

Finite temperature correlation function of two dissipative massive scalar fields: Thermofield approach

Marjan Jafari

Department of Physics, Faculty of Science, Imam Khomeini International University, P.O.Box 34148-96818, Qazvin, Iran

Abstract. The present paper aims at investigating the manner of two dissipative massive scalar fields. Two massive scalar fields that interact with a reservoir were considered and a reservoir was modeled by continuum Klein-Gordon fields. The Lagrangian of the total system was canonically quantized and the dynamics of the system was determined using the Euler-Lagrange equation. Then, the explicit form of the quantum massive scalar fields in long-time limit were observed. The propagator of the system and correlation functions were calculated at finite temperature in the thermofield dynamics formalism.

Keywords: massive scalar field, correlation function, dissipative, thermofield dynamics.

1 Introduction

Dissipative scalar field theories play an important role in physics and quantum field theory. For example in optomechanics, a scalar field is used to describe a movable mirror [1]. Another interesting example is Casimir effect; in Casimir physics, a scalar field is the fluctuating field interacting linearly with some external matter fields defined inside or over some specific surfaces [2]. One of the most important applications of scalar field is in astronomy where Bosonic stars are described by scalar massive field [3] and Soliton stars can also be investigated by scalar fields [4].

In some of the phenomena like casimir effect or investigating behavior of Bosonic stars, the correlation between objects are important [5, 6]. Therefore, it is useful to introduce the coupling of scalar fields in describing objects.

A very interesting approach to thermal field theories is thermofield dynamics [7, 8], which is used in many areas of physics. Thermo field dynamic is an operator-based approach. Present study is going to study examines the quantization of the scalar fields at finite temperature in the framework of the thermofield approach. In this method, a fictitious system without any interaction with the original system is introduced and a thermal vacuum state is constructed in the mixed space.

In this paper, a system consisting of two dissipative $1+1$ dimensional massive scalar fields is considered. We use the fundamental approach of describing dissipative by coupling to heat bath for the system. The scalar fields are coupled to heat bath and heat bath modeled by real Klein-Gordon field. The mass of scalar fields may be zero during or at the end of the calculations. The ideas of heat bath that modeled by real Klein-Gordon field is not new [9]. The paper is structured as following: In sec 2, a classical Lagrangian is proposed for the total system and using Euler-Lagrange equations, the dynamical equations for the scalar fields and heat bath are found. In sec 3, the system is canonically quantized and the memory

functions and coupling function of the system are defined in terms of the heat bath Green's function. The explicit form of the quantum massive scalar fields are derived. In sec 4, knowing the explicit form of the quantum fields, correlation functions of the system are obtained at finite temperature. in Sce 5, using thermofield approach, the system is thermalized and Green's functions of the system at finite temperature are derived. Finally, the results are discussed in sec 6.

2 Lagrangian

This section examines a system consisting of two scalar massive field interacting with a field as a reservoir which is defined by real scalar field $Y_\omega(x, t)$. Throughout the calculation, we assume that the fields are in 1 + 1 dimensional and use natural units. The covariant Lagrangian density of the system is as follows:

$$\begin{aligned} L = & \sum_{i=1}^2 \frac{1}{2} \partial_\mu \phi_i(x, t) \partial^\mu \phi_i(x, t) - \frac{1}{2} m_i^2 \phi_i^2(x, t) \\ & + \frac{1}{2} \int_0^\infty d\omega [\partial_\mu Y_\omega(x, t) \partial^\mu Y_\omega(x, t) - \omega^2 Y_\omega^2(x, t)] \\ & + \sum_{i=1}^2 \int_0^\infty d\omega f_i(\omega) \phi_i(x, t) Y_\omega(x, t), \end{aligned} \quad (1)$$

where $f_i(\omega)$ is homogeneous coupling function of scalar fields to reservoir and $\phi_i(x, t)$ is scalar massive field or main system. Using Euler-Lagrange equations, the classical equation of motion for the fields $\phi(x, t)$ and $Y_\omega(x, t)$ are obtained

$$(\partial_t^2 + \Omega_{k,i}^2) \phi_i(x, t) = \int_0^\infty d\omega f_i(\omega) Y_\omega(x, t), \quad (2)$$

$$(\partial^2 + \omega^2) Y_\omega(x, t) = f_i(\omega) \phi_i(x, t). \quad i = 1, 2 \quad (3)$$

where $\partial^2 = \partial_t^2 - \partial_x^2$ and $\Omega_{k,i}^2 = k^2 + m_i^2$.

