

## Large amplitude dust ion acoustic solitons: considering dust polarity and nonextensive electrons

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**Abstract.** The characteristics of arbitrary amplitude dust ion acoustic solitary waves (DIASWs) are studied in unmagnetized dusty plasmas whose constituents are cold fluid ions, nonextensive electrons and stationary negative/positive dust particles. The pseudopotential approach has been used to investigate the structure of localized waves. It is found that, solitary waves exist in a definite interval for the Mach number which depends sensitively to the electron nonextensivity and dust polarity. Our results can be useful to understand the properties of localized electrostatic disturbances that may occur in astrophysical and space dusty plasmas.

*Keywords:* Dust Ion Acoustic Soliton, Sagdeev Potential, Nonextensive Electrons, Dust Polarity

## 1 Introduction

The physics of dusty plasmas has received considerable attention in the past few decades [1]-[6]. The study of dusty plasma is important to understand the space environments and astrophysical phenomena, such as planetary rings, comets, the interstellar medium, the earth's ionosphere, and the magnetosphere as well as industrial plasma devices [1, 2], [7]-[12]. One of the important electrostatic dust-associated waves is the low-frequency dust ion-acoustic (DIA) waves. Shukla and Silin [13] in their pioneer work, have shown that under certain conditions, a dusty plasma (with negatively charged static dust) supports low-frequency dust ion-acoustic (DIA) waves (DIAWs) with phase speed much smaller (larger) than the electron (ion) thermal speed. Needed conditions are conservation of equilibrium charge density  $n_{e0} + Z_d n_{d0} = n_{i0}$ , and the strong inequality  $n_{e0} \ll n_{i0}$ , where  $n_{e0}$ ,  $n_{d0}$  and  $n_{i0}$  are, respectively, electron, dust, and ion number density at equilibrium,  $Z_d$  is the number of electrons residing onto the dust grain surface, and  $e$  is the magnitude of the electronic charge. The DIAWs have also been observed in laboratory experiments [14, 15]. Theoretically, Mamun and Shukla [16, 17] have investigated DIASWs in unmagnetized dusty plasmas consisting of cold ion fluid, isothermal electrons, and negatively charged static dust particles. Mamun [18] discussed the propagation of nonlinear one-dimensional DIASWs in an unmagnetized adiabatic dusty plasma containing adiabatic inertialess electrons, adiabatic inertial ions, and negatively charged static dust grains. Most of these studies are based on the presence of negatively charged dust in the plasma. However, in many environments dust particles are positive [19]-[22]. Space plasma observations indicate clearly the presence of ion and electron populations which are far from their thermodynamic equilibrium. In the experiment for measuring the ion acoustic waves, the energy distribution of electrons

