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# Effect of the temperature profile of the accretion disk on the structure of jets and outflows around protostars

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**Abstract**. Magnetic fields play an important role in creating, driving, and in the evolution of outflows and jets from protostars and accretion disks. On the other hand, the temperature profile of the accretion disks may also affect the structure of the magnetic field and outflows. In this paper, we use the self-similar method in cylindrical coordinates to investigate the effect of the temperature profile of the accretion disk ($T \propto r^{-1/2}$) in the formation of jets and outflows around protostars. The results show that in each specified angular momentum, there is a critical value for the ratio of the mass flux to the magnetic flux. For values less/larger than this critical value, taking into account the temperature profile in the disk results in a broadening/collimation of the jet, respectively. Outflows, on the other hand, are always broadened when accounting for the temperature profile of the disk.

*Keywords*: accretion discs – magnetic fields – (magnetohydrodynamics) MHD – methods: numerical – stars: protostars – ISM: jets and outflows.

## 1 Introduction

Outflows and jets are formed during the cloud collapse process around the poles of star-disk system [**1**]. Jets and outflows play an important role in the transfer of angular momentum to out of the disk using magnetic torques e.g., [2]. One of the first calculation to be made the study was performed by [3] in relation to the jet magnetohydrodynamic models for the motions based on the theory of in one-dimensional stationary axisymmetric MHD solar wind. The structure of these flows was investigated by [4] and [5] using the self-similar method in cylindrical coordinates around radio galaxies. But, the first analytical work was done by [6] in the study of magnetic lines out of the accretion disk. They established the treatment and launch of jets, near the disk, based on the theory of symmetric two-dimensional stationary cold MHD non-viscous flows using the self-similar method. They showed that close to the disk, the flow is probably driven by gas pressure in a hot magnetically dominated corona. In this way, magnetic stresses can extract the angular momentum from a thin accretion disk and thus enable matter to be accreted.

A self-consistent two-dimensional wind solution for the jets around an accretion disk, without assuming the self-similarity, was obtained by [7]. This theory was further developed by [8, 9] in the context of star formation and outflows/jets launched by accreting protostars. Many studies on cold jets and outflows have been done in recent years e.g., [10, 11, 12, 13, 14]. A conclusion from these works is that thermal pressure is not the main agent that cause the launching of jet and outflows in the protostars. In contrast, other works [15, 29] found that the presence of temperature profile is the mechanism by which jets and outflows can be launch from a protostellar accretion disk. On the other hand, there are many works that by taking the temperature profile, based on the existence of stationary axisymmetric warm MHD flow, consider the thermal pressure as the main force of acceleration of the outflows and jets e.g., [17, 18, 19, 20, 21].

In this paper, we are considering a disk which is geometrically thin and optically thick, with the assumption that the temperature is proportional to the disk radius as a power law with exponent $-1/2$ [22]. We anticipate that the Alfven speed scales as the Keplerian velocity in the accretion disk. In this approximation, the temperature power law with exponent$-1/2$ is necessary because it is the only value for which the luminosity is conserved. In this way, we use the self-similar method of [6] to investigate the effect of temperature profile of accretion disk around protostars on the structure of the magnetic field lines for the non-relativistic outflows and jets. These non-relativistic outflows and jets, which are derived from a disk-star system, will be launched, accelerated and collimated not only by electromagnetic forces but also by the pressure force. The two-dimensional equations of mass conservation, angular momentum and linear momentum of the outflows and jets in vertical direction of protostellar accretion disk are given in section 2. In section 3, with the assumption of self-similarity between radius and height, we investigate these conservation equations using the cylindrical coordinates. In section 4, the structure of magnetic field lines of outflows and jets in sub-Alfven speeds that are affected by the temperature profile of accretion disk and without it around protostars are obtained. Finally, in section 5, we conclude and give an outlook for further work.

## 2 MHD equations

According to the effect of temperature in the magnetosphere of Keplerian accretion disks, where magnetic fields are mainly poloidal and flows are sub-Alfven, outflows and jets can be driven not only by magnetic pressure but also by thermal pressure. In order to investigate the behavior of these outflows and jets, we assume infinite conductance of matter in close to the disk, and use the cylindrical coordinate (r, $\varphi$, z) with toroidal symmetry ($\frac{\partial}{\partial\varphi} = 0$).