3 Equation of motion

From the Lagrangian density (1), the conjugate momenta corresponding to the fields are defined by:

$$\begin{aligned} \Pi_\omega(x, t) &= \frac{\partial L}{\partial \dot{Y}_\omega(x, t)} = \dot{Y}_\omega(x, t) \\ \pi_i(x, t) &= \frac{\partial L}{\partial \dot{\phi}_i(x, t)} = \dot{\phi}_i(x, t). \end{aligned} \quad (4)$$

The following equal-time commutation relations can be required on the fields and their conjugate momenta to quantized the theory canonically

$$[\hat{\phi}_i^\dagger(x, t), \hat{\pi}_j(x', t)] = i\hbar \delta(x - x') \delta_{ij}, \quad (5)$$

$$[\hat{Y}_\omega^\dagger(x, t), \hat{\Pi}_{\omega'}(x', t)] = i\hbar \delta(\omega - \omega') \delta(x - x'), \quad (6)$$

The Hamiltonian of the system is as follows:

$$\begin{aligned}
 H = & \sum_{i=1}^2 \int_{-\infty}^{\infty} dx \left[\frac{1}{2} (\hat{\pi}_i^2(x, t) + (\partial_x \hat{\phi}_i(x, t))^2 + m^2 \hat{\phi}_i^2(x, t)) \right. \\
 & + \frac{1}{2} \int_0^{\infty} d\omega (\hat{\Pi}_\omega^2(x, t) + (\partial_x \hat{Y}_\omega(x, t))^2 + m^2 \hat{Y}_\omega^2(x, t)) \\
 & \left. - \sum_{i=1}^2 \int_0^{\infty} d\omega f_i(\omega) \hat{\phi}_i(x, t) \hat{Y}_\omega(x, t) \right] \quad (7)
 \end{aligned}$$

The Heisenberg equations for the coupled fields-environment dynamics is as follows:

$$(\partial_t^2 + \Omega_{k,i}^2) \phi_i(x, t) = \int_0^{\infty} d\omega f_i(\omega) Y_\omega(x, t), \quad (8)$$

$$(\partial^2 + \omega^2) Y_\omega(x, t) = f_i(\omega) \phi_i(x, t). \quad i = 1, 2 \quad (9)$$

the formal solution of above equation is

$$\hat{Y}_\omega(x, t) = \hat{Y}_\omega^N(x, t) - \int dx' \int dt' G_\omega(x - x', t - t') (f_1(\omega) \phi_1(x', t') + f_2(\omega) \phi_2(x', t')) \quad (10)$$

from Eq (2) we have

$$(\partial^2 + \omega^2) G_\omega(x - x', t - t') = -\delta(x - x') \delta(t - t'), \quad (11)$$

Where $G_\omega(x - x', t - t')$ is the Green's function that satisfied above equation and

$$G_\omega(x - x', t - t') = -\frac{1}{2} \Theta(t - t' - |x - x'|) J_0(\omega \sqrt{(t - t')^2 - |x - x'|^2}) \quad (12)$$

$\Theta(x)$ is Heaviside step function and $J_0(x)$ is Bessel function of the first kind and zero order. $Y_\omega^N(x, t)$ is noise field or quantum vacuum fluctuating field which can be written in terms of annihilation and creation operators as follows

$$Y_\omega^N(x, t) = \int \frac{dk}{\sqrt{4\pi\omega_k}} [\hat{a}_k(\omega) e^{i(kx - \omega_k t)} + \hat{a}_k^+(\omega) e^{-i(kx - \omega_k t)}] \quad (13)$$

where $\omega_k = \sqrt{k_0^2 + \omega^2}$ and the annihilation and creation operators satisfy the usual commutation relations

$$[\hat{a}_k(\omega), \hat{a}_{k'}^+(\omega')] = \delta(k - k') \delta(\omega - \omega') \quad (14)$$