may not be actually the Maxwellian one and hence, it is hard to determine a valid electron temperature [23]. The non-Maxwellian velocity distributions for electrons in plasma have been observed experimentally, in situations where the temperature gradient was steep [24, 25]. A few examples of physical systems where the standard Boltzmann Gibbs approach seems to be inadequate are self-gravitating systems and some kinds of plasma turbulence. It has been shown that the experimental results, for electrostatic plane wave propagation in a collisionless thermal plasma, point to a class of Tsallis velocity distribution described by a nonextensive  $q$ -parameter smaller than unity [26]. Over the last two decades, a great deal of attention was paid to nonextensive statistic mechanics based on the deviations of the BoltzmannGibbsShannon (BGS) entropic measure. A suitable nonextensive generalization of the BGS entropy for statistical equilibrium was first recognized by Renyi [26] and subsequently proposed by Tsallis [27], suitably extending the standard additivity of the entropies to the nonlinear, nonextensive case where one particular parameter, the entropic index  $q$ , characterizes the degree of nonextensivity of the considered system ( $q=1$  corresponds to the standard, extensive, BGS statistics). Indeed, many physical systems that cannot be explained correctly in the classical statistical description found their convincing description within the framework of nonextensive statistics. As is well known, the Maxwellian distribution in the BoltzmannGibbs statistics is believed to be valid universally for the macroscopic ergodic equilibrium systems. However, for systems with long-range interactions, such as plasmas (Coulombian long-range interaction) and gravitational systems, where nonequilibrium stationary states exist, the Maxwellian distribution might be inadequate for the description of these systems. The parameter  $q$  that underpins the generalized entropy of Tsallis is linked to the underlying dynamics of the system and measures the amount of its nonextensivity. In statistical mechanics and thermodynamics, systems characterized by the property of nonextensivity are systems for which the entropy of the whole is different from the sum of the entropies of the respective parts. In other words, the generalized entropy of the whole is greater than the sum of the entropies of the parts if  $q < 1$  (superextensivity), whereas the generalized entropy of the system is smaller than the sum of the entropies of the parts if  $q > 1$  (subextensivity). Nonextensive statistics was successfully applied to a number of astrophysical and cosmological scenarios, which include stellar polytropes [28], the solar neutrino problem [29], peculiar velocity distributions of galaxies [30] and generally systems with long-range interactions and fractals such as spacetimes. Cosmological implications were discussed in [31] and recently an analysis of plasma oscillations in a collisionless thermal plasma was provided from  $q$ -statistics in [32]. Sahu et al. [33] have investigated the effect of nonextensive ions on the dust acoustic waves in an electron depleted dusty plasma. Recently, Bacha et al. [34] have extended the analysis of Shukla and Silin [13] to study the dust-ion acoustic solitary and shock waves in a nonextensive plasma. More recently, Alinejad [35] studied the effect of dust polarity on dust ion-acoustic localized structures in a superthermal dusty plasma. But no evidence yet has been investigated dust polarity on DIAS waves in a nonextensive dusty plasma. Therefore, in this model we consider the nonextensive electrons along with stationary dust particles and study the effect of dust polarity on the structures of DIAS waves by deriving the Sagdeev potential. We also obtain a definite interval for the Mach number in which solitary waves exist and depend sensitively on the electron nonextensivity and dust polarity. The manuscript is organized as follows. The basic equations governing the dusty electronegative plasma system under consideration are given in Sec. 2. The basic features of the dust ion-acoustic waves are investigated in Sec. 3, whereas those of the DIA solitary waves are investigated in Sec. 4. A brief discussion is finally presented in Sec. 5.

## 2 Basic equations

We consider an unmagnetized dusty plasma system consisting of cold ion-fluid, uniformly distributed massive negative (positive) dust particles and nonextensive distributed electrons. The charge neutrality at equilibrium requires  $n_{i0} = \alpha Z_d n_{d0} + n_{e0}$ , where  $n_{i0}$ ,  $n_{d0}$  and  $n_{e0}$  represents the equilibrium number densities of the ions, dust and electron particles, respectively.  $Z_d$  displays the absolute value dust charges and  $\alpha = \pm 1$ , where positive sign is for negative dust and negative sign is for positive dust particles. The nonlinear dynamics of the dust-ion-acoustic (DIA) waves in such a dusty plasma system is governed by:

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nu) = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = -\frac{\partial \phi}{\partial x} \quad (2)$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_e - \mu n + \alpha \frac{n_{d0}}{n_{e0}} Z_d \quad (3)$$

where  $n$  is the ion number density normalized by its equilibrium value ( $n_{e0}$ ).  $u$  is the ion fluid speed, normalized by  $c_i = \sqrt{T_e/m_i}$  and  $\phi$  is the electrostatic wave potential normalized by  $(T_e/e)$ , where  $T_e$  is the electron temperature. The time  $t$  and the distance  $x$  are normalized by the ion plasma frequency  $\omega_{pi}^{-1} = \sqrt{\frac{m_i}{4\pi n_{i0} e^2}}$  and the Debye radius  $\lambda_{Di} = \sqrt{\frac{T_e}{4\pi n_{i0} e^2}}$ , respectively, while  $\mu = \frac{n_{i0}}{n_{e0}}$ .