In this coordinate, the fluid velocity $\mathbf{V}(r)$ is related to the field strength $\mathbf{B}(r)$ by the following relation (e.g., [23])

$$\mathbf{V} = \frac{k}{4\pi\rho}\mathbf{B} + (\omega \times \mathbf{r}), \tag{1}$$

where the undetermined constant $k/4\pi$ (which can be interpreted as the ratio of the mass flux to the magnetic flux) and the angular velocity $\omega$ satisfy

$$(\mathbf{B} \cdot \nabla\ )k = (\mathbf{B} \cdot \nabla)\omega = 0. \tag{2}$$

According to isorotation of matters in Keplerian disk and flux freezing of magnetic field in stationary flows with high conductivity, $\omega$ and k are two constants of the motion, which remain constant along the magnetic field lines ejected from disk. The momentum equation is

$$\rho(\mathbf{V} \cdot \nabla)\mathbf{V} = -\nabla p - \rho\nabla\psi_g + \frac{1}{4}(\nabla \times \mathbf{B}) \times \mathbf{B}\ , \tag{3}$$

where $\psi_g$ is the gravitational potential. Using the toroidal component of equation (3), we have

$$\ell = \omega r^2 - \frac{rB_\varphi}{4\pi k\rho}. \tag{4}$$

The specific energy equation is

$$e = \frac{1}{2}\mathbf{V}^2 + \psi_g + h - \frac{r\omega B_\varphi}{4\pi k\rho}, \tag{5}$$

Where h is enthalpy per unit mass and $\frac{r\omega B_\varphi}{4\pi k\rho}$ represents work done on the streaming gas by the magnetic torque per unit mass.

In equations (4) and (5), the specific angular momentum and specific energy are two constants of motion as

$$(\mathbf{B}\cdot\nabla\ )e = (\mathbf{B}\cdot\nabla)\ell = 0. \tag{6}$$

The gas density is obtained from the continuity equation:

$$\nabla\cdot(\rho\mathbf{V}) = 0. \tag{7}$$

# 3 Self- similar solutions

We take into account the magnetic stresses as important factor for transporting of angular momentum from the Keplerian accretion disks. In these disks, the specific angular momentum and energy are scaling as their Keplerian counterparts. Since the Alfven velocity is proportional to $B \propto r^{-\frac{5}{4}}$. with $\rho \propto r^{-3/2}$, and the Kepler velocity is proportional to $r^{-\frac{1}{2}}$, we have $B \propto r^{-\frac{5}{4}}$. For the magnetic field lines meeting the disk at radius $r_0$, we introduce the self-similar scaling

$$\mathbf{r} = [\xi(\chi), \varphi, \chi]r_0, \tag{8}$$

$$\mathbf{V}\ = [\xi'(\chi)f(\chi), g(\chi), f(\chi)](\frac{GM}{r_0})^{(\frac{1}{2})}. \tag{9}$$

In equations (8) and (9) the subscript "0" indicates the quantity evaluated at the disk surface where $\chi = 0$ and $\xi = 1$, and a prime denotes differentiation with respect to $\chi$.

We also introduce the dimensionless parameters as follows:

$$\epsilon \equiv \frac{e}{(\frac{GM}{r_0})}, \tag{10}$$

$$\lambda \equiv \frac{\ell}{(GMr_0)^{\frac{1}{2}}}, \tag{11}$$

$$w \equiv k(1+{\xi_0'}^2)^{\frac{1}{2}}\frac{(\frac{GM}{r_0})^{\frac{1}{2}}}{B_0}, \tag{12}$$

$$\tau \equiv \frac{5}{2}\frac{K}{m}(\frac{r_0}{GM})T, \tag{13}$$

for the specific energy, specific angular momentum, ratio of mass flux to magnetic flux and temperature (according to introduce enthalpy and pressure defined as $h = \frac{5}{2}\frac{KT}{m}$ and $P = \frac{KT}{m}\rho$ where $K$ is the Boltzmann constant and $m$ is the particle mass), respectively. In these equations, $w$, $\lambda$, $\epsilon$ and $\xi_0'$ are constant parameters ($\frac{dw}{w\chi} = 0, \frac{d\lambda}{d\chi} = 0, \frac{d\epsilon}{d\chi} = 0$ and $\frac{d\xi_0'}{d\chi} = 0$) and $\tau$ is a function of $\xi(\chi)$. Here, we assume that in the thin disks the temperature profile is $T \propto \xi^{-\frac{1}{2}}$ [22].