From Eqs. (8,9,13), we can obtained

$$\begin{aligned}
 (\partial^2 + m_1^2) \phi_1(x, t) &= \int d\omega f_1(\omega) [Y_\omega^N(x, t) - \int dx' \int dt' G_\omega(x - x', t - t') (f_1(\omega) \phi_1(x', t') + f_2(\omega) \phi_2(x', t'))], \\
 (\partial^2 + m_2^2) \phi_2(x, t) &= \int d\omega f_2(\omega) [Y_\omega^N(x, t) - \int dx' \int dt' G_\omega(x - x', t - t') (f_1(\omega) \phi_1(x', t') + f_2(\omega) \phi_2(x', t'))], \quad (15)
 \end{aligned}$$

where the memory functions γ and γ'' or the susceptibility of the mediums are defined by

$$\begin{aligned}
 \gamma(x - x', t - t') &= \int_0^{\infty} d\omega f_1^2(\omega) G_\omega(x - x', t - t'), \\
 \gamma''(x - x', t - t') &= \int_0^{\infty} d\omega f_2^2(\omega) G_\omega(x - x', t - t'), \quad (16)
 \end{aligned}$$

and γ' is correlation of the two subsystems

$$\gamma'(x-x', t-t') = \int_0^\infty d\omega f_1(\omega) f_2(\omega) G_\omega(x-x', t-t'), \quad (17)$$

and the noise currents are

$$\begin{aligned} J_N(x, t) &= \int_0^\infty d\omega f_1(\omega) Y_\omega^N(x, t) \\ J'_N(x, t) &= \int_0^\infty d\omega f_2(\omega) Y_\omega^N(x, t) \end{aligned} \quad (18)$$

by substituting (16) and (18) into (15), we obtaine

$$\begin{aligned} (\partial^2 + m_1^2)\phi_1(x, t) + \int \int dx' dt' [\gamma(x-x', t-t')\phi_1(x', t') + \gamma'(x-x', t-t')\phi_2(x', t')] &= J^N(x, t) \\ (\partial^2 + m_2^2)\phi_2(x, t) + \int \int dx' dt' [\gamma''(x-x', t-t')\phi_2(x', t') + \gamma'(x-x', t-t')\phi_1(x', t')] &= J'^N(x, t) \end{aligned} \quad (19)$$

To solve Eq.(19), taking the fourier-Laplace transform of both sides of Eq.(19), we obtained

$$\begin{aligned} \phi_1(k, s) &= \frac{(k^2 + s^2 + m_1^2 + \tilde{\gamma}(k, s))J'_N(k, s) - \tilde{\gamma}'(k, s)J_N(k, s)}{(k^2 + s^2 + m_2^2 + \tilde{\gamma}''(k, s))(k^2 + s^2 + m_1^2 + \tilde{\gamma}(k, s)) - \tilde{\gamma}'^2(k, s)} \\ \phi_2(k, s) &= \frac{(k^2 + s^2 + m_2^2 + \tilde{\gamma}''(k, s))J_N(k, s) - \tilde{\gamma}'(k, s)J'_N(k, s)}{(k^2 + s^2 + m_1^2 + \tilde{\gamma}(k, s))(k^2 + s^2 + m_2^2 + \tilde{\gamma}''(k, s)) - \tilde{\gamma}'^2(k, s)} \end{aligned} \quad (20)$$

that we are assume $\phi_i(k, 0) = \phi'_i(k, 0) = 0$ and

$$\begin{aligned} \tilde{\gamma}(k, s) &= - \int_0^\infty d\omega \frac{f_1^2(\omega)}{s^2 + \omega^2 + k^2}, \\ \tilde{\gamma}'(k, s) &= - \int_0^\infty d\omega \frac{f_1(\omega)f_2(\omega)}{s^2 + \omega^2 + k^2}, \\ \tilde{\gamma}''(k, s) &= - \int_0^\infty d\omega \frac{f_2^2(\omega)}{s^2 + \omega^2 + k^2} \end{aligned} \quad (21)$$

and fourier-Laplace transform of the noise fields are given by

$$\begin{aligned} J^N(k, s) &= \int_0^\infty d\omega f_1(\omega) \sqrt{\frac{\pi}{\omega_k}} \left[\frac{1}{s + i\omega_k} \hat{a}_k(\omega) + \frac{1}{s - i\omega_k} \hat{a}_k^+(\omega) \right] \\ J'^N(k, s) &= \int_0^\infty d\omega f_2(\omega) \sqrt{\frac{\pi}{\omega_k}} \left[\frac{1}{s + i\omega_k} \hat{a}_k(\omega) + \frac{1}{s - i\omega_k} \hat{a}_k^+(\omega) \right] \end{aligned} \quad (22)$$