Nonextensive distribution for electrons is modelled using the following q-distribution function given by Lima et al [32].

$$f_e(v_e) = C_q \left\{ 1 + (1 - q) \left[ \frac{m_e v_e^2}{2T_e} - \frac{e\phi}{T_e} \right] \right\}^{\frac{1}{q-1}} \quad (4)$$

where  $\phi$  stands for the electrostatic potential, the parameter  $q$  measures the strength of nonextensivity and the remaining variables/parameters have their usual meaning. It may be noted that  $f_e(v_e)$  is the particular distribution that maximizes the Tsallis entropy and therefore follows to the laws of thermodynamics. The constant of normalization  $C_q$  is given by

$$C_q = n_{e0} \frac{\Gamma\left(\frac{1}{1-q}\right)}{\Gamma\left(\frac{1}{1-q} - \frac{1}{2}\right)} \sqrt{\frac{m_e(1-q)}{2\pi T_e}} \quad -1 < q \leq 1 \quad (5)$$

$$C_q = n_{e0} \left(\frac{1+q}{2}\right) \frac{\Gamma\left(\frac{1}{q-1} + \frac{1}{2}\right)}{\Gamma\left(\frac{1}{q-1}\right)} \sqrt{\frac{m_e(q-1)}{2\pi T_e}} \quad q \geq 1 \quad (6)$$

For  $q < -1$ , the q-distribution is unnormalizable. In the extensive limiting case ( $q \rightarrow 1$ ), the q-distribution reduces to the well-known MaxwellBoltzmann distribution. Also, for  $q > 1$ , the q-distribution function exhibits a thermal cutoff on the allowable maximum value of the electron velocity, which is given by :

$$n_e = \sqrt{\frac{2T_e}{m_e} \left( \frac{e\phi}{T_e} + \frac{1}{q-1} \right)} \quad (7)$$

By integrating the  $q$ -distribution over the velocity space, following nonextensive hot electron number density is derived [32]:

$$n_e = [1 + (q - 1)\phi]^{\frac{(q+1)}{2(q-1)}} \quad (8)$$

Now, making use of the transformation  $\zeta = x - Mt$  (where  $M$  is the Mach number = solitary wave speed/ $C_i$ ) along with the appropriate boundary conditions for localized perturbations ( $\phi \rightarrow 0, u \rightarrow 0$  and  $n \rightarrow 1$  as  $\zeta \rightarrow \infty$ ), one can obtain

$$\frac{1}{2} \left[ \frac{d\phi}{d\zeta} \right]^2 + V(\phi) = 0 \quad (9)$$

where  $V(\phi)$  is the Sagdeev potential given by

$$V(\phi) = \frac{2}{3q-1} \left\{ 1 - [1 + (q-1)\phi]^{\frac{3q-1}{2(q-1)}} \right\} + \mu M^2 \left\{ 1 - \left[ 1 - \frac{2\phi}{M^2} \right]^{\frac{1}{2}} \right\} - \delta\phi \quad (10)$$

Here,  $\delta = \pm \frac{n_{d0} Z_d}{n_{e0}}$  where positive (negative) sign being for negative (positive) dust. We note that (6) can be regarded as an energy integral of an oscillating particle of unit mass. The first term of the energy integral can be regarded as the kinetic energy of the unit mass at position  $\phi$  and time  $\zeta$ , whereas  $V(\phi)$  is the potential energy. On the other hand, for the existence of solitary solutions, these conditions should be satisfied:  $V(\phi) = 0$ ,  $\frac{dV(\phi)}{d\phi} = 0$  and  $\frac{d^2V(\phi)}{d\phi^2} < 0$  at  $\phi = 0$ , and there exists a nonzero  $\phi_m = \phi_0$ , at the maximum value of  $\phi$ ,  $V(\phi_m) = 0$ , and also  $V(\phi) < 0$  for  $\phi$  lying between 0 and  $\phi_m$ . For the existence of localized structures requires that the Mach number satisfy the relation

$$\frac{d^2V}{d\phi^2} < 0 \Rightarrow M^2 \geq \frac{2}{\mu(q+1)}. \quad (11)$$

From this relation, it can be seen that the lower limit is

$$M_{min} = \sqrt{\frac{2\mu}{q+1}} = \sqrt{\frac{2(1+\delta)}{q+1}} \quad (12)$$

