the corrected temperature is positive (negative) when the α parameter is large and positive (negative). So, the corrected temperature has stable phase for the values of large α with positive correction, but it has non-physical state when α is large and negative.

From equation (7), the corrected physical mass for the static black hole in $f(R)$ gravity is given by,

$$\widehat{M} = \beta_1 \ln(r_+) + 4\pi P r_+ - \frac{1}{3r_+^3} + \frac{\alpha^2}{40\pi^2 r_+^5}\left(1 + \frac{5}{2}\beta_1 r_+ + \frac{40}{3}\pi P r_+^2\right). \tag{19}$$

In Fig. 3, we plot the behavior of correct physical mass in terms of horizon radius for the corresponding black hole. As we know, here we considered the α as a quantum correction. Here, in Fig. (3.a), one can see that there exists a critical point for physical mass in the range of $r_+ = 0.37$, when α is small. By increasing parameter α, the critical point of physical mass increases and shift to the right of the horizon radius. Also, in Fig. (3.b), we see the mass increases by the large values of α.

In the next section, we will discuses the corrected thermodynamical structure in extended phase space and obtain the critical points and the stability of the static black hole in $f(R)$ gravity.

4 Thermodynamical structure in extended phase space

In this section, we are going thermodynamical structure in extended phase space when we apply the same time effect of first-order correction entropy and temperature in a static black hole in $f(R)$ gravity. At first, we study extended phase space and investigate the $P - V$ criticality of such black hole. We obtain the critical quantities such as $r_{+,c}$, P_c, and T_c. To start, we review some basic thermodynamic properties of static black hole in $f(R)$ gravity. As we said, the pressure is associated to the cosmological constant,

$$P = -\frac{\Lambda}{8\pi} = \frac{3}{8\pi\ell^2}, \tag{20}$$

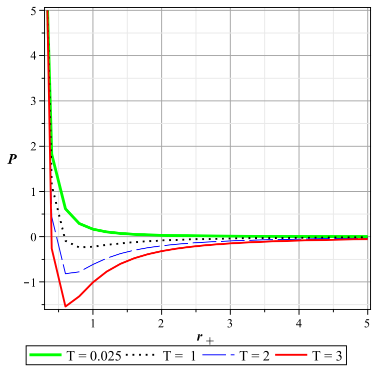

Figure 4: Differentiation P in terms of r_+ for $\alpha = 0.5$, $\beta = 1.2$ and all values of T .

by rearranging Eq. (16), we obtain the equation of state as following,

$$P = \frac{\pi r_+}{\left(\alpha + 2\pi r_+^2\right)}\widehat{T} - \frac{1}{8\pi r_+^2} - \frac{\beta_1}{4\pi r_+}, \tag{21}$$

We draw pressure diagram with respect to horizon radius in figure (4). We see that, criticality points exist for $T < T_c$ (dot line) and one can say the static black hole in $f(R)$ gravity have Van der waals-like behavior.

The critical points $(r_{+,c}, P_c, T_c)$ for the static black hole in $f(R)$ gravity in the phase transition with the corrected entropy and temperature obtained by the following condition,

$$\frac{\partial P}{\partial r_+} = \frac{\partial^2 P}{\partial r_+^2} = 0, \tag{22}$$

where,

$$2\pi r_+^2 + 2\alpha\beta_1 r_+ + 3\alpha = 0, \tag{23}$$

and by using condition

$$\alpha \gg \frac{6\pi}{\beta_1^2}, \tag{24}$$

one can obtain the critical values,

$$r_{+,c} = \frac{-3}{2\beta_1}, \tag{25}$$

and,

$$P_c = \frac{\beta_1^2}{6\pi}\frac{\left(2\alpha\beta_1^2 - 3\pi\right)}{\left(2\alpha\beta_1^2 - 9\pi\right)}, \qquad \widehat{T}_c = \frac{\beta_1}{54\pi^2}\frac{\left(2\alpha\beta_1^2 + 9\pi\right)^2}{\left(9\pi - 2\alpha\beta_1^2\right)}. \tag{26}$$

Now, the critical ratio is computed to be,

$$\rho = \frac{P_c \nu_c}{T_c} = 27\pi \frac{\left(2\alpha\beta_1^2 - 3\pi\right)}{\left(2\alpha\beta_1^2 + 9\pi\right)^2} \tag{27}$$

When

$$\frac{\left(2\alpha\beta_1^2 - 3\pi\right)}{\left(2\alpha\beta_1^2 + 9\pi\right)^2} = \frac{1}{72\pi}, \tag{28}$$

the above product will be as a usual relation $\frac{P_c \nu_c}{T_c} = \frac{3}{8}$. In that words, to solving equation (28), we have two solution for the corrected α parameter as following,

$$\alpha_1 = 24 \frac{\pi}{\beta_1^2}, \qquad \alpha_2 = 3 \frac{\pi}{\beta_1^2}. \tag{29}$$

Then, with using of condition (24), we will see that α_1 is regard. This shows when we have effect the quantum corrections in static black hole in $f(R)$ gravity, it has Van der waals-like behavior.