The inverse laplace transform of the fields are

$$\begin{aligned} \hat{\phi}_1(k, t) &= \int_0^t dt' \int_0^\infty d\omega \sqrt{\frac{\pi}{\omega_k}} \beta(k, t', \omega) (\hat{a}_k(\omega) e^{-i\omega_k(t-t')} + \hat{a}_{-k}^+(\omega) e^{i\omega_k(t-t')}) \\ \hat{\phi}_2(k, t) &= \int_0^t dt' \int_0^\infty d\omega \sqrt{\frac{\pi}{\omega_k}} \alpha(k, t', \omega) (\hat{a}_k(\omega) e^{-i\omega_k(t-t')} + \hat{a}_{-k}^+(\omega) e^{i\omega_k(t-t')}) \end{aligned} \quad (23)$$

where $\alpha(k, t, \omega)$ and $\beta(k, t, \omega)$ are

$$\begin{aligned} \beta(k, t', \omega) &= L^{-1} \left[\frac{(s^2 + k^2 + m_2^2 + \tilde{\gamma}''(k, s))f_1(\omega) - \tilde{\gamma}'(k, s)f_2(\omega)}{(s^2 + k^2 + m_2^2 + \tilde{\gamma}''(k, s))(s^2 + k^2 + m_1^2 + \tilde{\gamma}(k, s)) - \tilde{\gamma}'^2(k, s)} \right] \\ \alpha(k, t', \omega) &= L^{-1} \left[\frac{(s^2 + k^2 + m_1^2 + \tilde{\gamma}(k, s))f_2(\omega) - \tilde{\gamma}'(k, s)f_1(\omega)}{(s^2 + k^2 + m_2^2 + \tilde{\gamma}''(k, s))(s^2 + k^2 + m_1^2 + \tilde{\gamma}(k, s)) - \tilde{\gamma}'^2(k, s)} \right] \end{aligned} \quad (24)$$

where L^{-1} is the inverse Laplace transform operator. The explicit form of the fields $\hat{\phi}_i(x, t)$ at large-time limit are as follows:

$$\begin{aligned}\hat{\phi}_1(x, t) &= \int_{-\infty}^{\infty} \frac{dk}{2\pi\omega_k} \int_0^{\infty} d\omega \tilde{\beta}(k, \omega, -i\omega_k) (\hat{a}_k(\omega) e^{i(kx - \omega_k t)} + \hat{a}_{-k}^+(\omega) e^{i(kx - \omega_k t)}) \\ \hat{\phi}_2(x, t) &= \int_{-\infty}^{\infty} \frac{dk}{2\pi\omega_k} \int_0^{\infty} d\omega \tilde{\alpha}(k, \omega, -i\omega_k) (\hat{a}_k(\omega) e^{i(kx - \omega_k t)} + \hat{a}_{-k}^+(\omega) e^{i(kx - \omega_k t)})\end{aligned}\quad (25)$$

where

$$\begin{aligned}& \int_0^{t \gg \frac{1}{\omega_k}} \alpha(k, t', \omega) e^{i\omega_k t'} dt' = \tilde{\alpha}(k, \omega, s = -i\omega_k) \\ &= \frac{(m_1^2 - \omega^2 + \tilde{\gamma}(k, -i\omega_k)) f_2(\omega) - \tilde{\gamma}'(k, -i\omega_k) f_1(\omega)}{(m_1^2 - \omega^2 + \tilde{\gamma}(k, -i\omega_k))(m_2^2 - \omega^2 + \tilde{\gamma}''(k, -i\omega_k)) - \tilde{\gamma}'^2(k, -i\omega_k)}\end{aligned}\quad (26)$$

and

$$\begin{aligned}& \int_0^{t \gg \frac{1}{\omega_k}} \beta(k, t', \omega) e^{i\omega_k t'} dt' = \tilde{\beta}(k, \omega, s = -i\omega_k) \\ &= \frac{(m_2^2 - \omega^2 + \tilde{\gamma}''(k, -i\omega_k)) f_1(\omega) - \tilde{\gamma}'(k, -i\omega_k) f_2(\omega)}{(m_2^2 - \omega^2 + \tilde{\gamma}''(k, -i\omega_k))(m_1^2 - \omega^2 + \tilde{\gamma}(k, -i\omega_k)) - \tilde{\gamma}'^2(k, -i\omega_k)}\end{aligned}\quad (27)$$

Knowing the explicit form of the quantum fields, in the next section the two point function and correlation function will be obtained.