The upper limit of  $M$  ( $M_{max}$ ) can be found by the condition  $V(\phi_0) \geq 0$ , where  $\phi_m = \phi_0 = \frac{M^2}{2}$  is the maximum value of  $\phi$ . Thus, we have

$$V(\phi_0) = V\left(\frac{M^2}{2}\right) = \mu M^2 + \frac{2}{3q-1} \left\{ 1 - \left[ 1 + (q-1)\frac{M^2}{2} \right]^{\frac{3q-1}{2(q-1)}} \right\} - \delta \left( \frac{M^2}{2} \right) \geq 0 \quad (13)$$

The range of the parameter  $M$ , for which dust ion acoustic solitary waves can be established, is restricted significantly by the conditions (8) and (9). For a fixed value of  $q$ , the soliton solutions may exist only for values of the Mach number satisfying  $M > M_{min}$ . The existence condition,  $M > M_{min}$  implies the existence of solitary waves, traveling at a speed exceeding the sound speed in the medium. It is clearly seen that the lower limit  $M_{min}$  is greater than the lower limit in a plasma without dust particles ( $\mu = 1$ ). Eq. (8) presents that  $M_{min}$  increases significantly with increasing negative dust particles ( $\delta > 0$ ). It is also clear that the lower limit of Mach number decreases in presence of positive dusty plasma ( $\delta < 0$ ). Note that the simple value  $M_{min} = 1$  is recovered for IA waves in electron-ion plasmas with Maxwellian electrons. Eq. (8) also shows that in the presence of nonextensive electron

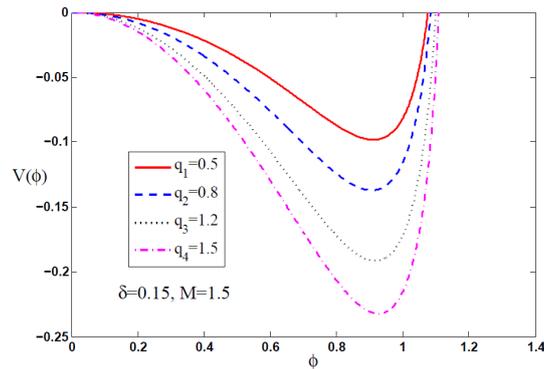


Figure 1: Variation of the Sagdeev pseudo-potential  $V(\phi)$  with  $\phi$  for values of  $M = 1.5$ ,  $\delta = 0.15$  and different values of  $q$ .

with  $-1 < q < 0$  ( $q > 0$ ), the lower limit  $M_{min}$  is greater (smaller) than its Boltzmannian counterpart ( $q = 1$ ). We can obtain  $M_{min}$  from (8) and  $M_{max}$  from (9) by numerical computation. In the next section, we try to obtain the range of  $M$  for different values of  $\mu$  and  $q$ . In fact, we want to know how the value of  $M$  changes when plasma parameters are changed.

Another quantitative study can be performed through the small amplitude approximation using the reductive perturbation method [36]. Results agree with our presented outcomes in the small amplitude wave propagation. Also, there has been presented large amplitude analysis for DIA waves in a dense plasma in which the pressure is significant [37]. Our results are in agreement with parts of results of this paper.

### 3 Discussion

The pseudo-potential  $V(\phi)$  plays a crucial role in the formation of solitons. Figures 1 and 2 show the behavior of  $V(\phi)$  as a function of  $\phi$  for different values of the positive and negative  $q$ -parameter, respectively. Other parameters are  $\delta = 0.15$  and  $M = 1.5$ . It can be seen that a well-structured potential appears for  $\phi > 0$  side in Fig. 1 and also  $\phi < 0$  in Fig. 2 for certain values of the Mach numbers. So, both compressive and rarefactive solitons can be created in the plasma model under consideration. Fig.1 (Fig.2) clearly demonstrate that the compressive (rarefactive) soliton amplitude increases (decreases) by increasing the positive (negative) nonextensive parameter. The dependence of the compressive and rarefactive soliton characteristics on the negative (positive) dust concentration have been shown in figures 3-6. One can observe from figures 3 and 4 that, the amplitude of both compressive and rarefactive solitons decreases with increasing values of the negative dust density ( $\delta > 0$ ). We note that, when the negative dust density is increased, the number density of ions reduces continuously. Thus, a decrease in ion density causes decrease in the amplitude of the SWs (for both positive and negative potential). This result is in agreement with the result reported earlier in [38] for magnetized nonextensive dusty plasmas. Decreasing soliton amplitude in our plasma with negative dust concentration is a similar behavior obtained earlier for an unmagnetized dusty plasma with trapped electrons [39, 40, 41] and also for superthermal electrons [35]. Figures 5 and 6 exhibit effects of positive dust concentration on the amplitude of compressive and rarefactive DIA solitons. These figures show that the amplitude of compressive and rarefactive DIA solitons increases when positive dust