The density is related to the field geometry through the continuity condition

$$\rho\xi fJ = (\rho f)_0, \tag{14}$$

where

$$J = \xi - \chi\xi'. \tag{15}$$

Now, with respect to the self-similar relations, we come back to the MHD equations and investigate them with consideration of the temperature effect. Combining the specific energy and specific angular momentum equations, (4) and (5) for flows close to the disk in which they are sub-Alfven, with respect to the boundary conditions $g_0 = 1$ and $f_0 = 0$, we obtain

$$\begin{aligned} \epsilon - \lambda &= 1/2[f^2U + g^2 - 2g\xi] - S + \tau \\ &= \frac{-3}{2} + \tau, \end{aligned} \tag{16}$$

where

$$U = (1 + \xi'^2), \tag{17}$$

$$S = (\xi^2 + \chi^2)^{-\frac{1}{2}}. \tag{18}$$

We introduce $Q = 2S - 3 + \xi^2$ that $-1/2Q$ is the gravitational plus centrifugal potential difference between the points $(\xi, \chi)$ and $(1, 0)$. Using the square of the poloidal Alfven Mach number,

$$n = \frac{k^2}{4\pi\rho} = wf\xi J, \tag{19}$$

by eliminating the z-component of the magnetic field from the equation (1) and (4), we can obtain a quartic equation for $f(\chi)$ as

$$Q - f^2U = \left(\frac{n(\xi^2 - \lambda)}{\xi(1 - n)}\right)^2. \tag{20}$$

Using the self-similar relation in momentum equation in the z-direction, we will drive to the equation

$$an' + b\xi'' + c = 0, \tag{21}$$

where

$$\begin{aligned} a &= \frac{2}{(Jw\xi)^2}(2n^3J\xi - Q(J\xi)^3w^2 - \frac{\tau(J\xi w)^2}{5n}), \\ b &= \frac{2}{(Jw\xi)^2}(2n^3\xi\chi(n-1)), \\ c &= \frac{2}{(Jw\xi)^2}(2\xi'J(-n^4 + n^3) - (\xi\xi' + \chi)S^3(\xi J)^3w^2(n+1)n \\ &\quad + (\xi J)^3\xi\xi'w^2n + \frac{1}{5n}\tau'(J\xi w)^2), \end{aligned} \tag{22}$$

and to the differential equation (20) as

$$a'n' + b'\xi'' + c' = 0, \tag{23}$$

where

$$\begin{aligned} a' &= \frac{1}{(Jw\xi)^2}(n-1)(n^3J\xi U - Q(\xi J)^3w^2), \\ b' &= \frac{1}{(Jw\xi)^2}n^3(n-1)^2(\chi\xi + \xi^2\xi'), \\ c' &= \frac{1}{(Jw\xi)^2}(2n^3(Jw\xi)^2(\xi^2 - \lambda)J\xi' - n(n-1)^2(Jw\xi)^2 \\ &\quad \times(Q\xi' + (1 - S^3)\xi'\xi^2 - \chi\xi S^3). \end{aligned} \tag{24}$$

Now, by combining equation (21) and (23), we obtain a second-order differential equation for $\xi(\chi)$ as

$$
\begin{aligned}
((n-1)^2 n^3 Q(\xi J)^3 w^2(2\chi\xi-(\chi\xi+\xi^2\xi'))+(n-1)^2 n^6 J\xi(2(\chi\xi+\xi^2\xi')-2\chi\xi U)\\
-(n-1)^2 n^3(\chi\xi+\xi^2\xi')\frac{1}{5n}\tau'(J\xi w)^2)\xi''+((n-1)(Q(J\xi)^3 w^2\\
-n^3 UJ\xi)(2\xi' J(n^3-n^4)(-(\chi+\xi\xi')S^3(\xi J)^2 w^2 n(n+1)\\
+(\xi J)^2\xi'\xi w^2 n+\frac{1}{5}\tau'(\xi J)^2 w^2)+(2n^3 w^2(\xi J)^2((\xi J)^2-\lambda)J\xi'-n(n-1)^2\\
\times(\xi J)^2 w^2(Q\xi'+(1-S^3)\xi^2\xi'-\chi\xi' S^3))(2n^3 J\xi-Q(J\xi)^3 w^2-\frac{\tau(J\xi w)^2}{5n})=0 \qquad (25)
\end{aligned}
$$