4 correlation function

We can calculate the two point function of each scalar fields and correlation function of the scalar massive fields at finite temperature. A common assumption is that the environment is in a thermal state. In the thermal equilibrium, the normal modes of the systems have the expectation values:

$$\langle \hat{a}_k^+(\omega) \hat{a}_{k'}(\omega) \rangle = \delta(k - k') \delta(\omega - \omega') N(\omega), \quad N(\omega) = [\exp(\frac{\hbar\omega}{K_B T}) - 1]^{-1} \quad (28)$$

Using Eq.(25), the two-point function of the scalar massive fields at finite temperature is obtained

$$\begin{aligned}& \frac{1}{2} \langle \hat{\phi}_1(x, t) \hat{\phi}_1(x', t') + h.c \rangle \\ &= \int_{-\infty}^{\infty} \frac{dk}{4\pi} \int_0^{\infty} d\omega |\beta(k, \omega, -i\omega_k)|^2 \frac{1}{\omega_k} \coth(\frac{\hbar\omega}{2K_B T}) \cos[k(x - x') - \omega_k(t - t')], \\ & \frac{1}{2} \langle \hat{\phi}_2(x, t) \hat{\phi}_2(x', t') + h.c \rangle \\ &= \int_{-\infty}^{\infty} \frac{dk}{4\pi} \int_0^{\infty} d\omega |\alpha(k, \omega, -i\omega_k)|^2 \frac{1}{\omega_k} \coth(\frac{\hbar\omega}{2K_B T}) \cos[k(x - x') - \omega_k(t - t')],\end{aligned}\quad (29)$$

and correlation function of two scalar massive fields is as follows:

$$\begin{aligned}& \frac{1}{2} \langle \hat{\phi}_1(x, t) \hat{\phi}_2(x', t') + h.c \rangle \\ &= \int_{-\infty}^{\infty} \frac{dk}{4\pi} \int_0^{\infty} d\omega \beta(k, \omega, -i\omega_k) \alpha(k, \omega, -i\omega_k) \\ & \times \frac{2}{\omega_k} \coth(\frac{\hbar\omega}{2K_B T}) \cos[k(x - x') - \omega_k(t - t')],\end{aligned}\quad (30)$$

5 Thermofield approach

In this section, we use thermofield formalism to thermalized the system because it provides us with much information we need.

In this formalism, it is necessary to introduce a tilde space as well as the complete system which is a combination of original and tilde systems. To construct the thermal propagator of the system, the doublet can be presented as follows:

$$\Phi_1 = \begin{pmatrix} \varphi_1 \\ \tilde{\varphi}_1 \end{pmatrix} \quad \Phi_2 = \begin{pmatrix} \varphi_2 \\ \tilde{\varphi}_2 \end{pmatrix} \quad (31)$$

The propagator for each subsystem is

$$iG_1(x-y) = \langle 0, \tilde{0} | T(\Phi_1(x,t)\Phi_1(y,t)) | 0, \tilde{0} \rangle \quad (32)$$

that G in the present case is 2×2 matrix and fourier transform of the $\Phi_1(k)$ is as follows

$$G_1(k) = \begin{pmatrix} \frac{|\beta|^2}{\omega_k} & 0 \\ 0 & \frac{|\beta|^2}{\omega_k} \end{pmatrix} \quad (33)$$

The thermal propagator in the formalism of the thermofield dynamics at temperature T is obtained

$$\begin{aligned} iG_{1,\lambda}(x-y) &= \langle 0, \lambda | T(\Phi_1\Phi_1) | 0, \lambda \rangle \\ iG_{1,\lambda}(k) &= \begin{pmatrix} \frac{|\beta|^2}{\omega_k} (2 \sinh^2 \theta_k(\lambda) + 1) & \frac{2|\beta|^2}{\omega_k} \sinh \theta_k(\lambda) \cosh \theta_k(\lambda) \\ -\frac{2|\beta|^2}{\omega_k} \sinh \theta_k(\lambda) \cosh \theta_k(\lambda) & \frac{|\beta|^2}{\omega_k} (2 \sinh^2 \theta_k(\lambda) + 1) \end{pmatrix} \end{aligned} \quad (34)$$

that $\cosh \theta_k(\lambda) = \frac{1}{\sqrt{1-e^{-\lambda\omega}}}$ and $\sinh \theta_k(\lambda) = \frac{e^{-\lambda\omega}}{\sqrt{1-e^{-\lambda\omega}}}$ and $\lambda = \frac{1}{K_B T}$. The finite temperature Green's function can be separated into a sum of two terms, the first corresponding to zero temperature and the second dependent on temperature. In the next step, we can write the propagator for the total combined system at zero temperature following:

$$G_{\text{comb}}(x-y) = \langle 0_1, \tilde{0}_1, 0_2, \tilde{0}_2 | T(\Phi\Phi) | 0_1, \tilde{0}_1, 0_2, \tilde{0}_2 \rangle \quad \Phi = \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} \quad (35)$$