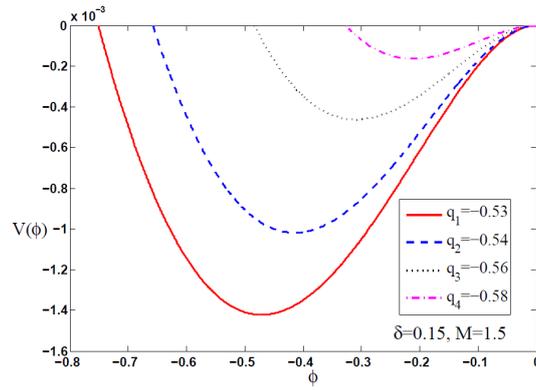


Figure 2: Variation of the Sagdeev pseudo-potential  $V(\phi)$  with  $\phi$  for values of  $M = 2.5$ ,  $\delta = 0.4$  and different values of  $q$ .

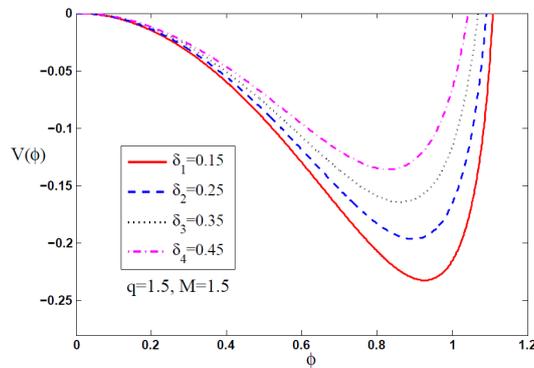


Figure 3: Variation of  $V(\phi)$  with compressive  $\phi$  for  $q = 1.5$ ,  $M = 1.5$  and different values of negative polarity of dusts ( $\delta > 0$ ).

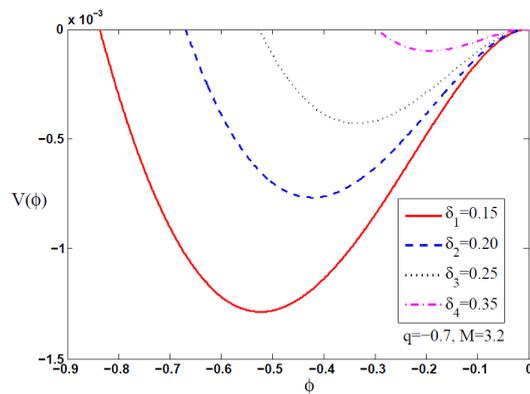


Figure 4: Variation of  $V(\phi)$  with rarefactive  $\phi$  for  $q = -0.7$ ,  $M = 3.2$  and different values of negative polarity of dust particles ( $\delta > 0$ ).

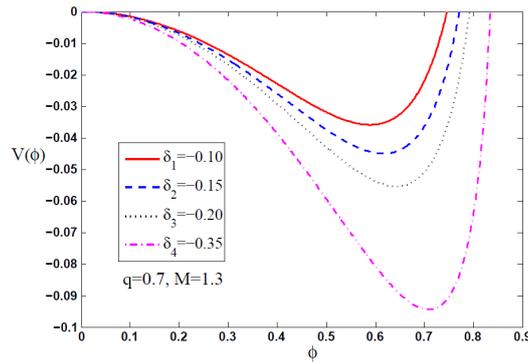


Figure 5: Variation of  $V(\phi)$  with compressive  $\phi$  for  $q = 0.7$ ,  $M = 1.3$  and different values of positive polarity of dust ( $\delta < 0$ ).