This equation shows the structure of magnetic field lines that carries excess angular momentum from accretion disk magnetosphere by outflows and jets. If we set $\tau(\chi)=0$ and $\tau'(\chi)=0$ in equation (25), we recover the equation of the outflows and jets in the absence of a temperature profile in the disk. According to the differential equation (25), we plan in this paper to investigate the effect of three parameters $\lambda$, $w$ and $\tau$, on the structure of the sub-Alfven flows.

# 4 Result

We restrict our study for heights less than the Alfven surface in which $n=1$ and the structure of magnetic field lines are mainly poloidal. This surface is located in the range of dimensionless height $\chi=4.5$. With this restriction, according to figure 2 of [24], two parameters $\lambda$ and $w$ are in the range of $\lambda>1$ and $w<1$. Also, there are two boundary conditions: $\xi(0)=1$ and $\xi'(0)=\xi_0'$, where $\xi_0'$ shows the slope of magnetic field lines and ejected matters from accretion disk ($\xi_0'=\tan\theta_0$ where $\theta_0$ is the polar angle in respect to the disk axis). The magnetic field lines and matters that are ejected with angles $\theta<30°$ are known as the jets and those with angles between $30°-58°$ are called as outflows [25]. We consider two typical cases with $\xi_0'=0.3$ for a jet and $\xi_0'=1$ for an outflow. The results with parameters $\lambda=10$ and $w=0.03$ (according to figure 2 of [24]) are shown in Fig. 1. According to this figure, we see that effect of temperature profile on the jets are greater than outflows.

The results show that for each value of $\lambda$, there is a critical value $w_{crit}$ in which for $w<w_{crit}$, temperature profile of accretion disk leads to broadening of jets while for $w>w_{crit}$, it causes collimation of jets. Namely, for each angular momentum $\lambda$ there is a critical value for the ratio of mass flux to magnetic flux. A typical case of the effect of temperature profile of accretion disk on a jet, with $\lambda=10$ and $w=0.03$ (for broadening) and $w=0.04$ (for collimation) is shown in Fig. 2. This critical value depends on the value of $\lambda$ that is depicted in Fig. 3. As can be seen, with increasing angular momentum, the ejected critical value of ratio of mass flux to the magnetic flux decreases.

# 5 Conclusions

Here, we investigated the effect of temperature profile of accretion disk around protostar on the structure of magnetic field lines and ejected matters as outflows and jets, using the self-similar method. Our study is restricted to heights less than Alfven surface. Fig. 1 shows the behavior of the outflows and jets for two cases with consideration of temperature profile

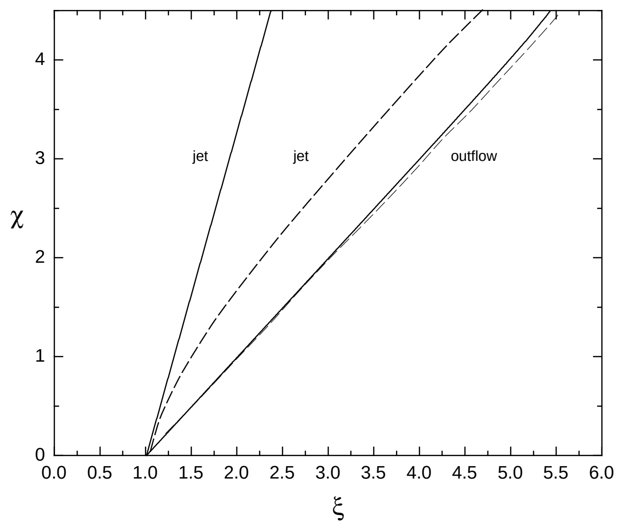


Figure 1: The jet with $\xi_0' = 0.3$ and outflow with $\xi_0' = 1$, without temperature (solid curve) and with temperature (dashed curve) for typical values $\lambda = 10$ and $w = 0.03$.