That G_{comb} in the present case is 4×4 matrix as follows:

$$G_{\text{comb}}(\mathbf{k}) = \begin{pmatrix} \frac{|\beta|^2}{\omega_k} & 0 & \frac{\beta\alpha}{\omega_k} & 0 \\ 0 & \frac{|\beta|^2}{\omega_k} & 0 & \frac{\beta\alpha}{\omega_k} \\ \frac{\beta\alpha}{\omega_k} & 0 & \frac{|\alpha|^2}{\omega_k} & 0 \\ 0 & \frac{\beta\alpha}{\omega_k} & 0 & \frac{|\alpha|^2}{\omega_k} \end{pmatrix} \quad (36)$$

So the thermal propagator is:

$$G_{\text{comb},\lambda}(\mathbf{k}) = \begin{pmatrix} \frac{|\beta|^2}{\omega_k} & 0 & \frac{\alpha\beta}{\omega_k} & 0 \\ 0 & \frac{|\beta|^2}{\omega_k} & 0 & \frac{\alpha\beta}{\omega_k} \\ \frac{\alpha\beta}{\omega_k} & 0 & \frac{|\alpha|^2}{\omega_k} & 0 \\ 0 & \frac{\alpha\beta}{\omega_k} & 0 & \frac{|\alpha|^2}{\omega_k} \end{pmatrix} + 2 \begin{pmatrix} \frac{|\beta|^2}{\omega_k} \sinh^2 \theta(\lambda) & \frac{|\beta|^2}{\omega_k} \sinh 2\theta(\lambda) & \frac{\alpha\beta}{\omega_k} \sinh^2 \theta(\lambda) & \frac{\alpha\beta}{\omega_k} \sinh 2\theta(\lambda) \\ \frac{|\beta|^2}{\omega_k} \sinh 2\theta(\lambda) & \frac{|\beta|^2}{\omega_k} \sinh^2 \theta(\lambda) & \frac{\alpha\beta}{\omega_k} \sinh 2\theta(\lambda) & \frac{\alpha\beta}{\omega_k} \sinh^2 \theta(\lambda) \\ \frac{\alpha\beta}{\omega_k} \sinh^2 \theta(\lambda) & \frac{\alpha\beta}{\omega_k} \sinh 2\theta(\lambda) & \frac{|\alpha|^2}{\omega_k} \sinh^2 \theta(\lambda) & \frac{|\alpha|^2}{\omega_k} \sinh 2\theta(\lambda) \\ \frac{\alpha\beta}{\omega_k} \sinh 2\theta(\lambda) & \frac{\alpha\beta}{\omega_k} \sinh^2 \theta(\lambda) & \frac{|\alpha|^2}{\omega_k} \sinh 2\theta(\lambda) & \frac{|\alpha|^2}{\omega_k} \sinh^2 \theta(\lambda) \end{pmatrix} \quad (37)$$

In the same way, the finite temperature Greens function of the total system can be separated into a sum of two terms, one corresponding to zero temperature and the other dependent on temperature. Off diagonal matrix elements show the correlation functions of two fields. Although two fields do not have direct interaction with each other, their correlation function is not zero. This point can be utilized in describing quantum entanglement of the two subsystem.

6 Conclusion

The quantum theory of two massive scalar field in the framework of canonical quantization was investigated. Two quantum dissipative equation were obtained for massive scalar fields and the explicit form of the quantum scalar fields in long-time limit were found. Knowing the explicit form of the quantum fields, the correlation function of the fields were calculated at finite temperature. In the last step, using the thermofield dynamics method, the propagators of the system were found. Despite the fact that two fields do not have direct interaction with each other, their correlation function was not observed as zero. This point can be utilized in describing the quantum entanglement of the two subsystem.

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