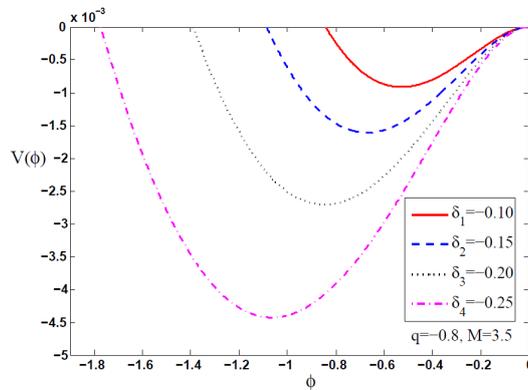


Figure 6: Variation of  $V(\phi)$  with rarefactive  $\phi$  for  $q = -0.8$ ,  $M = 3.5$  and different values of positive polarity of dust particles ( $\delta < 0$ ).

concentration increases. Increasing the soliton amplitude due to the increase in the positive dust concentration in figures 5 and 6 can be explained by the basis of restoring force which is developed during ion oscillations for the excitation of the wave. This is consistent with the reported result in [35] and [39] for positive dusty plasmas with trapped and superthermal electrons, respectively. The existence of domains of allowable Mach numbers for DIA soliton are depicted in figures 7-10, for different values of negative and positive dust concentration. In each figure, the lower (upper) curve depicts the variation of  $\delta$  with  $M_{min}$  ( $M_{max}$ ). Let us investigate the effect of the negative and positive dust concentration on the domain of allowable Mach numbers for compressive solitons (see figures 7 and 8). It is clear that the appropriate range of Mach number in which compressive solitons can be created, implicitly depends on the negative ( $\delta_-$ ) and/or positive ( $\delta_+$ ) dust concentrations. It can be seen that the velocity of the dust-ion acoustic solitons with negative polarity is greater than that of positive polarity. Figures 7 and 8 also demonstrate that velocity of DIA solitary wave increases (decreases) as negative (positive) dust concentration increases. It can be easily found that the range of Mach number for compressive solitons in the presence of negative dusts is larger than that for the positive dusts. The effect of negative ( $\delta_+$ ) and positive ( $\delta_-$ ) dust concentration on the velocity of rarefactive solitons have been shown

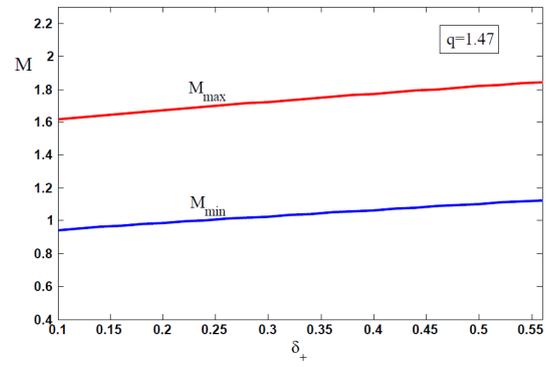


Figure 7: The range of Mach number with  $\delta_+$  in compressive soliton case for  $q = 1.47$ .

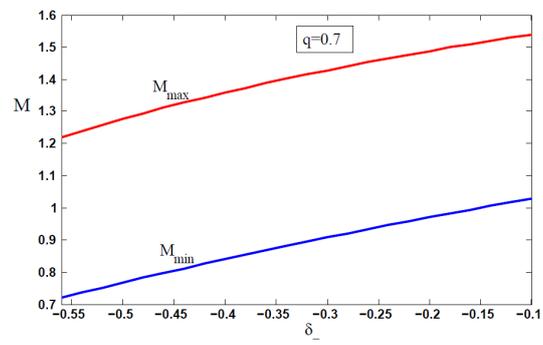


Figure 8: The range of Mach number with  $\delta_-$  in compressive soliton case for  $q = 0.7$ .