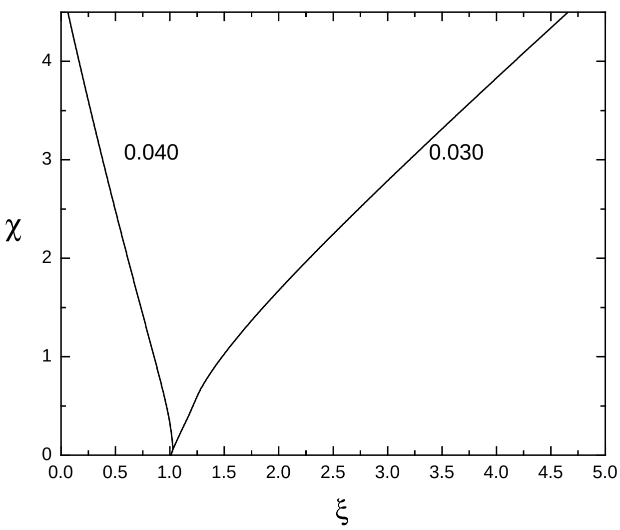


Figure 2: Effect of temperature profile of accretion disk on a typical jet with $\lambda = 10$ and $w = 0.03$ (for broadening) and $w = 0.04$ (for collimation).

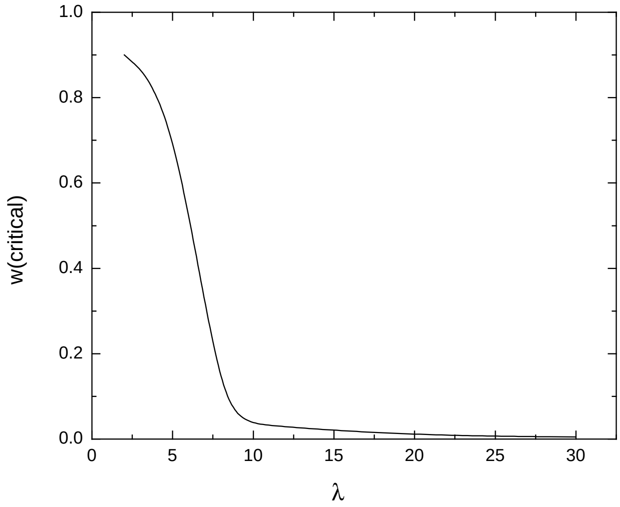


Figure 3: The critical value of ratio of mass flux to magnetic flux versus specific angular momentum $\lambda$.

and without it. This figure shows that the effect of temperature profile of accretion disk on the jets is effectively greater than outflows.

Viscosity and turbulence in the accretion disks are responsible for the outward angular momentum transport ([26]). In this paper, the effect of viscosity is ignored so that change of specific angular momentum $\lambda$ cannot affect the structure of magnetic field lines and ejected matters from accretion disk. On the other hand, the ratio of mass flux to magnetic flux, $w$, depend on the energy of matters ejected from disk through magnetic field lines. This energy can be effective on hoop stresses and the centrifugal acceleration of these lines toward the rotation axis of disk, and it also causes collimation and broadening of the magnetic field lines ([1]). Thus, we investigated the structure of magnetic field lines of outflows and jets in a constant $\lambda$ and with different values of $w$. We found in the studies that different values of $w$ cannot make a particular change in the formation of magnetic field lines of outflows, while it is effective on the jets.

Effect of temperature profile of accretion disk on a typical jet, with $\lambda = 10$ and $w = 0.03$ (for broadening) and $w = 0.04$ (for collimation), is shown in Fig. 2. This figure shows that for each specific angular momentum $\lambda$, there is a critical value of ratio of mass flux to magnetic flux. For $w$ less than this critical value, temperature profile of accretion disk cause broadening of the jets while for the values greater than this critical value, jets will be collimated. In Fig. 3, we see that for each specific angular momentum, there is a critical value of ratio of the mass flux to magnetic flux that with increasing $\lambda$, its value decrease.

There are some technical observations to find the bumps within accretion disks and jets and outflows around it. For example, near-infrared and Millimeter-wavelength observations, maser emission ([27]), and scattering of radiation by dust and grains ([28]). Also, the results of this research can be compared with some observational/numerical works (e.g., [24][15][29][30]).

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