5 Conclusion

In this paper, we first considered the static black hole in $f(R)$ gravity. We applied the effect of first-order correction entropy and temperature and investigated the $P-V$ critically and stability of such black hole. We have plotted the corrected Hawking temperature in terms of horizon radius with respect to correction coefficient α, and we showed the quantum correction effect. We found that the quantum correction is affected at small horizon radius. In here, we showed the corrected temperature is the same with uncorrected temperature when the α parameter is positive and small. But, when the α negative coefficient is affected by the temperature, the temperature changes from physical to non-physical state in the range of small r_+. As we know, the corrections exist for any black hole, but they are important for a small black holes and negligible for the large black holes. The advantage of a static black hole in $f(R)$ gravity is its holographic picture, which is a van der Waals fluid. Then, we investigated the corrected thermodynamical structure in extended phase space and obtained the $P-V$ criticality of such black hole. We have shown that in the presence of corrections, there is still a van der Waals fluid as a dual picture. The critical values of event horizon radius for phase transitions are shown due to the corrections of entropy and temperature. However, there are enough stable regions to see quantum gravity effects before the phase transition of a van der Waals fluid. This means that there is a minimum radius, which is known critical radius. Finally, one can say that the above mentioned corrections for the corresponding black hole lead us to have stable system.

References

[1] Zhao. L., Xu. W., & Zhu. B. 2014, Commun. Theor. Phys., 61, 475

[2] Sadeghi. J., Pourhassan. B., & Farahani. H. 2014, Commun. Theor. Phys., 62, 358

[3] Sadeghi. J., & Farahani. H. 2014, Int. J. Theor. Phys., 53, 3683

[4] Bekenstein, J. D. 1973, Phys. Rev. D, 7, 2333

[5] Heckler, A. F. 1997, Phys. Rev. D, 55, 480

[6] Das, S., Majumdar, P., & Bhaduri, R. K. 2002, Class. Quant. Grav., 19, 2355

[7] Nojiri, S., Odintsov, S. D. 2011, Phys. Rept., 505, 59

[8] Chabab, M., Moumni, H. El., Iraoui, S., Masmar, K., & Zhizeh, S. 2018, LHEP, 02, 05

[9] Moon, T., Myung, Y. S., & Son, E. J. 2011, Gen. Rel. Grav., 43, 3079

[10] Chen, S., Liu, X., Liu, Ch., & Jing, J. 2013, Chin. Phys. Lett., 30, 060401

[11] Qiang, Gu., Gu, Li., & Jie-Xiong, M. 2016, Phys. Rev., D, 12, 124021

[12] Sheykhi, A. 2012, Phys. Rev. D, 86, 024013

[13] Hendi, S. H., & Momeni, D. 2011, Eur. Phys. J. C, 71, 1823

[14] Pourhassan, B., & Faizal, M. 2015, Eur. phys. Lett, 111, 40006

[15] Sadeghi. J., Pourhassan., B., & Rostami. M. 2016, Phys. Rev. D, 94, 064006

[16] Cardy, J. L. 1986, Nucl. Phys. B, 270, 186

[17] Kaul, R. K., & Majumdar, P., 1998, Phys. Lett. B, 439, 267

[18] Faizal. M., & Pourhassan, B. 2015, Phys. Lett. B, 751, 487

[19] Carlip, S. 2000, Class. Quant. Grav, 17, 4175

[20] Govindarajan, T. R., Kaul, R. K., & Suneeta, V. 2001, Class. Quant. Grav, 18, 2877

[21] Mann, R. B., & Solodukhin, S. N. 1998, Nucl. Phys. B, 523, 293

[22] Solodukhin, S. N. 1998, Phys. Rev. D, 57, 2410

[23] Faizal, M., & Khalil, M. M. 2015, Int. J. Mod. Phys. A, 30, 1550144

[24] Ali, A. F. 2012, J. High Energy Phys., 09, 067

[25] Sen, A. 2013, J. High Energy Phys., 04, 156

[26] Das, S., Kaul, R. K., & Majumdar, P. 2001, Phys. Rev. D, 63, 044019

[27] Kaul, R. K., & Majumdar, P., 2000, Phys. Rev. Lett., 84, 5255

[28] Majumdar, P. 2000, Pramana, J. Phys, 55, 511

[29] Cavaglia, M. & Das, S. 2004, Class. Quant. Grav, 21, 4511

[30] Falls, K., & Litim, D. F. 2014, Phys. Rev. D, 89, 084002

[31] Hawking, S. W. 1977, Commun. Math. Phys., 55, 133

[32] Zhang, M. 2018, Nucl. Phys. B, 935, 170

[33] Kubiznak, D., & Mann, R. B. 2012, J. High Energy Phys., 07, 033

[34] Dutta, S., Jain, A., & Soni, R. 2013, J. High Energy Phys., 12, 060

[35] Saffari, R., & Rahvar, S. 2012, Phys. Rev. D, 85, 024017

[36] Rostami, M., Sadeghi, J., Miraboutalebi, S., & Pourhassan, B. 2019, [arXiv: 1903.03437v1]