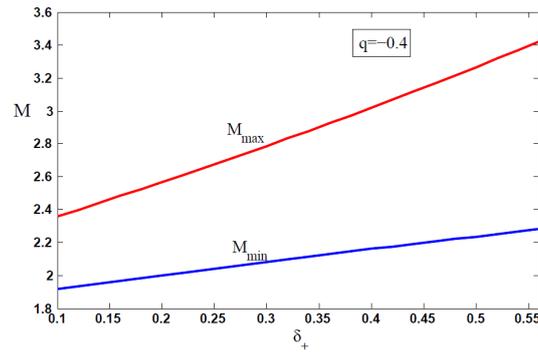


Figure 9: The range of Mach number with  $\delta_+$  in rarefactive soliton case for  $q = -0.4$ .

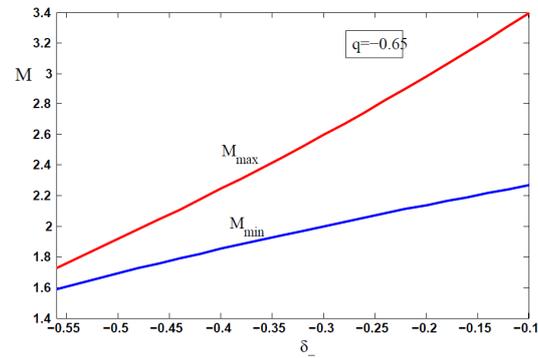


Figure 10: The range of Mach number with  $\delta_-$  in rarefactive soliton case for  $q = -0.65$ .

in Figures 9 and 10. We observe that the range of the possible Mach number increases (decreases) when negative (positive) dust concentration increases. It is also obvious that the velocity of rarefactive solitons increases (decreases) by increasing the negative (positive) dust concentration. Figure 9 shows that the range of rarefactive soliton velocity increases as negative dust concentration ( $\delta_+$ ) increases, while it decreases with increasing values of positive dust concentration ( $\delta_-$ ) as shown in the figure 10. Moreover, the range of the Mach number in the presence of negatively charged dust grains is larger when compared with what can find for positively charged dust grains. It can be concluded that the effect of negative (positive) dust concentration on the behavior of Mach number for rarefactive and compressive DIA solitons are completely different.

## 4 Conclusion

We have addressed the problem of dust ion acoustic oscillations in an unmagnetized collisionless plasma consisting of cold positive ions, nonextensive electrons and stationary positive and negative dust particles. The Sagdeev potential is derived and stability conditions have been investigated. Our results show that the compressive (rarefactive) soliton amplitude increases (decreases) by increasing the positive (negative) nonextensive parameter. We have also found that the amplitude of solitons increases (decreases) by increasing values of negative (positive) dust concentration. The domain of allowable Mach numbers depends dras-

tically on the dust polarity. It is shown that the range of the Mach number in the presence of negatively charged dust grains is larger when compared with one can see for positively charged dust grains. It is also found that the compressive soliton velocity is greater than that for rarefactive case. However for both compressive and rarefactive DIA solitons, the range of possible Mach number increases (decreases) with increasing the negative (positive) dust concentration, but numerical results show that the effect of dust concentration on the range of Mach number for the case of rarefactive soliton is larger than that for the compressive soliton. The above results show that the basic features of DIA waves are significantly modified by the effects of positive (negative) dust grains in the presence of nonextensive electrons. We know that DIA waves [42] are more feasible than the DA waves [43] to observe in laboratory dusty plasma. Moreover, the existence of DIA localized disturbances in space environments, particularly in dusty ionosphere or mesosphere [44] where dust can be positively charged by electron ejection from their surface due to the photoelectric effect with solar light [45], is also predicted. Furthermore, the present work can provide a possibility to develop more refined theories of nonlinear DIA solitary waves that may exist in space and laboratory dusty plasma systems. The ranges of different plasma parameters used in this investigation are very wide ( $0.1 < \delta < 0.55$ ,  $-0.8 < q < 1.74$  and  $1.3 < M < 3.5$ ), and are relevant to astrophysical and space dusty plasmas [35, 46, 47].

There are several works which can be done in the future. In some astrophysical situations, distribution of electrons is out of equilibrium, but it can be nonthermal or superthermal. Such media should be investigated separately. Results will be different from our results certainly. It is interesting to compare differences and similarities to find better knowledge about the effects of statistical distributions on the behavior of plasmas.

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